



Preliminary Study of Creative Problem Solving on Open-ended Mathematical Problems

Lim Keng Keh, Univeristi Teknologi Malaysia, Malaysia.

Zaleha Ismail, Universiti Teknologi Malaysia, Malaysia.

Abstract

This paper aims to find out how undergraduate engineering students solve open-ended mathematical problems. More specifically the paper provides an insight into the effects of using Creative Problem Solving (CPS) when solving open-ended mathematical problem. This preliminary study was carried out in a local Malaysian university. Students (n= 25) had to solve open-ended mathematical problem collaboratively with the use of pen, paper and calculators. Intervention through Creative Problem Solving strategies was used as a guide to help the students to solve such problems. After they had solved the problems they were interviewed individually. The results showed that engineering students use different ways to solve open-ended mathematical problems.

Keywords: Creative Problem Solving, Open-ended Mathematical Problem



INTRODUCTION

This paper is concerned with undergraduate engineering students and the strategies they use when solving open-ended mathematical problems. This paper essentially documents a small scale ‘pilot study’ into engineering students’ use of a technique for solving open-ended mathematical problems, known as ‘Creative Problem Solving’ (CPS). To begin this paper I provide an introduction to CPS.

Creative Problem Solving (CPS) is a learning strategy designed to promote problem solving ability and was pioneered by Osborn (1953), a ‘creative theorist’. He proposed seven stages of creativity: problem orientation; preparation; analysis; hypothesis; incubation; synthesis and verification. His work was later expanded and organized into a five-step process model by Parnes (1967). Parnes’ five-step process model comprised: fact-finding, problem-finding, idea-finding, and acceptance-finding. In 1985, Isaksen and Parnes further innovated on Parnes’ (1967) process model by adding a new stage known as ‘mess-finding’. They also changed ‘fact-finding’ to ‘data-finding’ having realized that effective problem-solving did not only depend on facts. They also explored the premise of divergent and convergent thinking.

The literature indicates that there are certain benefits of using CPS: most notably it appears to encourage:

- (a) a dynamic balance between critical and creative thinking;
- (b) the stimulation of effective teamwork and cooperation;
- (c) the focusing of attentions in the required direction;
- (d) the framing of the problems by building motivation and commitment to solving it.

(Isaksen, Dorval & Treffinger, 2011).

Further, CPS appears to enable the problem solver to ‘stretch’ their imagination and thus stimulate the creation of a number of ‘unusual’ creative ideas to solve a problem. In effect it helps turn creative ideas into workable and usable solutions (Isaksen, Dorval & Treffinger, 2011). Isaksen, Dorval & Treffinger (2011) argue that CPS can help one to understand their problem solving goals and thus avoid being distracted by many ‘other’ opportunities and



challenges in new or unknown situations. In simple terms CPS provides a flexible framework, organizing tools and strategies to solve problems (Isaksen, Dorval & Treffinger, 2011).

Having made these preliminary comments with respect to CPS, I now turn to the application of CPS in the context of engineering students ($n= 25$) in a local Malaysian University. In order to achieve such a goal I begin with an outlining of methodology before revealing data and making conclusions.

METHODOLOGY

Qualitative research methods were used in the study as it enabled the researcher to view what students were doing, given the practical nature of the study and its mathematical problem solving context. Further, as the sample size was relatively small ($n=25$ students) a more detailed quantitative study with statistical calculations would not have proved viable. Three data collection methods were used to gain a depth of data based on how engineering students carrying out processes associated with Creative Problem Solving. These methods included an observation, semi-structured interviews and a review of documents used by students during such tasks. These three different yet inter-related methods enabled the researcher to triangulate data collected and to increase its validity. The collection of data occurred during and as a result of a single open ended mathematical problem solving session. Students were provided with an open ended mathematical problem and asked to solve them. The researcher then observed students in action and collected or noted any documents generated by students, to supplement the observations and then followed it all up with a series of one-on-one semi-structured interviews with students.

Video-recording was used during observations as it enabled the capture of 'all' the activities of the students as they carried out the process of problem solving and which might be missed by the researcher in a single live viewing. What students worked out ---or wrote--- on a whiteboard was also recorded in the observation. Further, a particular focus of the observations was the videoing of the 'step-by-step procedures' that students used to solve a given problem. Audio-taping was used during the semi-structured interview with students. After the students had completed standard problem solving task, they were interviewed individually. The interview schedule was framed by the core elements of CPS (as outlined previously) and contextualized to what each student did during observation. Oral responses were later transcribed. The students



were told before interview that they could feel free to express their opinions of the experience and importantly to vocalize how they felt and what they thought of during the solving of the mathematical problem. Students' sketches and graphs were collected as documents. These documents provided evidence of their results and the procedures applied to gain such results. Further, the computer program used in their problem solving, as attached in Appendix G , was kept as document.

There were three processes of qualitative data analysis, namely : 'collecting' , 'noticing', and 'thinking' (Lewins, Taylor & Gibbs, 2005). After the task was completed by students and interviews completed, all the data was 'collected' together for processing. First the various data sources were sorted and arranged so as to create a logical picture of what had occurred in each student. Then, 'noticing' was used to identify a series of recurring data themes and aligned categories of responses. After 'noticing' the data, these initial themes created the stimulus to further sort the data into sub themes and categories that informed the overall study aim: to gain an insight into engineering student's use of CPS while solving open ended mathematical problems. The last step is 'thinking' and this is used primarily to make sense of the sub themes and to generalize and develop same into findings.

RESULT AND DISCUSSION

There were ten sub-themes found from data analysis, namely:

- (1) developing the solution,
- (2) generation of ideas to solve the problem,
- (3) selection of best solution to solve the problem,
- (4) transformation of ideas into practical solution,
- (5) evaluation of solution,
- (6) use of strategies to solve open-ended mathematical problems,
- (7) use of creativity to solve open-ended mathematical problems,
- (8) use of convergent thinking to solve open-ended mathematical problems,
- (9) use of divergent thinking to solve open-ended mathematical problems and
- (10) belief in solving open-ended mathematical problems.



I discuss each in turn.

1. Developing of solution

All students first read the problem and then sought to understand it before they identified what they thought were all the ‘unknowns’ --such as fact, data or information--- in order to develop their solutions.

“Okay, first read the question and understand the question roughly and then, okay we need to identify all the information given, the unknown, how many equation can form.” (Student Yap from interview)

Then, they tended to list all the facts, data and information needed to find out any restrictions. They reviewed what the question required or stipulated and then listed any further things they needed in order to solve the problem.

“Okay, first I will read the question first and then I will identify all the unknown, unknown variable, unknown variable inside the question that it will list out all of it and then I will look at the question and we will. I will develop some equations that it will lead to the situation of the solution like....solution.” (Student See from interview).

Student Yap said: *“How about we list out all the restriction and limitation so that we can see the problem clearer”* (Student Yap, from observation).

Students indicated that they could get all the facts, data and information of the problem either from a library book or internet and used same to analyse the problem. They mentioned how they discussed the problem with their teammate and together reviewed the mathematical theory before they developed the equations to solve the problem.

“I, firstly, I discuss with my friend. Okay.... and after that if we after the discussion, we have any problem we go to internet to search for the data and get the other information.” (Student Gob from interview).



” Ah....I discuss with my friend or I go to the library, claim some books to review the theory and the data from the book” (Student Gob from interview).

2. Generation of ideas to solve the problem

Students engaged in discussions amongst themselves or in a group to generate ideas to solve the problem. They used brainstorming to solve the problem, accepting only what they together thought was a good idea. They wrote down their ideas on paper.

“How do I generate? Ah.... a. create ideas, I brainstorm.....brainstorming with my friends, then we gather all the ideas and review the ideas and theory we have discuss and then collect the good one and leave the bad one.” (Student Gob, from interview).

Student See suggested others to solve the problem by using programming. He said:
“How about you try in another method? We do this trial and error in another method?” He generated new ideas to solve the problem by using programming. (Student See, from observation).

3. Selection of best solution to solve the problem

Students devised a table and listed out all the possible methods to solve the problem in order to find out the best solution to solve the problem.

“Okay... just like mm....may be, we can devise a table, then list out all the possible method to solve the problem, then write down the pros and cons of using those methods, then vote for all the method and the highest mark ..”

Then, they listed out all the pros and cons of using identified methods and compared the different methods based on efficiency, accuracy, flexibility, simplicity, originality, convenient and time saving.

“Okay, first, we will do the question, different solution first, method first and then we will compare with each other. We discuss about it. So, we think that which one is the easiest one. Which one is the fastest



one and the most efficient one, then we will select the one which are, are..., the best one, the best solution. The fastest solution and the efficient solution.” (Student Seebai, from interview).

Then, they identified the method to solve the problem and selected the best method by voting.

“Okay...I think we will vote it. Voting, by voting...are..., So.....ar. We will leave out the method first and we will vote. We get the most number of voting that is the decision we make.” (Student Seebai, from interview).

4. Transformation of ideas into practical solution

Students implemented the solution by listing out all the solution so that they can transform their ideas into practical solution. They solved the problem by drawing a graph, calculating on paper or running a computer programme. Importantly for this study they indicated that they used their imagination to project the problem as the real world problem.

“Okay, we will come up with paper first and then list out all the, list out all the solution and then if the question needed, ar..... to draw a graph., we draw a graph. And then calculating, we work it on the paper, ar..., then we get the answer.” (Student See, from interview).

“Transfer,....m..... for example, we select like the programming, then we .. you know, .with the equation we get, actually involve both the simultaneous and the trial and error if we use the programming method, then we try to design our code (Student Yap, from interview).

“I transform the idea like a.....like I don't do, I don't like solve the problem basically by using the theory that I have learnt. Sometime, I will just likeah..... like imagine that problem into the real, the real world problem and then I solve it by, solve it by my idea.” (Student See, from interview).

5. Evaluation of solution

The students evaluated their answer by substituting it into a mathematical equation to make sure the answer was valid.



"I basically check my result based on the , based on the problem and sometime I will check the answer whether it is valid or not. If the question asks for real number, then my answer is not real number, then my result is wrong. I can know my result is wrong." (Student See, from interview).

"We actually sub. in the..er.....a.....the answer we got.to redo it so that the right hand side is equal to the left hand side which means. ..a.... equation and the answer is valid itself". (Student Lee, from interview).

The students worked backward in order to check whether the answer made sense, whether the answer meet the limitations and restrictions. They also checked one by one for any error in the equation.

"I substitute it all the answer inside the equations. So, if the, if the answer in the left hand side of the equation and the answer in the right hand side of the equation is the same one, then it is correct. And then we will check the sign, positive, negative and if got right, wrong or not." (Student Seebai, from interview).

"Counting from the back, backward method like .. ,working backward to get the answer." (Student Lee, from interview).

6. Use of strategies to solve open-ended mathematical problems

The students showed that they solved the problem by using inspection, discussion, calculating and programming with the tools of computer, calculator, pencil and paper.

"Okay, there are a lot of strategies, so I will inspect the question first and then we see, so firstly, mostly, we need to use trial and error." (Student Seebai, from interview).

"Well, we use a.. three strategies like trial and error, simultaneous equation and programming with C plus plus." (Student Lee, from interview).

The students indicated that they had to 'expand' their mind to use creativity. In effect to think 'out of the box' and thus select several perhaps new and creative ways to solve the problem.



“We have to change our mind set, cannot think must think out of box , must be creative, be open-minded so that you can, you know devise more solution, different way , way to solve the question.”
(Student Yap, from interview).

7. Use of creativity to solve open-ended mathematical problems

They students indicated that they did use the element of creativity to solve open-ended mathematical problems, indicating they didn't 'fix their mind' on one solution: tending to explore various ways to solve the problem. This was exemplified by using computer technology and graphs. They also didn't appear to limit themselves on a specific mathematical theory to solve the problem. Therefore, they discussed with each other ideas and tried many ways to solve the same problem. They used a multi-disciplinary of study to help them in problem solving.

“M.. for example,.....a.... if it is a maths question, then instead of just using the theory in maths, ...we use.. ah.....you know technology in problem solving, for example in these days programming, so this is considered a multi-disciplinary problem solving method, so this considered quite innovative and creative method.” *(Student Yap, from interview).*

“I will, I don't I won't fix my mind on that problem, the problem with only one fixed solution.”
(Student See, from interview).

Student See saw another perspective when he solved the problem. He told us:

“The question seems easy but when we try to solve it not that easy .because the possible outcome and the possible way is. So, I think I learn to see another question, another kind of question is not from the textbook.... is outside the text book and open, open box. Ya! We need to think out of the box. “
(Student See, from observation).

“.....Does not need to stick to one solution and use our creativity.” *(Student Gob, from observation).*

Student Chai also said open-ended mathematical problem don't fix on one solution and we have to use creativity to solve it. He said:



“Because I think this kind of question is not traditional and teacher... the traditional...because what teacher taught us is.... you do this and formula to get this answer.....so, this kind of question doesn't, doesn't fit one or more solution, ways to fix it, to get the answer., just like what he says.....so, it can make us become more creativity.” (Student Chai, from observation).

8. Use of convergent thinking to solve open-ended mathematical problems

The students indicated that they used convergent thinking as they tried to use the same method from the beginning until the end.

“So, ar.... I will focus on one method first. So, we will use this method from beginning until the end. Like, first inspect the question, identify all the variables, build up the equation, solve it, do it, make it, come out with the answer.” (Student See, from interview).

They figured out the answer to make sure that it is logic, make sense and didn't make any mistake.

“We just focus on one way, one way to solve the problem we didn't think about other possibility, then we just focus on one way and then make sure the way is make sense, logic and didn't make, we don't have any mistake.” (Student Gob, from interview).

9. Use of divergent thinking to solve open-ended mathematical problems

Students indicated that they used divergent thinking and didn't restrict themselves on one solution. They found out more than one answer for the problem by using different strategies.

“Here involves lots of strategies, may be, then try to figure out as much as possible .. er.....method of solving this kind of question, then try to think out of the box.” (Student Yap, from interview)

“After, we done the convergent thinking in solving mathematical problem. then, we try to figi..figure it out whether is there any other alternative way to solve the mathematical problem. I think that is considered as divergent thinking.” (Student Gob, from interview).



They listed out many methods to solve the problem.

“An open-ended mathematical problem there are not only got one way to do it, there are many methods. So we will list all the methods first and then will do it one by one.” (Student See, from interview).

10. Belief in solving open-ended mathematical problems

Students believed that there was more than one way to solve open-ended mathematical problems. They demonstrated that they can and did use different methods to solve a problem: so long as their solution was logic and made sense to them. It appears that the methods employed could be long, short, hard or easy and therefore that there are many alternative ways to solve an open-ended mathematical problem. They indicated that open-ended mathematical problems were related to daily life and were based on a level of difficulty.

“Actually mathematical problem is very related with our daily life..., just like when we want you to ... when you want to go to one destination, you can go by walking it, we can take bus, you can take taxi. There is not only restrict one way to do it, so just the mathematics, a question we can use different methods to do it. The question only faster or slower, the complex one or the simple one, so depending on how you do it, how you make it.” (Student See, from interview).

“Because mm.. open-ended question does not necessary just contain one answer, just like what we learn in our class the conventional, traditional mathematics problem, so that should be more than one way and more than one answer.” (Student Yap, from interview).

Student Yap regarded open-ended mathematical problems as interesting. He said: “This question is interesting, I shocked because what... the mathematical question we solve ...is like a ...one solution. May be one answer. But this problem is like a.... .. lots of possible answers. So,... is a very good question for us. (Student Yap, from observation).

CONCLUSION AND REMARKS

The results of this study, whilst narrow and limited to one problem and one engineering student cohort, showed that engineering students, when exposed to CPS as a teaching strategy for



solving open ended mathematical problems, tended to use different ways to solve open-ended mathematical problems. This marries with Hancock (1995), who argues that there are multiple approaches to solving the open-ended mathematical problems, with little constraints on their solving. The use of open ended problems had another effect in that they tended to help students to learn how to use different strategies to solve open-ended mathematical problems (Klavir and Hershkovitz, 2008). Students demonstrated that they can and did use creativity to solve open-ended mathematical problems. They did this by exploring various ways to solve the problem and was exemplified by the use of computer technologies and devices such as graphs. Studies by Felder (1987) suggest that people have to 'explore themselves' when seeking to be creative in solving open-ended problems: they ultimately learn various ways to gather and arrange the required information and a host of strategies to solve the open ended mathematical problem. Students indicated that they learnt that open-ended mathematical problems can generate more than one solution and thus increase their creativity when persuing such solutions in modes as outlined in CPS. Shimada (1997) says that mathematical problems with only one correct answer are very hard to develop creativity and thus are antithesis of a required CPS environment (Dreyfus & Eisenberg, 1966; Ginsburg, 1996).

ACKNOWLEDGMENT

The authors would like to thank the Universiti Teknologi Malaysia (UTM) and Ministry of Higher Education (MOHE) Malaysia for their financial support in making this project possible. This work was supported by the [Fundamental Research Grant Scheme](#) [4F187-FRGS] initiated by MOHE.

REFERENCES

Dreyfus, T., & Eisenberg, T. (1996). On different facets of mathematical thinking. In R. J. Sternberg & T. Ben-Zeev (Eds.), *The nature of mathematical thinking* (pp. 253–284). Mahwah, NJ:Lawrence Erlbaum.



- Felder, R. M. (1987). On creating creative engineers. *Engineering education*, 77(4), 222-227.
- Ginsburg, H. P. (1996). Toby's math. In R. J. Sternberg & T. Ben-Zeev (Eds.), *The nature of mathematical thinking* (pp. 175–282). Mahwah, NJ: Lawrence Erlbaum.
- Hancock, C. L. (1995). Enhancing mathematics learning with open-ended questions. *The Mathematics Teacher*, 88(6), 496.
- Isaksen, S. G., Dorval, K. B., & Treffinger, D. J. (2011). *Creative approaches to problem solving: A framework for change*. Kendall Hunt Publishing Company.
- Klavir, R., & Hershkovitz, S. (2008). Teaching and evaluating 'open-ended' problems. *International Journal for Mathematics Teaching and Learning*, 20(5), 23.
- Lewins, A., Taylor, C., & Gibbs, G. R. (2005). What is qualitative data analysis (QDA)? *Online QDA*. Online: onlineqda.bud.ac.uk/Intro_QDA/what_is_qda.php. {Accessed 19 July 2008}.
- Osborn, A. F. (1953). *Applied imagination, principles and procedures of creative thinking*.
- Parnes, S.J. (1967). *Creative Behavior Guidebook*. New York. Scribner.
- Shimada, S. (1997). The significance of an open-ended approach. In J. P. Becker & S. Shimada (Eds.), *The open-ended approach: A new proposal for teaching mathematics* (pp. 1–9). Reston, VA: National Council of Teachers of Mathematics.