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Crystallographic Pattern Analysis of the Loom Woven Clothes of Abra

Reymond S. Vasquez[a], Novelyn B. Valera[b], Jonathan P. Zales[c], [a,b,c]Abra State Institute of Sciences and Technology, Abra, Philippines, Email: [a]reymondvasquez990@yahoo.com, [b]zv032405@gmail.com, [c]jonathanpurisimazales1205@gmail.com

This paper presents the symmetry analysis, geometry concept and crystallographic patterns present in the loom-woven clothes of Abra, particularly the binakol kuskusikos, sungkat, laylayon, piningitan and baag. It aimed to associate these concepts in the teaching of mathematics. Also, mathematics teachers can use these patterns as a real-world application of the ideas in it to arouse students' interests and for them to find it as an enjoyable and meaningful subject. Observation was utilised to determine the geometric ideas and symmetry analyses present in the woven clothes. Gathering and capturing photos of the finished products were also utilised to determine the crystallographic patterns embedded in the woven cloths. These crystallographic patterns are analysed through the International Union of Crystallography. The woven cloths exhibit geometric concepts such as points, lines and planes. Further, they display symmetry analysis such as reflection, rotation and translation, and uncover the crystallographic patterns, frieze and wallpaper patterns. The woven cloths of Abra showed similar classification under the frieze revealed and wallpaper patterns. The most prevailing pattern in the frieze group was pmm2. Other frieze patterns were pma2, pm11, p1m1, p1a1, p112, and p111. Likewise, pg, pm, cm, pgg, p2, pmg, cmm, pmm, p4, p4g, and p4m were unveiled to be wallpaper patterns with p2 being the dominant pattern.

Keywords: *Crystallography, Geometry, Symmetry Analysis, Loom Woven Clothes, Abra, Philippines.*

Introduction

The world is constantly changing and developing. Beauty is seen for now but soon fades as the years go by. Some inventions are effective today but will soon be developed into more advanced technology. Some knowledge and philosophies will remain and accepted in this generation, but for the next generation, some people will contradict these philosophies and will propose another philosophy. The only thing to cope with this kind of world is through education. Education is the cure of ignorance and the bridge between research and inventions. It is an important medium to acquire knowledge and skills. It is through reading books and other materials for learning to know all the concepts. But the curriculum is constantly changing and developing also. Now, it focuses more on the application of these concepts to enhance students' ability to apply their knowledge by creating something useful for society.

In this matter, mathematics is the most powerful tool to enhance students' ability to apply concepts in the real world. These mathematical concepts have something to do with one of its branches which its concepts are usually applied everywhere. The shape of the mountain, roads houses, fields, and rivers all have something to do with geometry. Geometry is a branch of mathematics that deals with the measurement and relationship of points, lines, angles, surfaces, and solids (Merriam Webster, 2018). It is concerned with the shape of individual objects, spatial relationships among various objects, and the properties of surrounding space.

On the other hand, the repetitive beautiful pattern in our surroundings will explain the concept of crystallography. Wallpaper is an example of a real-world item that is seen every day in most homes across the nation. Some people will look at wallpaper as an incredible pattern because of rotation, reflection, translation, and glide reflection that are present on the wallpaper. And crystallography is a branch of geometry that uses symmetry analyses, rotation, reflection, glide reflection, and translation. These symmetry analyses create a pattern that helps us organise our world conceptually. The repetition of a basic unit of a pattern in a given distance in one direction is called the translation. The term that describes a basic unit of a given point of the lattice in the plane being rotated, is rotation (Sani, et al., 1988 cited by Liwas, et al., 2016). The reflection of a basic unit about a line in a plane is termed as a mirror reflection. Lastly, glide reflection is a combination of a translation and a mirror reflection (Washburn, 1988 cited by Liwas, et al., 2016). Symmetry analysis has two divisions, the wallpaper and the frieze pattern. One-dimensional patterns that are repeated along a line in one direction are called the frieze pattern that makes use of rotations, reflections, and translations. Wallpaper patterns, on the other hand, are two-dimensional patterns that are repeated in two directions (Washburn, 1988 cited by Liwas, et al., 2016).

In education, there are many reasons for the students' lack of interest and success in mathematics. These reasons include minimal visual skills, difficulty in understanding basic concepts, hardship in relating these concepts with another one, and students' lack of exposure to life outside of their neighbourhood. To know more about these issues, Parnell (2018) added that in the classroom, teaching is a matter of putting students in the classroom and attempting to fill their heads with facts through lectures and textbooks, and there is a little attempt of connecting what students are learning with the world inwhich they will be expected to work and spend their lives.

However, the emphasis of education today is the application of these concepts in the real world. One way of gaining the interest of the students is through experiencing the application of these mathematical concepts. Furthermore, as added by the National Academy of Sciences, Engineering and Medicine; students need to experience mathematical ideas in the context in which they naturally arise – from simple counting and measurements to applications in business and science. As mentioned also, learning mathematics through applications can lead to exceptional achievement. Also, today's educators are expected to nurture in-depth understanding and a passion of innovation and will seek to maximise learning experiences so the students can enhance the skills needed to pilot a rapidly evolving world (Digital Newspaper Replicas, 2018)

In this matter, one way of preparing the students to live and connect their mathematical knowledge to the real world is through weaving. Through weaving, students will be able to apply their creative skills in making their designs and patterns. Weaving involves the production of fabric or cloth by interlacing two distinct sets of yarns or threads at a right angle (The history of weaving part 1- Asia 2014). This is a perfect workpiece to apply symmetry analyses, geometric concepts, and crystallographic patterns that will boost the confidence of the students to engage in a skill-developing activity. Through this exposure to weaving, students will not just be more equipped and knowledgeable on the application of these mathematical concepts in the real world, but able to transfer knowledge to other mathematical areas. Weaving is an activity that will let both the teacher and the student bridge the gap between these mathematical concepts and the real world. It will not only bridge the gap but will go a step further to bind the knowledge, exposure, and experience to develop a skill that can be used as one of the strongest tools to create something useful for society.

This study was undertaken to apply mathematical concepts specifically the geometric concept, symmetry analyses, and crystallographic patterns into the real world.

Theoretical/Conceptual Framework

Weaving is the intertwining of the vertical and horizontal yarn to create a textile. Upright loom weaving is the technique used by the weavers of Bulbulala, La Paz and Lumubang, San Juan, Abra. This kind of weaving makes used of a loom machine to weave textiles. As mentioned by

Liwas, Garcia, & Trinidad, (2016) in their article "Crystallographic Patterns in the Weavings of Sagada and Bontoc", the thread is first warped arranging it vertically in the warping frame before it is put in the machine. Gan-ay is termed as the method of warping. The threads are then further arranged by inserting a certain number of threads (depending on the pattern) in a comb-like structure called the heedless. To produce the symbols and designs of weaving, several heedles are interchanged. The reed compacts the threads horizontally to interlock the threads so that the patterns and symbols remain intact. The reed also helps in the precision of the weaving as it acts like a comb that maintains the number of threads. The shuttle serves the purpose of the sikwan (from back-strap weaving). They use the karilyasan to put the threads in the shuttle. The machine can produce 4 metres of weaving with a width of .54 inches each day. After the weavings are finished, they are sown to create products such as bags.

Since weaving is such a popular tradition in the country, the best way to preserve this culture is through its association with the mathematical concepts that are learned by the students in school. To prove that this is one of the well-known traditions of the country, Llames (2013) stated that Kalinga and Gaddang are famous with their gilamat and ikkat weaving. Gilamat is a plain albeit old type that is twilled and decorated with silk embroidery rather than cotton, an indication of class delineation. While ikkat weaving is a technique wherein before the threads are woven, it first uses a resist dyeing process similar to tie-dye on either the warp or weft to make a pattern or design.

Furthermore, Rapanut, Nathaniel & Baylas IV (2012) added that the northern kankana-ey is known for its wakes, bedbed, and tapis. Their designs and patterns are very famous because it reflects their culture. The feast or ritual wear is the bagket while women use the wakes for daily wear. A cloth made up of abel (cloth) or kuba (bark) is one that Kankana-ey men wear to cover their short hair and they decorate this with feathers, leaves, carabao horns is called bedbed.

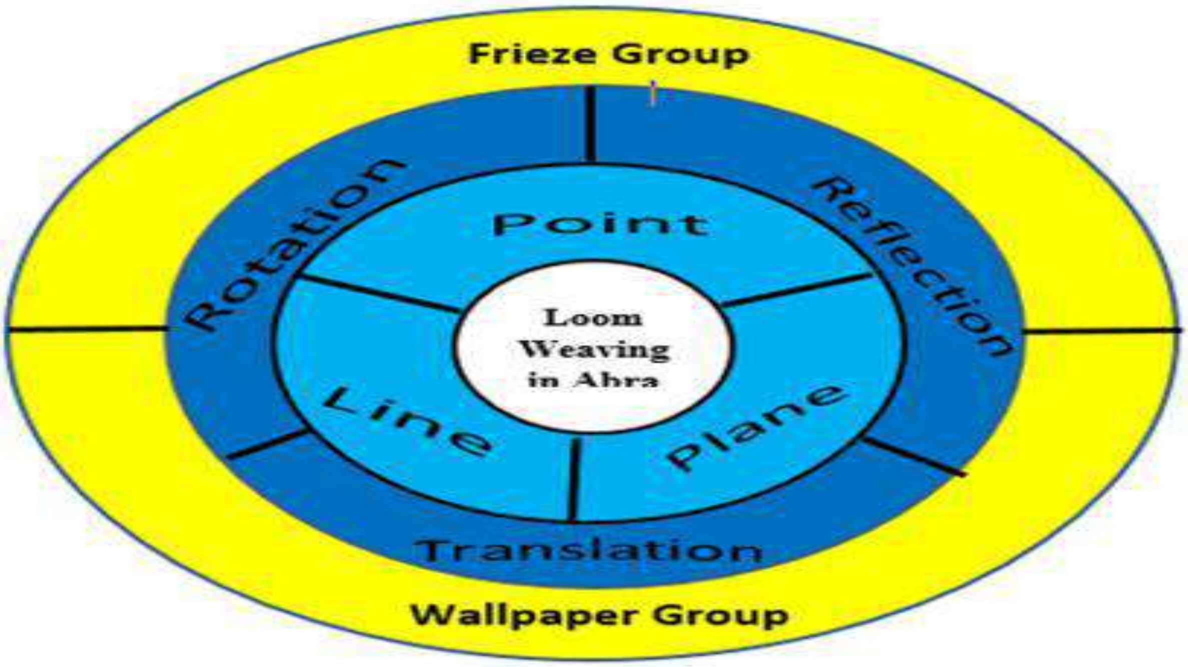
Moreover, the binakol weaving of Abra is one of the trademarks of the province. The finish products of the abel are made as bags, dresses, wallets, and others. Some are used for weddings and festivities.

On the other hand, Sani, et al, (2012) define crystallography as a branch of mathematics that studies geometric descriptions and internal arrangements of crystals. He stated that it is divided into two symmetry patterns, known as the wallpaper and the frieze pattern. A frieze pattern is one-dimensional and goes in one direction. While a wallpaper pattern is two-dimensional and goes in two directions. These patterns use the four isometries; rotation, mirror reflection, translation, and glide reflection. The existence of these patterns creates unique creative patterns and designs of woven cloth in weaving.

Rapanut, Baylas IV, & De Las Penas, (2012) discussed in their *Weaving Symmetry of Northern KanKana-ey* that crystallographic patterns are present in the *ules or tapiz, wakes,* and *bedbed* of the northern Kankana-ey. Furthermore, De las Penas, et. al (2014) mentioned in their "Color Symmetry in the Hand Woven of the Jama Mapun" that their mats consist of repeating patterns of five colours that utilise symmetry groups under wallpaper patterns. Moreover, crystallographic patterns are present in the funeral textiles and garments of northern Luzon, particularly the *lamma* and *tiktiko* where the patterns are in the wallpaper and frieze group, as mentioned by De las Penas & Salvador (2016) in their article "*Mathematical and Anthropological Analysis of Northern Luzon Funeral Textile"*

Conceptual Framework

Figure 1. The paradigm of the study



The innermost circle contains the loom weaving in Abra, particularly in La Paz and San Juan. The researchers believe that the patterns and designs of abel of each place are different. The abel weaving of Bulbulala is innovated from Iloko while the abel weaving of Lumubang was influenced by Cordillerans.

The next circle contains primary geometric concepts, points, lines (parallel, perpendicular, curved) and planes (shapes and polygon). These primary geometric concepts are usually seen in the surroundings. Hence, combination and arrangements of these concepts are applied in loom weaving through the creation of abel designs and patterns.

The third circle shows the symmetry analysis (rotation, translation, mirror reflection and glide reflection). Symmetry is always present when patterns are spoken about. Hence, its four motions will make each abel pattern creative, attractive, and meaningful.

The outermost circle presents the crystallographic patterns (frieze and wallpaper patterns). The researchers believe that these two classifications of the pattern are present in the abel woven cloth of Bulbulala, La Paz and Lumubang, San Juan, Abra

Objectives

The study aimed to apply mathematical concepts in a real-world scenario like weaving and aimed to let the students engage and be exposed to the real-life application of mathematics.

Specifically, it sought answers to the following questions:
1. What are the geometric concepts found in the loom woven clothes of Bulbulala, La Paz, Abra and Lumubang, San Juan, Abra?
2. What symmetric analysis is embedded in the loom-woven cloth of Bulbulala, La Paz, Abra and Lumubang, San Juan, Abra?
3. What crystallographic patterns are included in the loom-woven cloth of Bulbulala, La Paz, Abra and Lumubang, San Juan, Abra?

Expected Output

The results of this investigation are of great help to the learners to increase their engagement to the learning process as they gain valuable insights into real-life applications of the skills they are being taught. This study will provide learners with a vision of what is possible, providing a meaningful framework for both academic and personal development. Besides, it will catch the learner's interest in the application of these mathematical concepts and actively seek to satisfy their curiosities. And through this curiosity, they might be able to create something useful for the greater good.

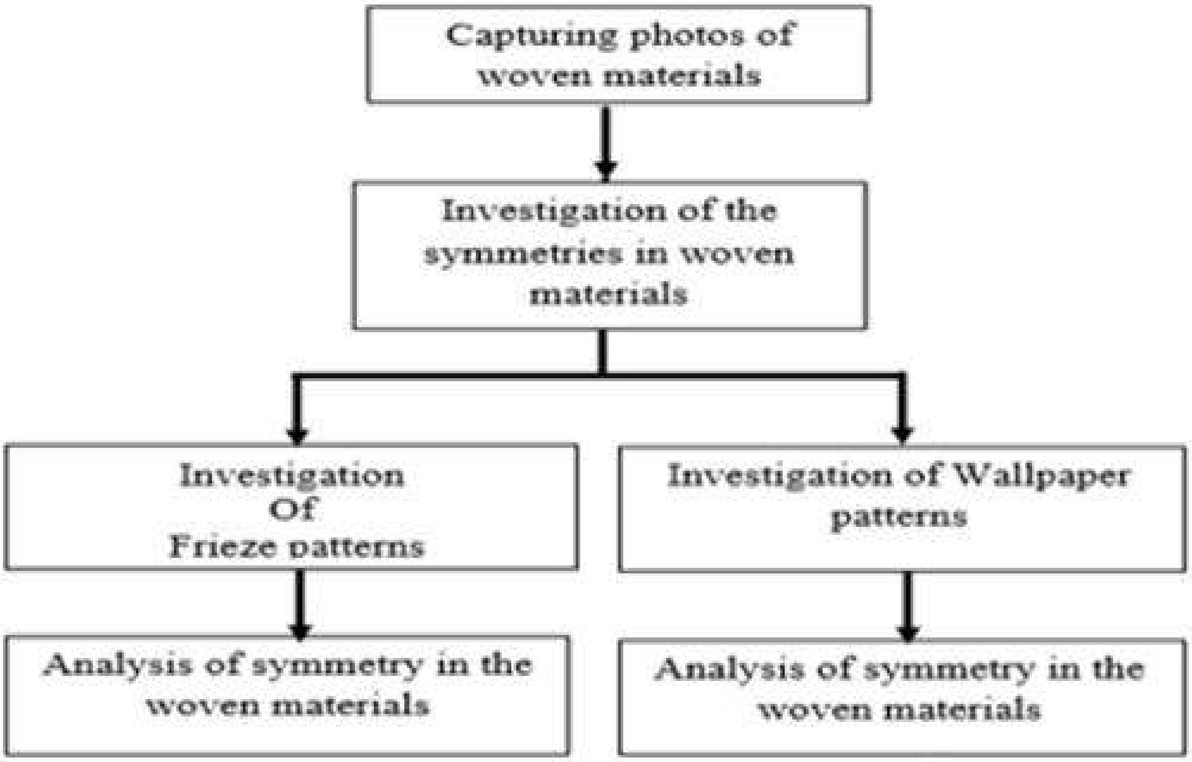
Moreover, the results of this study are of great help to mathematics educators as they will serve as a springboard to create a bridge between mathematical concepts and the real world. By these, educators may have a key to open a door of new investigations. To future researchers, the results of this investigation would be beneficial for them because this will serve as their springboard on how they will also formulate their research.

Methodology

Research Design

Mixed methods of research are utilised, specifically the ex post facto analysis design and ethno-mathematics. Ethno-mathematics is used to seek answers on the different geometric concepts. It is used also to discover symmetry analyses. While, ex post facto analysis is the analysis and manipulation of presented data. It is used to expose the different crystallographic patterns present in the woven cloth of Bulbulala, La Paz and Lumubang, San Juan Abra.

Figure 2. Data gathering procedure of the study



Population and Sample

The population of this study was the four loom weaving centres of Bulbulala, La Paz and Lumubang, San Juan, Abra.

Instrument

The study used the analysation of the documented woven materials through IUC (International Union of Crystallography) notation. Observation notes were utilised and photos of the woven cloth were taken to strengthen the study.

Data Analysis

The analysation of the crystallographic pattern in the woven cloth of Bulbulala, La Paz, Lumubang, San Juan Abra utilises the different crystallographic codes for frieze and wallpaper patterns that were presented by Washburn (1988).

These crystallographic codes are as follow:

Table 1: Crystallographic codes for Frieze Group

FRIEZE GROUPS	**DESCRIPTION**
pmm2	Vertical and horizontal reflections are present in the pattern
pma2	Vertical reflections and 180 – degree rotations are present in the pattern
pm11	Vertical reflections are present in the pattern
p1m1	Horizontal reflections are present in the pattern
p1a1	Glide reflections are present in the pattern
p112	180-degree rotations are present in the pattern
p111	There are no reflections or rotations in the pattern

Table 1 shows the different crystallographic frieze patterns with their corresponding descriptions as cited by Liwas, J.K., et al 2016 in their article and originally introduced by Washburn (1988). These are one-dimensional patterns present in the woven cloth which use rotation, reflection, glide reflection, and translation, hence, result in seven crystallographic codes: *pmm2, pma2, pm11, p1m1, p1a1, p112,* and *p111.*

Table 2: Crystallographic code for Wallpaper group

Wallpaper Groups	Description
p1	There are no reflections or rotations in the pattern
Pg	A glide reflection is present in the pattern
Cm	Reflections with glide reflections in an axis which is not on a reflection axis are present in the pattern
Pgg	180-degree rotation with glide reflection is present in the pattern
p2	180-degree rotation is present in the pattern
Pmg	A 180-degree of rotation and reflection are present in the pattern

Cmm	180-degree rotation and reflections in two directions are present in the pattern
Pmm	180-degree rotation and reflections which are in two directions and its rotation centres are on reflection axes are present in the pattern
p4	A 90-degree rotation is present in the pattern
p4g	A 90-degree rotation and reflection are present in the pattern
p4m	A 90-degree rotation and reflection which intersect at 450 are present in the pattern
p3	120-degree rotation is present in the pattern
p3m1	120-degree rotation and reflection are present in the pattern
p31m	120-degree rotation and reflection which have its rotation centers on the reflection axes are present in the pattern
p6	A 60-degree rotation is present in the pattern
p6m	A 60-degree rotation and reflection are present in the pattern

Table 2 presents the different crystallographic wallpaper patterns with their corresponding description to identify the pattern embedded in the woven cloth of Bulbulala, La Paz, Lumubang, San Juan Abra. Hence, it results in seventeen different crystallographic wallpaper codes:*p6m, p6, p31m, p3m1, p3, p4m, p4g, p4, pmm, cmm, pmg, p2, pgg, cm, pm, pg,* and *p1 (*Washburn, *2018)*

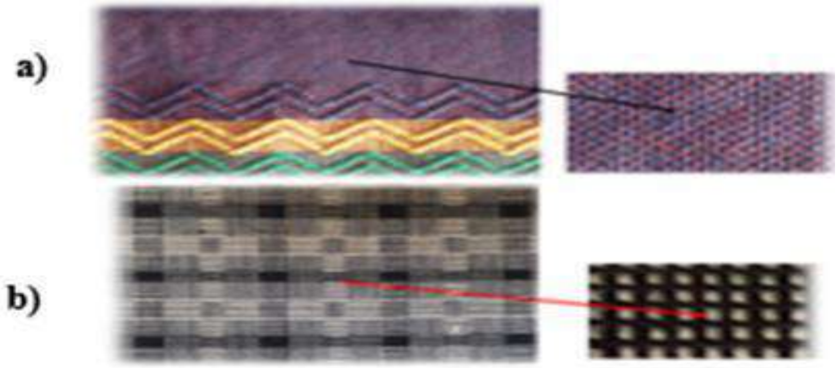
The International Union of Crystallography defines the symbols, as stated by Liwas, Garcia and Trinidad (2016) in their article "Crystallographic Patterns in the Weavings of Sagada and Bontoc, Mountain Province", where the first character in the international symbol is donated by the primitive or centred cell. The symbol *n integer* denotes the highest order of rotation that corresponds to the second character in the international symbol. The symbol, *m;g; or l* denotes the symmetry axis normal to the x-axis which corresponds to the third character in the international symbols. The *m* stands for the mirror, indicating a reflection on the axis. The *g* stands for glide reflection, indicating a glide reflection on the axis. Lastly, *l* indicates no reflection at all. The symbols *1;2;3;4; or 6* denote the fourth character. If the highest order of rotation is at 180^{o} then the symbol is 1 or 2. If it is at 45°, then the symbol is 4. Finally, when the symbol is either a 3 or 6, then the highest order of rotation is at 60°. If the third and fourth characters are not present on the full international symbol, then the group contains no reflection or glide reflection.

Results and Discussion

The following figures present the different geometric concepts present in the loom weaving of Abra.

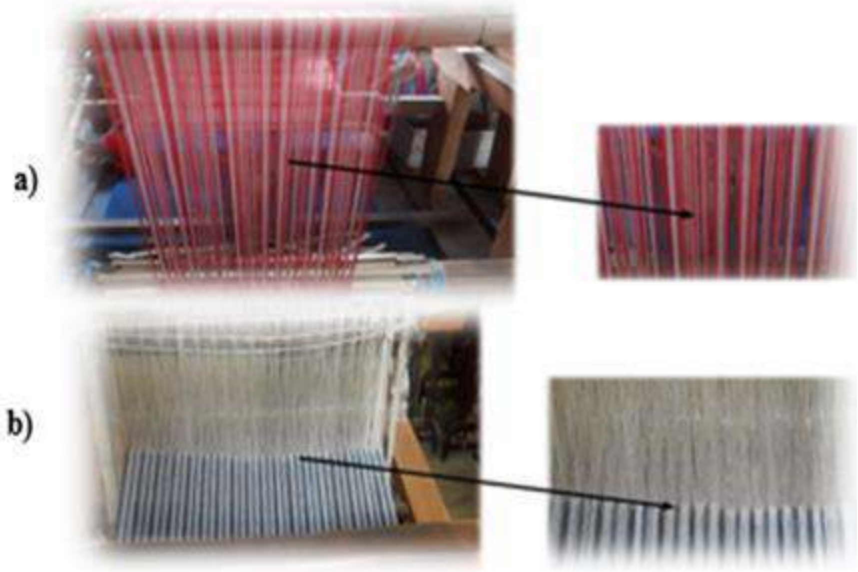
Point is one of the undefined terms in geometry. It has no length, width, or thickness. It is just a little dot but can create a beautiful pattern just like chaos. The points are seen in the woven cloth as you look at it closelt. Looking at the woven cloth is just like plain cloth but it was made up of a group of points. The points are described as red and violet (See figure 3a). Some were arranged vertically and horizontally that made a square pattern and draw the vertical and horizontal strip pattern of the cloth (See figure 3b). Point is not just a simple dot that they know in the classroom, but when applied to real-life, that small dot will transform into a beautiful and attractive design and pattern that will be attractive.

Figure 3. Plane Clothes



On the other side, lines are shown in the process of loom weaving. Before the thread is put into the machine it was first warp in the warping frame arranging the threads vertically forming parallel lines (See figure 4a). This method is termed as *gan-ay* and the thread is called *ur-ay*. The interlacing of thread using the comb-like structure (called heedless) formed perpendicular or intersecting lines (see figure 4b). The reed compacts the thread horizontally (forming parallel lines) to interlock the patterns and designs. The *karilyasan* is a wheel-shaped object used to put threads in the *tagingting* (a short cylindrical shape bamboo) where threads are forming a straight line. The shuttle is a wood and boat-like shape containing the *tagingting* that was thrown perpendicularly to the *ur-ay* to form creative patterns.

Figure 4. Threads



In addition to the geometric concepts, the plane is shown in the woven cloths. The woven cloth shows a flat surface that describes a plane. A plane is a flat surface that extends endlessly in all directions. The *laylayon stripes* (a linear design of loom woven clothes that is formed by either intersecting or parallel vertical or horizontal line strips with different colours), *piningitan* (a cultural wraparound skirt for tinguian women that is used in weddings, ethnic dances, and festivities. It is a plain white cloth with red stripes at the edge and at the centre of the woven cloth), and *baag* (a local term for G-string) which are plane woven clothes. In weaving, the pattern or design itself describes a plane.

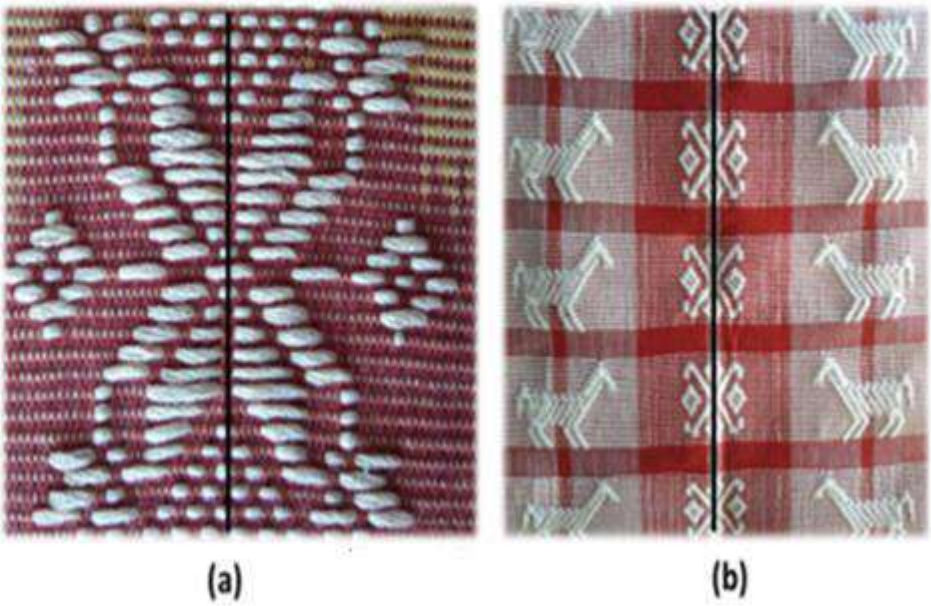
Figure 5. Plane clothes

What Symmetry Analysis is Embedded in the Loom Woven Clothes of Abra?

The creative patterns of the different woven clothes of La Paz and San Juan describe the use of symmetry analysis which makes the pattern revealed its beauty. Symmetry analysis specifically looks into the repetition of parts of a pattern that completes the pattern as a whole (Washburn, 2012). Symmetry is an underlying mathematical principle for the analysis of a repeated pattern in a textile or fabric. Here, the arrangement of the points, lines, and planes is reflected, rotated and translated.

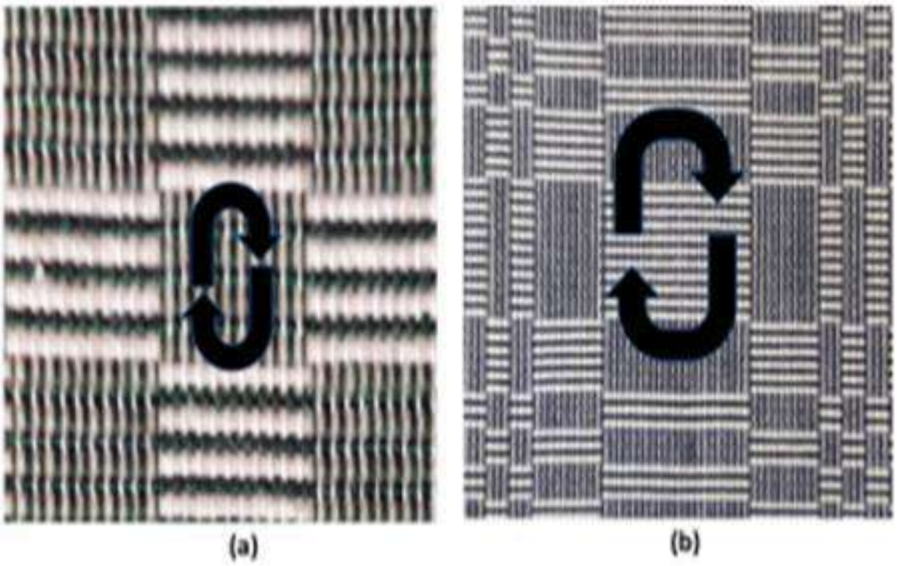
The foregoing figure presents symmetry. The frog pattern produces mirror images when bisected vertically (see figure 6a). The line of symmetry on the second figure (figure 6b) is drawn vertically at the centre of the diamond-like shape pattern that causes the horses to be reflected vertically. Same as through when the pattern is flipped horizontally, it produces two identical images of the horses and the diamond-like patterns.

Figure 6. Reflection



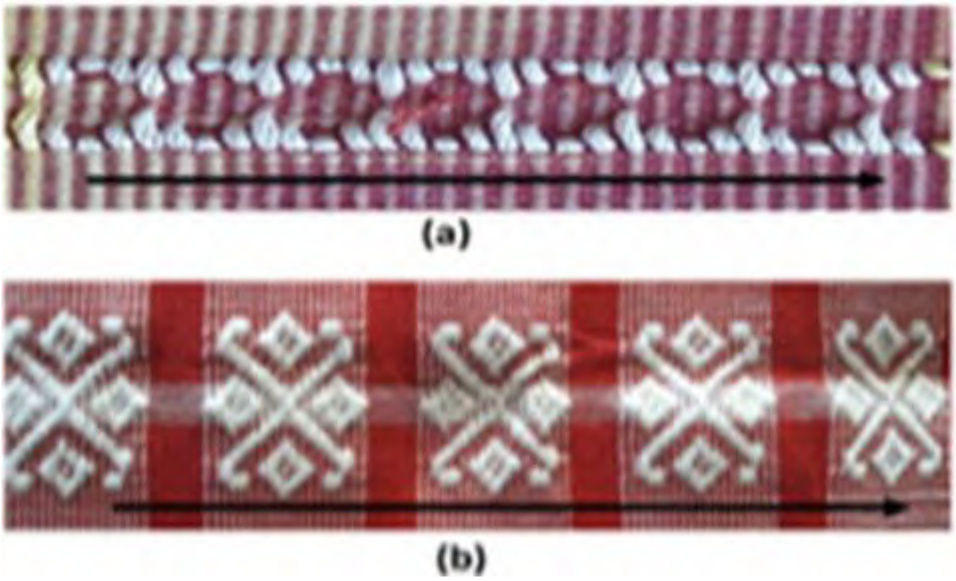
On the other side, the pre-existing figure describes a rotation. In figure (7a), the pattern can be rotated at 90^0 and 180^0, which when rotated at a fixed point it produces the same pattern. While the second figure (7b) has a 180-degree rotation. Rotation will emphasise the unique form of patterns in the woven cloth as if it is being rotated, the form of the pattern is not affected.

Figure 7. Rotation



Besides, translation is present in the woven cloth of La Paz and San Juan. The first pattern describes a circle which is being translated in one direction. The same image was produced (see figure 8a). Moreover, the diamond-like pattern is repeated and translated in one direction from left to right and are equidistant from one another. Hence, translation is the motion of an object in one direction with the same distance formed, without rotating and resizing the pre-image.

Figure 8. Translation



What Crystallographic Patterns are Included in the Loom-Woven Clothes of Abra?

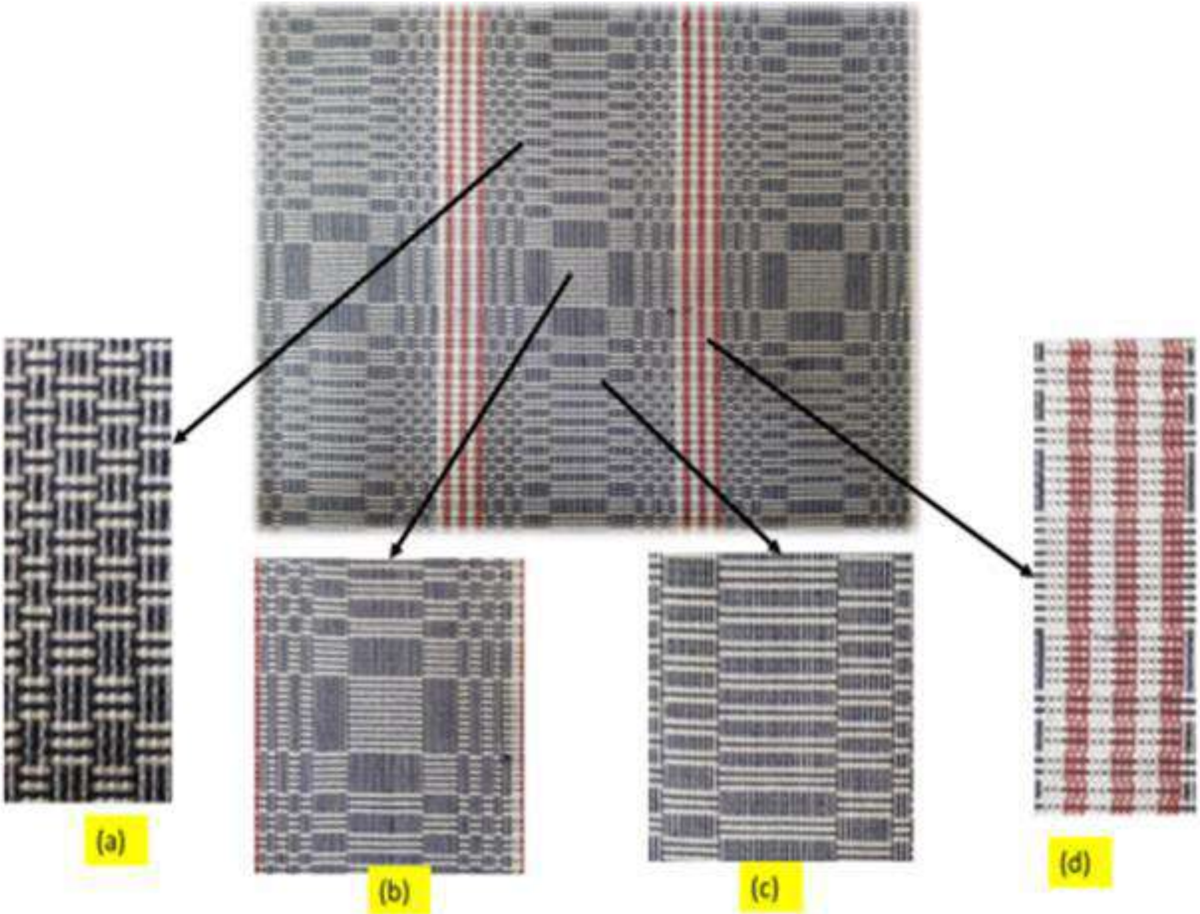
The following were the findings in the loom woven cloths of Bulbulala, La Paz, Abra.

a. ***Binakol/Binakul***

Binakol or Binakul is a textile pattern handwoven or a small scale in Ilocos also known as binakel, binakael, binakol, or binakul (meaning twill in English).

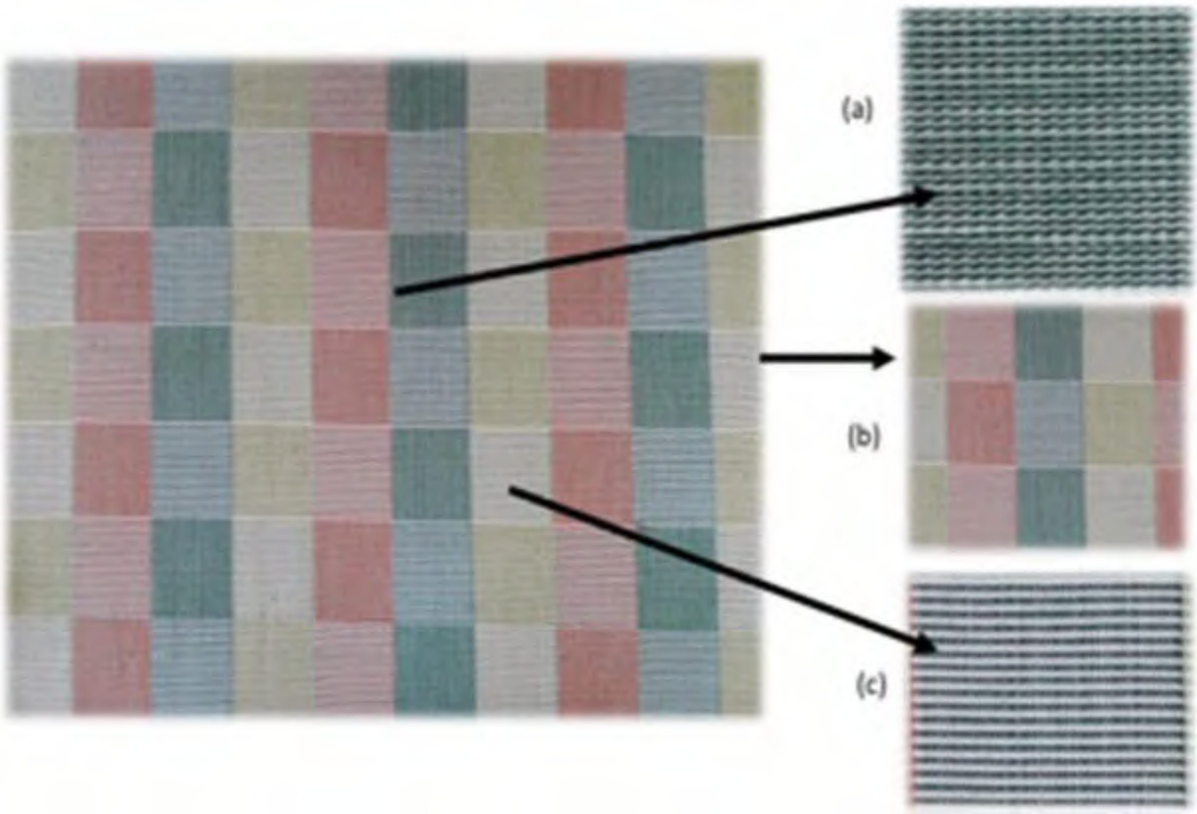
Kuskusikos (a design of binakol which has uniformly interlocked geometric patterns that result in psycheiclic optical art designs which are said to represent the wheel of life) is a kind of binakol comprised of two or more patterns. A glide reflection is shown in the pattern and is classified under the frieze group ***p1a1*** (see figure 9a). Another pattern is presented by a 180-degree rotation with vertical and horizontal reflection which is under ***pmm2.*** Vertical and horizontal reflections together with a 180-degree rotational symmetry around the central rectangle are present in the pattern of figure 9(b) under ***pmm2.*** Rotational symmetries have axes that are perpendicular to each other and that also intersect at a centre of 180^0 rotation. Frieze pattern ***pmm2*** is present in figure 9(c) as the pattern is being reflected vertically and horizontally. The degree of rotation is at 180 and classified under wallpaper pattern ***p2.*** The last figure presents a vertical and horizontal reflection under ***pmm2***.

Figure 9. Binakol Kuskusikos



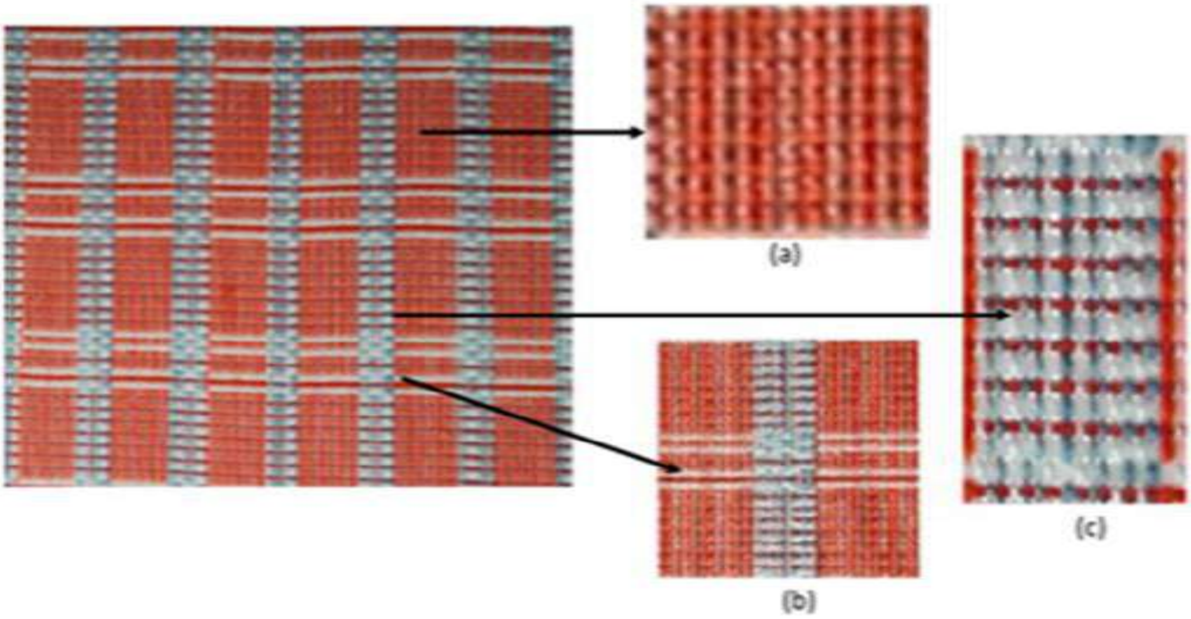
Translation of squares is seen in the pattern of ***Ik-ikamen*** (mat-like). ***Ik-ikamen*** is another pattern of ***binakol*** consisting of squares with different colours. A 90-degree rotation and reflection are described by a wallpaper pattern ***p4g (see figure 10a)***. There is no reflection in the pattern of the figure 10(b) which is under p111, however***, p2*** is present because it has a 180-degree rotation. The highest degree rotation of the last figure is 180^0 and is being reflected vertically and horizontally, which is under ***pmm2.***

Figure 10. Binakol Ik- ikamen



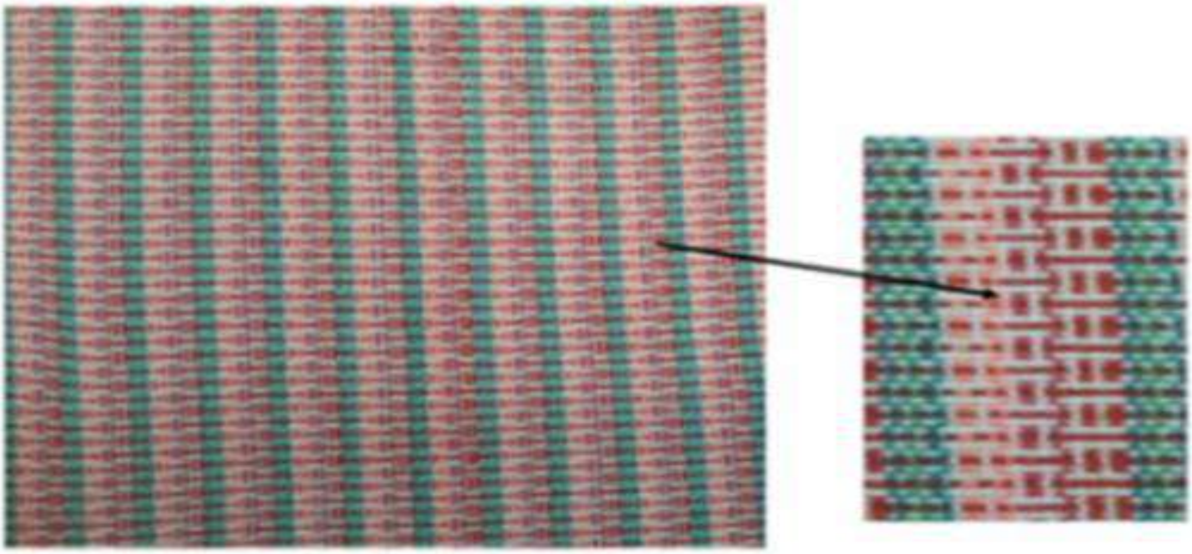
Kinkinelleng (field acre-like) is a binakol pattern composed of rectangles and strips. The rectangles are represented by a variety of orange threads. The thicker strips are represented by a light blue thread while the thin strips are represented by the red thread lines. The first pattern is classified under wallpaper pattern p4g because of the 90-degree rotation and reflections present (see figure 11a). The intersecting of the thicker and thinner strips formed four squares that are rotated at 180-degrees and reflected in two directions, describing a pattern under ***cmm*** (see figure 11b). Furthermore, the frieze group pmm2 is revealed through the vertical and horizontal reflection of the pattern, and a 180-degree rotation is seen which is under p2 (see figure 11c).

Figure 11. Binakol kinkinelleng



Bitbitwen (Star-like) is a pattern formed by intersecting and reflecting strips and threads. A glide reflection is present in the pattern and it is being classified under ***cm*** because the glide reflection is not on the reflection axis. It has a vertical and horizontal reflection with a 180-degree rotation which is under the frieze pattern ***pmm2.***

Figure 12. Bitbitwen

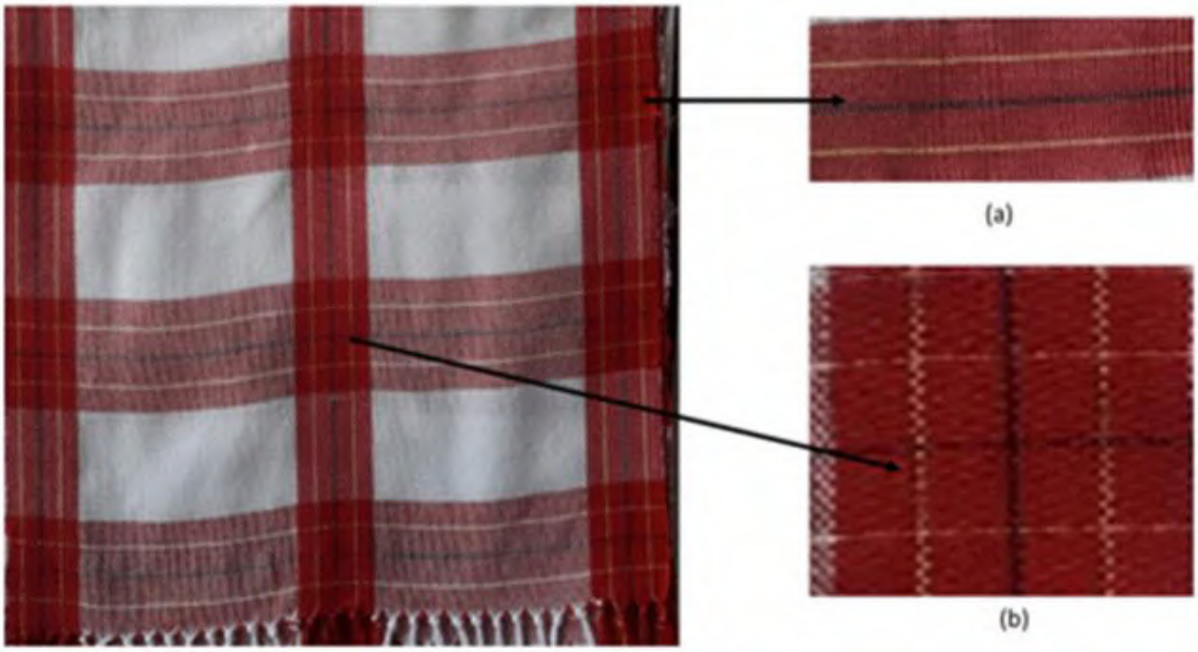


At the same time, a wrap-around skirt for women is called bankudo or piningitan. The cloth is plain white with red stripes at the edge and the centre. This cloth is used by Tinguians in weddings, ethnic dance, and festivities. The piningitan of Bulbulala is made up of red, blue, and yellow strips at the edge of the cloth. The centre is bisected by a thin red stripe. There is only one pattern being described by the strips of the cloth and that is the ***pmm2*** pattern. It is vertically and horizontally reflected.

Figure 13. Piningitan of Bulbulala La Paz

Forever Loom weaving in Bulbulala is a weaving centre where this cloth was made. This design was inspired by the Piningitan cloth of the said weaving centre. The stripe pattern is under ***pmm2*** because of the vertical and horizontal reflection that is the same as the stripe pattern of Piningitan (figure14a). Furthermore, the remarkable design of the cloth is rotated at 90 degrees which is under ***p4g*** (figure 14b).

Figure 14. Bahag is made up of piningitan of Bulbulala, La Paz Abra



Most of the design of Laylayon stripes are defined by stripe lines with different colours. The *Laylayon kantarinis* is described using thick and thin strip lines bisecting the cloth. This kind of design has one pattern only and that is the ***pmm2*** where the pattern is vertically and horizontally reflected. A 80-degree rotation is also present under ***p2***.

Figure 15. Laylayon kantarinis La Paz Abra

This ***Laylayon stripe*** has a combination of a yellow, blue, red, and white strips which are arranged vertically. The design is rotated at 180^0 that produces a pattern under ***p2*** and is reflected horizontally and vertically that result in ***pmm2.***

Figure 16. Laylayon kantarinis with a combination of blue, yellow, white and red stripes

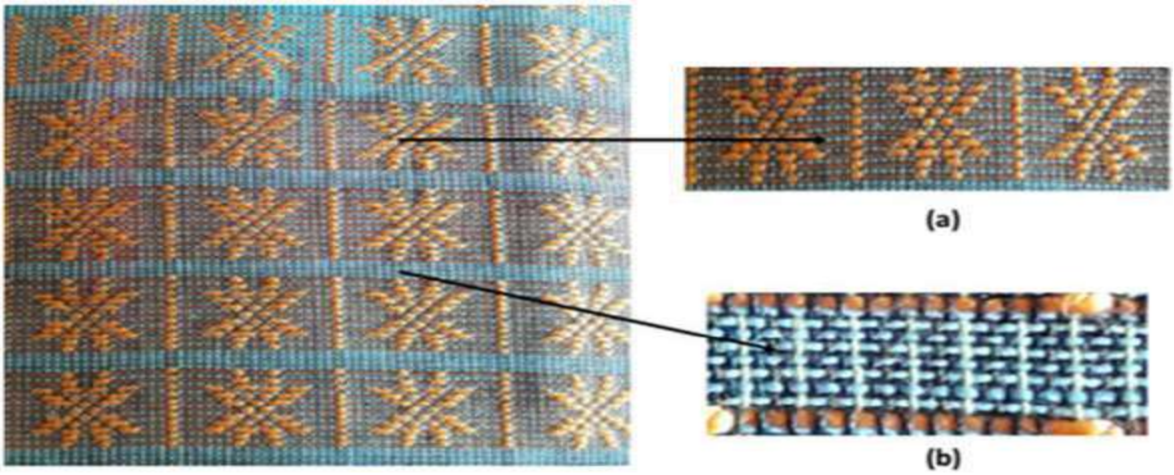
The following were also the findings from the loom woven cloths in Lumubang, San Juan, Abra

Sungkat

Sungkat is a trademark of Lumubang, San Juan, Abra. The pattern, textiles and contrasting colours emphasise their sungkat designs. Sungkat is innovated by the Cordilleran' weaving design. It usually uses objects to emphasise nature, and its beauty is abstractly seen.

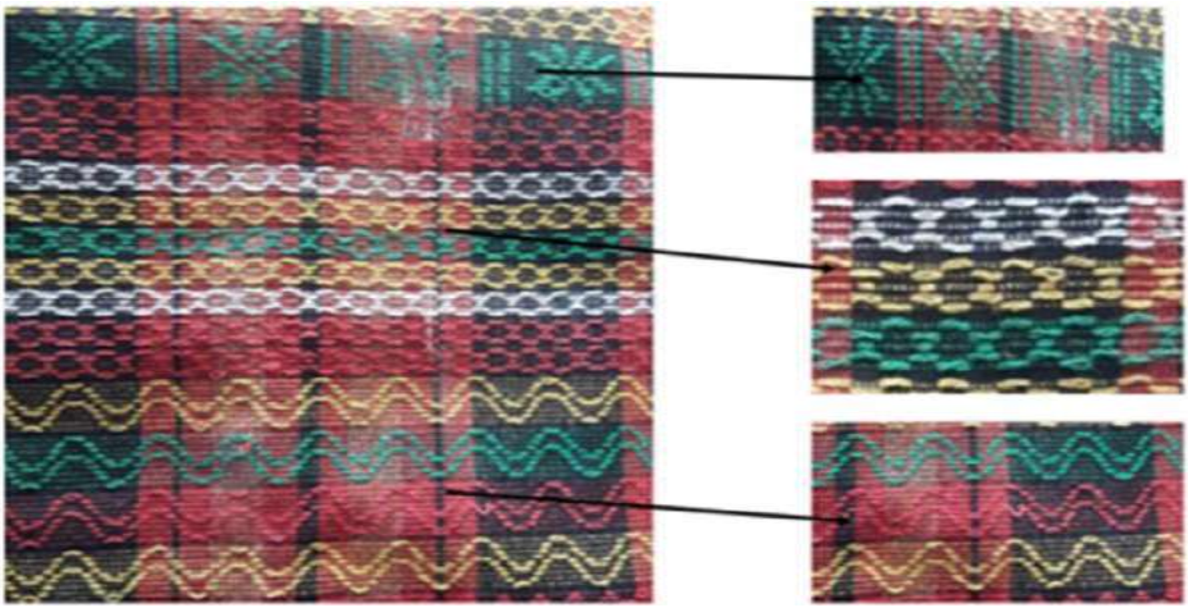
The ***sabsabong*** (flower-like) pattern design is described by the intersecting four yellow short lines that form a flower-like shape. A reflection and a 45^0 rotation are present in the ***sabsabong*** pattern which falls under the wallpaper pattern ***p4m*** (see figure 17*a*). At the same time, it unveils the frieze pattern ***pmm2***. The frieze pattern ***p1a1*** is revealed because glide reflection is present in the strips of the ***sabsabong*** pattern (see figure 17b). Also, ***p2*** is described through the 180-degree rotation of the pattern.

Figure 17. Sungkat- sabsabong pattern



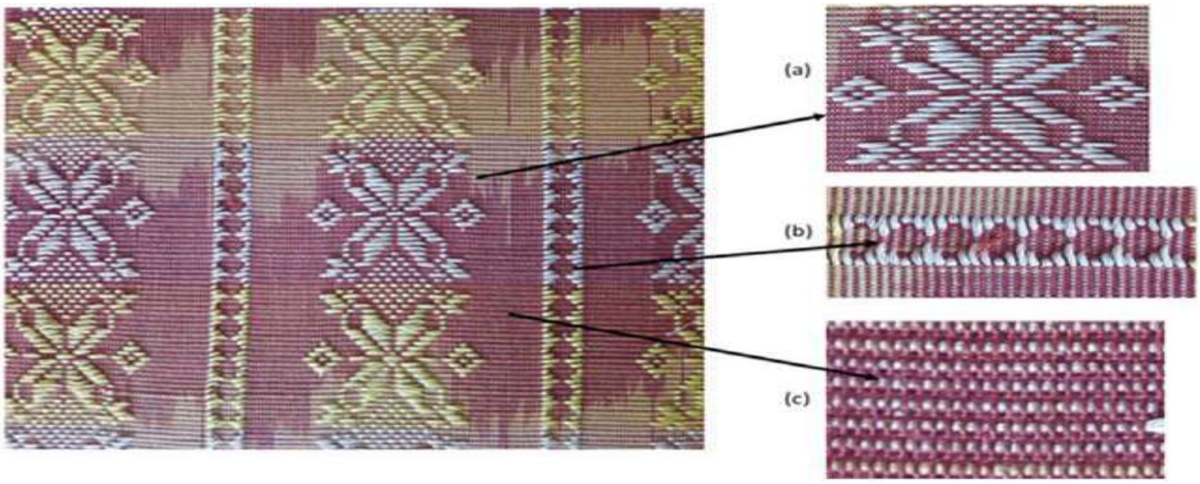
A perfect combination of three different sungkat patterns (sabsabong pattern, diamond-like pattern, and wavy pattern) is seen in the woven cloth of San Juan, Abra. The first strip shows a sabsabong pattern that has vertical and horizontal reflection under pmm2 (see figure 18*a*). Also, wallpaper pattern p4 is unveiled by the 45^0 rotation of the sabsabong pattern. Since reflections with glide reflections in an axis that is not on a reflection axis are present in the pattern, it is the wallpaper pattern ***cm*** (see figure 18b). Vertical reflection but no horizontal reflection is revealed by the wavy pattern that falls under ***pma2***. As the smallest rotational symmetry is 180^0 with vertical reflection, the pattern is under ***pmg*** (see figure 18c).

Figure 18. Woven cloth with a combination of three patterns



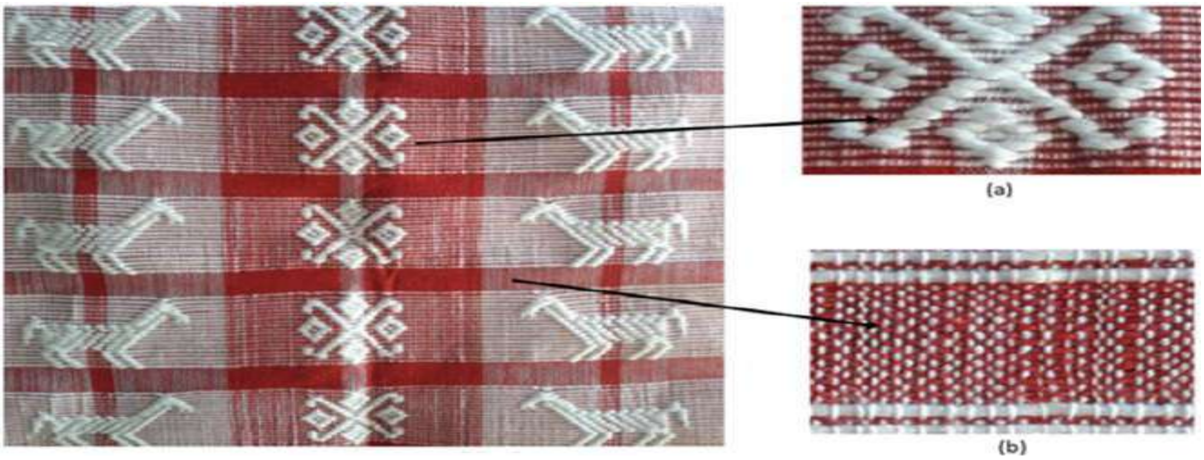
The frog-like pattern revealed the frieze pattern ***pmm2.*** Such a pattern is present also under ***pmm*** because reflection is in two directions, it is being rotated at 180^0 and the rotation centre is at the reflection axes (see figure 19a). In figure (b) and (c), horizontal and vertical reflections are present which is under the frieze pattern ***pmm2.*** Moreover, wallpaper pattern ***p2*** is seen in figure (c) as the pattern is rotated at 180 degrees.

Figure 19. Frog-like pattern



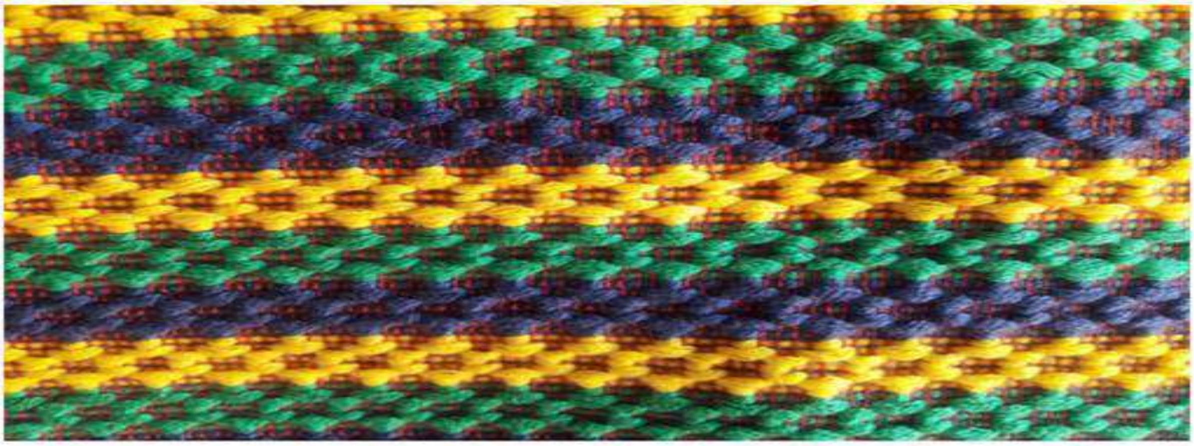
This pattern is called a ***horse-like pattern***. Frieze pattern ***pm11*** is shown and described by the horses that are reflected vertically. There is a 90^0 rotation of the diamond-like pattern, which is horizontally and vertically reflected and falls under ***p4g*** (see figure a). Frieze pattern ***p1a1*** is present since glide reflections are embedded in the strips of the cloth (see figure b). Also, vertical reflections are present which is under ***p1m1.***

Figure 20. Sungkat-horse-like pattern



This pattern is under ***pma2***, since vertical and 180-degree rotations are revealed in the pattern. Glide horizontal reflections unveiled the symmetry group of the pattern under ***pg***. 180-degree rotation and reflection of this pattern describes the wallpaper pattern ***pmg***.

Figure 20. Sungkat having a diamond-like pattern



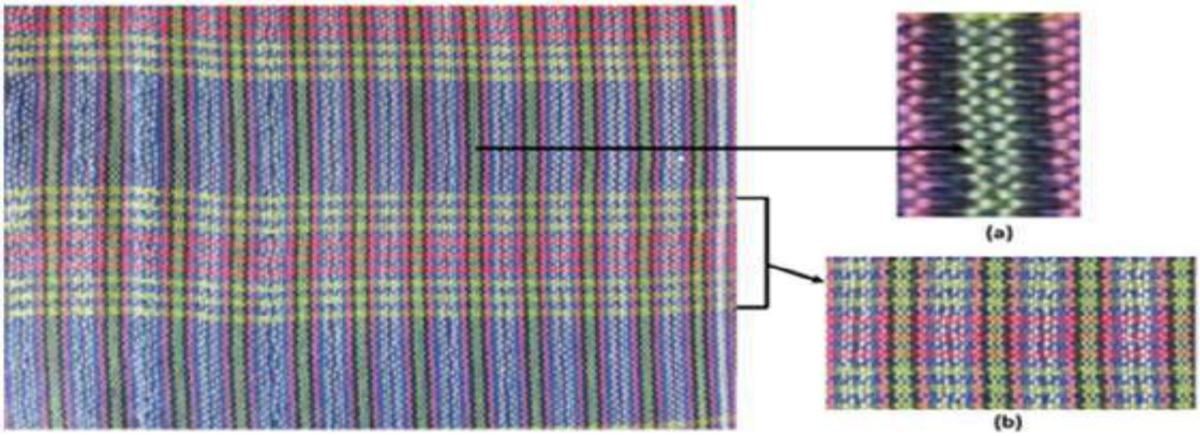
Since vertical reflections are present in the pattern and it has no horizontal reflections, the pattern is under ***pma2***. As the smallest rotational symmetry is at 180-degree and has only vertical reflection symmetry in a two-dimensional pattern, this pattern falls under ***pmg.***

Figure 21. Sungkat having zigzag pattern

Stripes

Stripes are one of the designs made by the weavers of Lumubang, San Juan, Abra. The beauty of the stripes is seen through the different colors of the strip lines. Most of the patterns present in the stripes are linear. The strips of this stripe are vertically and horizontally reflected and are determined by yellow, red and blue line colors under ***pmm2***. Symmetry group ***pgg*** is determined through a 180-degree rotation and glide reflections in the stripe pattern (see figure 22*a*). The intersection of the yellow line colors and red line colors are rotated at 180^0 which are in two directions and its rotation centers are on reflection axes. Thus, its crystallographic group is ***pmm*** *(*see figure *b).*

Figure 20. Stripes



The cloth is composed of green and red strip lines. The patterned strip is under ***pmm2*** and is being rotated at a 180-degree rotation which is under ***p2***. The pattern under ***p1m1*** is revealed through a vertical glide reflection as shown with the green and red, and a horizontal glide reflection is shown with white. Also, the symmetry group is under ***p4***, as the degree of rotation of the intersecting lines formed squares and is rotated at 90 degrees (see figure b).

Figure 23. Stripes shows red and green line colors

Furthermore, ***p111*** is revealed through this pattern since it has no reflection and rotation at all. This pattern belongs to wallpaper pattern **p1** also because there is no reflection and rotation at all.

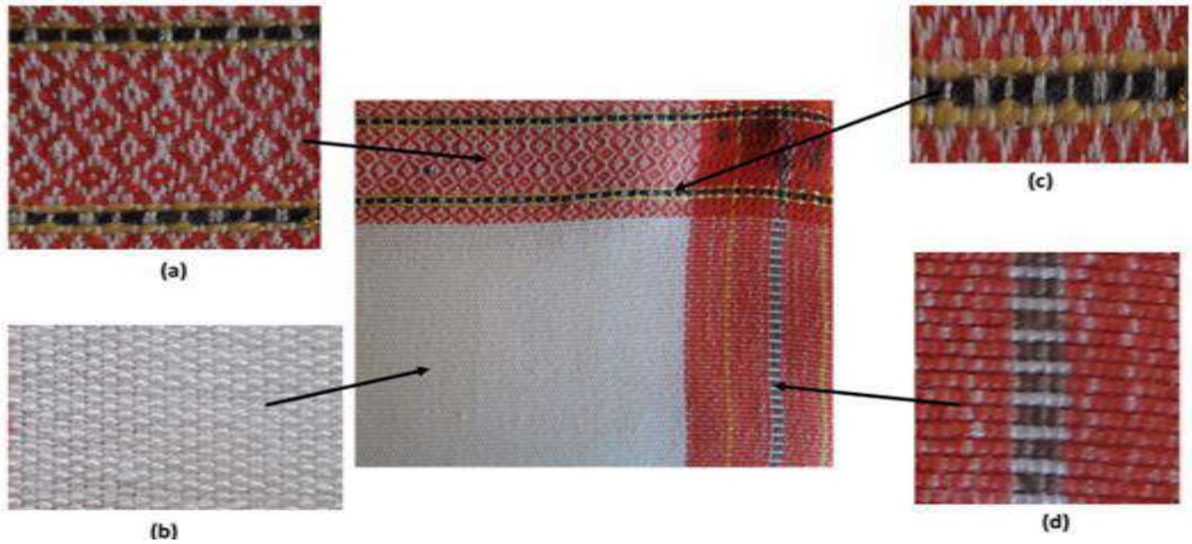
Figure 36. Plain Stripes

Piningitan

The Piningitan of Lumubang, San Juan, Abra is different from the Piningitan of La Paz, Abra in terms of strip designs and patterns, and the contrast of colors.

Figure 37(a) shows the pattern under ***p4g*** since the diamond patterns can be rotated at 90-degree rotation and have vertical and horizontal reflections (see figure 37a). In figure (b) glide reflection is revealed and its pattern is under ***pg.*** Vertical and horizontal reflections are shown by the black stripe pattern having two yellow lining colors which are under ***pmm2***, also, a 180-degree rotation is present which is under ***p2*** (see figure c**)**. Short white horizontal lines and black vertical lines located at the right side of the cloth are vertically and horizontally reflected, and so the pattern is under ***pmm2***. Moreover, it is under ***p2***, since a 180-degree rotation is present in the pattern (see figure d).

Figure 37. Piningitan made in Lumubang San Juan, Abra



The two municipalities used the same technique of weaving using loom machines. The processes are also the same in creating their own creative and trademark designs. Bulbulala is known for binakol weaving that includes the ***kuskusikos, ik-ikkamen, kinkinelleng,*** and ***bitbitwen patterns***. While Lumubang is known for their ***sabsabong pattern, horse-like pattern, wavy pattern, zigzag pattern, diamond-like pattern,*** and the ***frog-like pattern*** that belong to the ***sungkat*** weaving**.**

In the same way, ***laylayon stripes*** of the two municipalities revealed ***pmm2*** as the common crystallographic pattern that was created by the linear strips of the woven cloth. However, they differ with the contrasting colors of the stripe pattern. ***Laylayon stripes*** of San Juan have more intersecting line strips with a brighter colour than the parallel line strips of the ***laylayon stripes*** of Bulbulala.

Furthermore, the piningitan design of the two municipalities differs from one another. Linear patterns with darker colors are present in the strip of the piningitan of Bulbulala, La Paz, Abra. On the other hand, intersecting linear patterns with lighter colorus are revealed in the strips of the woven cloth of Lumubang, San Juan, Abra.

In general, ***pmm2*** is the dominant frieze pattern being displayed in the woven clothes of the two municipalities, while, the dominant wallpaper patterns revealed are under ***p2***. Other frieze patterns being displayed are ***pma2, pm11, p1m1, p1a1, p112***, and ***p111***. Other wallpaper patterns being exhibited are ***p4m, p4g, p4, pmm, cmm, pmg, pgg, cm, pm, pg***, and ***p1.***

Further, based on the result of the study, crystallographic patterns of loom weaving are a great help in bridging the concepts of mathematics into reality. It was found that geometric concepts, symmetry analyses, and crystallographic patterns are applied in weaving. Hence, this is such a powerful tool that can be used by mathematics teachers to connect the subject to the real world. It will serve as a springboard to open research and investigations to promote students' exposure and engagement.

This manifests that education is hands-on learning where the students need to actively participate to enhance their skills, knowledge and capabilities. Further, it implies that education is the instilling of the real-world application of mathematics to promote students' appreciation of the subject and to upgrade long term learning.

Conclusions

Because of the salient findings of the study, it is therefore concluded that:

1. The woven cloth of *binakul, sungkat, piningitan, and laylayon/stripes* exhibit geometric concepts such as points, lines and planes. Further, it displays symmetry analysis such as reflection, rotation and translation, and uncovers the crystallographic patterns, frieze and wallpaper patterns.

2. The woven cloths from La Paz and San Juan, Abra showed similar classification under frieze patterns and wallpaper patterns. The foremost frieze group in the woven clothes that were found was ***pmm2.*** Other revealed frieze patterns were ***pma2, pm11, p1m1, p1a1, p112***, and ***p111***. Hence, ***pg, pm, cm, pgg, p2, pmg, cmm, pmm, p4, p4g***, and ***p4m*** were unveiled to be wallpaper patterns in which the dominant one is under ***p2***.

Recommendations

In light of the above findings and conclusions, the researchers forward the following:

1. Math teachers may use loom weaving as real-life applications on their topics in geometric concepts and symmetry analysis.
2. Math teachers may use the loom-woven cloth as an aid for teaching geometric concepts, specifically the undefined terms, angles and geometric transformations.
3. Teachers may use hand-weaving as an activity for students to develop their skills, specifically their mathematical ability, creativity and craftsmanship.
4. Weaving done by students may be used to form a linkage of mathematics to art subjects.
5. Weaving is such a good springboard to open research and investigations to promote students' exposure and engagement.
6. Weaving is a real-world application that promotes students' appreciation of the subject and to upgrade long term learning.

7. A parallel study may be conducted on mathematical concepts on loom weaving and other craftmanship in Abra.

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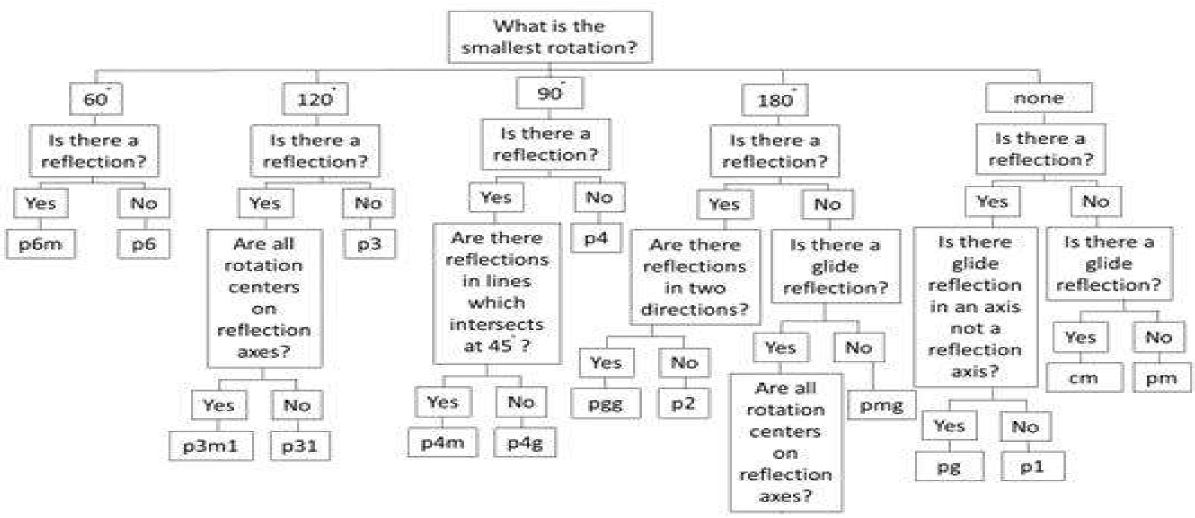
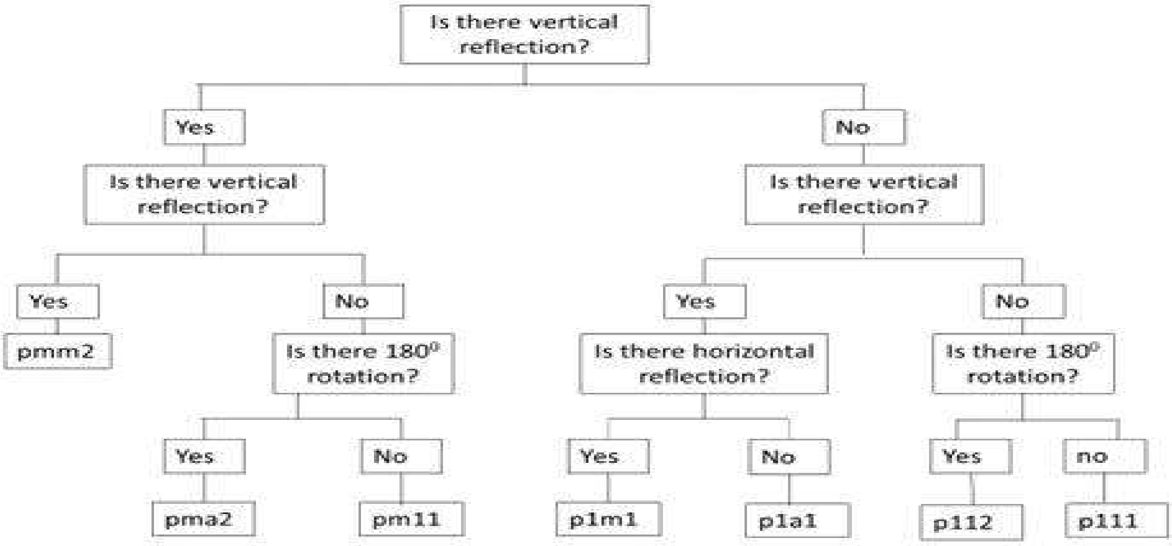
APPENDIX A

Flowcharts in the Identification of the Crystallography of the Loam – Woven Clothes of Abra



Flowchart in identifying the Wallpaper Patterns (Washburn,



Flowchart in identifying Frieze Patterns (Washburn, 1988)