

A Comparative Analysis of Instructional Strategies in Middle-Grade Mathematics Textbooks

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This study compared the quality of instructional activities of three series of selected middle-grade mathematics textbooks used in public and private schools in Pakistan. The aim was to analyse the extent to which the quality of instructional activities was appropriate for the development of mathematical proficiency in students. Geometry was selected for this study due to its challenging nature in mathematics curriculum. The procedure included the development of researched-based learning goals and indicators and, identification of evidences in textbooks relevant to the learning goals. These evidences were then compared with the indicators to check their appropriateness for the promotion of proficiency in geometry. The results showed that the New Syllabus Mathematics 2 contained comparatively effective instructional activities to support students in understanding key conceptual ideas in geometry. Mathematics 8 and COUNTDOWN contained less material for conceptual understanding and procedural fluency. These textbooks were focusing mostly on routine procedures with few connections to daily life. The results of this study will have greater significance for curriculum developers, textbook authors, textbook analysis experts and school administrators.

Key words: *Instructional strategies, Mathematics Curriculum, Mathematics Proficiency, Textbook Analysis*

Introduction

The textbooks are usually developed to provide support to teachers in their daily classroom teaching (Hussain & Shaheen, 2017). Most of the teachers rely on these textbooks and rarely use other resources to supplement their teaching in the classroom. The National Curriculum of Mathematics (MOE, 2006) lays much importance on the logical and coherent structure of instruction in mathematics. Studies have shown that the majority of mathematics teachers in schools follow the prescribed textbook when teaching mathematics in the classroom (Thomson & Fleming, 2004). Teachers use textbooks as a guide for their teaching of content and the teaching method (Van Den Heuvel-Panhuizen & Wijers, 2005). Qi, C., Zhang, X., & Huang, D. (2018) analysed different roles of teachers during the classroom teaching of geometric transformation and compared how uniquely teachers used textbook at various phases of classroom teaching. They found that the usage of textbooks by teachers' reaches the level of elaborating and creation with respect to the difference of years of teaching.

Fuson, Carroll, and Drueck, (2000) analysed the impact of standards based curriculum and traditional materials on students learning. The results revealed that students learning from Everyday Mathematics (EM), scored higher than students who studied from traditional materials on standard topics including place value and computation. Vincent & Stacy (2008) analysed a series of mathematics textbooks from eighth grade. By using criteria from the TIMSS Video Study, they examined the exercises and problems contained in the textbooks. Vincent and Stacey discussed that textbooks must help students make connections and recognise mathematical concepts, mathematical reasoning, and reflection (p. 102). It is evident that the textbook that is designed to build conceptual understanding through inquiry and student-centered learning strategies embedded with the variety of assessment methods are adopted in many schools across the world (Remillard J. T., 2005).

Therefore, it is imperative to analyse instructional strategies in mathematics textbooks before selecting it for the students. This study analysed the salient features of a series of selected mathematics textbooks using the procedure of the framework developed by (Hussain & Shaheen, 2017).

In this study, "Geometry" the key standard of mathematics from the middle-level mathematics curriculum that comprises 40% of the National Mathematics Curriculum (MOE, 2006), was chosen to compare the quality of imbedded instructional activities. The poor achievement of students in international level assessment programs like Third International Mathematics and Science Study (TIMSS), PISSA (Ubuz, Üstün & ErbaG, 2009; Mullis et al., 2000), and numerous other researches reveals that the geometry curriculum in middle school mathematics requires earnest consideration. A few researchers have recognised the distinctive features of geometry in mathematics textbooks, for example, problem features and presentations in geometry among textbooks across countries (e.g., Yang, D. C., Tseng, Y. K., & Wang, T. L. 2017; Hong & Choi, 2014; Zhu & Fan, 2006).

Numerous researchers found that in the geometry, the subcategories of measurement and space of PISA and TIMSS in students from various countries performed below achievement than the average level (Mullis et al., 2000; Ubuz, Üstün & Erbağ, 2009; Uzun, Bütüner & Yiğit, 2010). Similarly, the scores in geometry of U.S students in TIMSS were strongly related to certain courses a student took (for instance, regular mathematics versus algebra) and variability among schools (National Research Council.1999). These findings reveal that mathematics textbooks that incorporate effective instructional strategies have significant impact on students' learning in geometric concepts. However, despite having the importance of topic, little research has been conducted on how middle school mathematics textbooks should incorporate such strategies that help students develop proficiency in geometry. Hence, the purpose of this study was to examine how the selected mathematics textbook series use distinctive and quality activities to develop proficiency in geometry. The following research question guided this study.

How do the selected middle grade mathematics textbooks use a variety of and quality instructional strategies to develop proficiency in geometry?

Related Research

As mentioned above, geometry is considered to be crucial at the stage of middle level because it possesses 40% proportion in the National Mathematics Curriculum (MOE, 2006).

At this Stage, students use spatial visualisation skills to move between two- and three-dimensional shapes and their representations (Ina, Mullis, Michael, Pierre, & Alka, 2012). The diagrams and pictorial representations are necessary for devising and solving geometric problems, and these representations allow learners to describe key components of geometry during the process of problem-solving (Larson, Boswell, & Stiff, 2004; NCTM, 2000; Zodik & Zaslavsky, 2007). Numerous researchers argue that due to ineffective incorporation of abstract concepts of geometry in the textbooks and the direct introduction of these concepts and formulas without paying adequate emphasis on conceptual understanding, problem-solving and reasoning, makes the geometry a difficult area to learn (Karnasih & Soeparno, 1999; Soedjadi, 1991).

Corle (1964) found that students confronted difficulty in solving problems that contain volume concepts. Enochs & Gabel (1984) argues that the difficulty in the concepts related to volume might be because of the instructional order teachers adopt to teach the concept. Enochs & Gabel (1984) further discuss that students apply that concept to solve other geometric figures even though the concept doesn't work. For example, the formula "length \times width \times height" to find the volume of a rectangular solid can't be applied to the geometric figures such as cylinders. Bilbo and Milkent (1978) used an alternative strategy to teach volume concept to pre-service teachers without prior focusing on length and area. He found that students' understanding of volume concept was more successful as assessed by estimation, computation, and measurement test. Gabel, D. L and Enochs, L. G (1987) conducted similar research to investigate the

differences in learning of volume concept with equally focusing on length, area, and volume but changing the order in which the topics are taught. The result found that teaching the volume-first method was effective for all students. Gabel, D. L & Enochs, L. G (1987) also found that students with high spatial-visualisation ability learn the metric system more successfully with the instruction order of length, area, and volume. But if students have low spatial-visualisation capability, they suggested the ‘volume-first’ as preferable order to present the concept. The similar case is with textbooks in which three-dimensional concepts are presented using two-dimensional representations. Hershkowitz (1990) identified that numerous curricular programs neglect the need of actual three dimensional models for teaching spatial concepts. Students confront many difficulties to understand three-dimensional concepts through two-dimensional representation (Hershkowitz 1990). This indicates that spatial visualisation skills are important for students to solve the problems related to volume and surface area.

Maier (1996) noted that there is more focus on two- dimensional geometry than three-dimensional in middle level mathematics and because of this less emphasis on three-dimensional geometry, many students have difficulty visualising three- dimensional representations (Gutiérrez, Pegg and Lawrie, 2004). Numerous researchers also emphasise that the use and manipulation of geometric models are vital to improve student’s spatial abilities in geometry (e.g. Baki, Kosa, & Guven, 2009; Gittler & Gluck, 1998). Pittalis, Mousoulides, and Christou (2010) proposed a model to help students gain spatial abilities in three-dimensional geometry. The model emphasises the recognition and formation of nets, representation of three-dimensional figure, structuring of a three-dimensional array of cubes, and the recognition of the characteristics of three-dimensional objects.

Theoretical and Conceptual Framework

This research uses the framework developed by (Hussain & Shaheen, 2017) which is based upon mathematics proficiency standards (Kilpatrick, 2001) and formative assessment (Black & Wiliam, 1998) and this provides authentic procedure for the analysis of proficiency-based instructional activities. The framework evaluates how textbooks reflect the balance in the proposed six learning standards (Kilpatrick, 2001) and formative assessment (Black & Wiliam, 1998) and helps students meet the standards’ rigorous expectations. The framework uses learning goals for each standard and each learning goal is measured by a set of indicators, developed using constructed knowledge of research. Hence, the learning goals of the framework are consistent with the national and international curriculum goals for effective instruction. Only the first three learning standards of the framework were used for the current study to conduct in-depth analysis of selected topics rather than using all standards and doing a superficial examination of textbooks. A brief description of these proficiency standards supported by research literature is given below.

Conceptual understanding

“Conceptual understanding refers to an integrated and practical knowledge of mathematical concepts” (Kilpatrick et al. 2001:139). Students cannot comprehend mathematical concepts by just connecting bits of facts unless an obvious support in linking concepts is extended (Resnick, 1987). Students with conceptual understanding can form their knowledge into a coherent whole that helps them to integrate new concepts by synthesising those ideas to their previous concepts, (Bransford, J. D., Brown, A. L., & Cocking, R. R, 1999). An essential benchmark of conceptual understanding is having the knowledge to represent contextual situations in various ways and being able to know that how distinct representations can be suitable for different purposes (de Jong & Ferguson-Hessler, 1996; Star’s, 2005). The conceptual understanding has been transformed into the following two learning goals.

Goal 1. The subject matter develops mathematical concepts in students

This goal involves examining the models or basic skills in the textbooks and how these basic skills are gradually formalised with terminology. This goal examines subject matter in the textbook for pre-requisite concepts and/or skills. It is significantly more important for a mathematics textbook to address the prerequisites from the previous units so that students must make connections between standards and their prerequisites.

Goal 2. The textbook material assists students to understand the major mathematical concepts which are useful in particular contexts.

de Jong and Ferguson-Hessler, (1996) and Driscoll (1999) contend that for students to be able to use multiple representations, is not enough. This goal focuses on the connections and relationships among various representations. The goal involves finding pictorial models, diagrams, graphs, images and representations in the textbook.

Procedural Fluency

Procedural fluency is the second and a critical standard of mathematical proficiency. Procedural fluency refers to the ability of how someone performs and applies their skills flexibly, accurately, and efficiently (Kilpatrick et al. 2001:121; Star, 2005; Baroody J, Yingying F and Amanda, R. J, 2007). Deep procedural and deep conceptual understanding is inter-connected. The fluency in procedural knowledge is not possible without getting deep conceptual knowledge or vice versa (Baroody, 2003). Hence, students must be provided with such opportunities and experiences that help them integrate concepts and procedures.

Goal 3. The textbook contains and encourages use of a variety of procedures to build on fluencies and procedural skills in students.

Skill practice is important but abundant practice can be unproductive or it cause the loss of mathematical disposition (Isaacs, Andrew & Carroll, 1999). This goal examines how textbooks must contain a sufficient number of worked examples with clear explanation of the steps involved in the solution. This goal also involves the investigation of opportunities for the students in the textbook that encourages students to use their own strategies and methods for solving problems.

Strategic Competence

According to (kilpatrick et al. 2001), strategic competence refers to the ability of students that helps them to formulate mathematical problems, how to solve them, and represent them. According to NCTM (2000), problem-solving engages students in such tasks whose solution steps are not known in advance (p. 52). (Polya, 1975) developed a four stage heuristics in order to engage students in problem-solving: understanding the problem, devise planning, execution of the plan, and reflecting. According to Kilpatrick et al. (2001), flexibility is the fundamental characteristic for strategic competence.

Goal 4. The textbook provides explorative content and guide teachers to engage students in mathematical investigation.

This instructional goal involves examining the opportunities in the textbooks for real life problem-solving skills. The goal also examines instructional strategies in the textbooks for the development of deep strategic competence in Geometry such as explorative tasks, alternative strategies, variety of complex problems (Hussain & Shaheen, 2017), non-routine problems, contextual tasks (NCTM, 2000), graphs and manipulative models (Kilpatrick, 2001).

Goal 5. Subject matter contains making connections within and across the subject to develop strong mathematical understanding in students.

This goal involves examining the textbooks for such mathematical problems that provide ample opportunity for students to use the learned ideas in new situations and also, that require the generalisation of the concepts, process, and skills in similar situations.

Methodology

Selection of Textbooks

Three books, one from public and two from private school's use, were selected for this study. These textbooks were selected on the basis of its maximum adoption by public and private

schools. The textbook “Mathematics 8” (M8), developed and published by Punjab Text Book board, was selected for analysis from textbooks in use of public schools. M8 was adopted by more than 60 % of all students in public schools across Pakistan (PBS, 2017). Similarly, “New Syllabus Mathematic 2” (NSM2) and “COUNTDOWN New Addition Level Eight Math’s” (CD) were selected from private schools use. NSM2 is published by Oxford University Press and authorised by the Singapore Ministry of Education and the University of Cambridge International Examinations (CIE). The textbook “NSM2” is not only taught in private schools in Pakistan but also in other countries across the world. The CD is published by Oxford University Press for schools in Pakistan. It is also widely used by private English medium schools which are registered with local national examination boards.

Procedure of Textbook Analysis

Area and volume from the Geometry portion were selected to conduct comparative analysis in the depth of instructional quality in the selected textbooks. The selected textbooks contained slightly different content topics in the ‘Area and Volume’ chapter in geometry portion, however, four topics i.e. Surface area of the Cone, Surface area of the Sphere, Volume of the Cone, and Volume of the Sphere were identified as common content topics of geometry in each textbook series. Hence, this study focused on identifying the evidences of quality instructional strategies in the selected textbooks related to the above four content topics of geometry. The focused topics were thoroughly studied page by page to determine the evidences of effective activities related to the instructional goals.

Table 1

A sample of Evidence-Based Typical Sighting and Rating for Selected Textbooks

Goals	Indicators	Ratings			Activity/Evidence	Page No.
		Cone	Sphere	Average		
2. The material supports students to understand key conceptual ideas of Mathematics which are useful in Particular context.	a. The material includes conceptual problems and conceptual discussion questions.	2	1	1.5	A formation of cone from Pyramid as pyramid is the topic of previous level. As the number of sides of regular polygon increases infinitely, what will the polygon become eventually? Do it yourself(Activity for conceptual development).	209,210,215
	b. The material features opportunities to identify correspondences across Mathematical representations.	2	2	2	Real and colored images of cones and spheres and its relationship with other geometric objects.	211,215,221
	c. The material uses pictorial models, diagrams, manipulative for important concepts and skills and interrelate them with varied representation of concepts.	2	1	1.5	Difference between pyramid and cone. Diagrams about comparison between pyramid and cone, Different ways of constructing nets of cone	209, 210, 214,215
	d. The material includes application of facts and definitions and compares, contrast and integrate related concepts and principles.	3	1	2	c) Example 5, example 6. S) Example 12, example 12, Example 14, example 15, Example 16, example 17	212
	Average			1.75		

A scoring scheme was developed to assign ratings for indicators. On the basis of how well it addresses indicators, each activity was rated on a 4-point scale from 0 to 3 (3= High potential, 2= Medium potential, 1= Low potential, 0= No potential). The score for each goal was calculated by averaging the ratings of indicators for each topic. Textbooks were separately

analysed by using indicators to determine the average score of effective activities/evidences related to each instructional goal for each focused topic (Table 1). This was repeated for each standard to produce overall ratings of instructional ranking for each textbook. Finally, a performance grading criteria was developed to define the potential of each textbook for the development of mathematics proficiency (Table 2).

Table 2

The Criteria Used to Measure the Potential of Textbooks for Mathematical Proficiency

Performance Grading	Criteria	
Cumulative %	Mean	
High potential	2.4 & above	80 and above
Medium potential	1.5 to 2.4	50-80
Low potential	0.5 to 1.5	20-50
No potential	Less than 0.5	Less than 20

Reliability

The instructional goals and the corresponding indicators are based on the principles of proficiency in mathematics, developed by using effective instructional strategies supported by the large research evidence. Two experienced teachers and one PhD research student was involved in the process of textbooks analysis. The two experienced teachers had previous experience with the “Development of Framework for the analysis of proficiency-based mathematics instructional materials”. The PhD researcher student attended a session with experienced teachers before the analysis begins. In that session, he learned the scoring scheme and analysis procedure. Each reviewer applied the procedure to analyse the instructional strategies for the small topic of “Area of the Circle”. The agreement rate was 80% on each indicator. Then we proceeded to evaluate complete analysis of the focused topics in the selected textbooks series. Each analyst used the same instructional indicators and methodology to reach the ratings and was required to keep the complete record of evidence for each textbook that was analysed. After completing individual analysis, the analysts came up with specific evidence from the textbooks to justify each of their ratings. The overall reliability rate was 90% on scores of each of the indicator and the relational rankings of the textbook series on the ratings. The disagreement on any evidence and its rating was discussed mutually and developed a consensus on a common rating. Despite a consistent judgment by the reviewers, we think that this comparative study of mathematics textbooks may have both potential implications and limitations for how it will be used.

Results and Discussions

Conceptual Understanding

Table 3 compares the variation of average ratings of instructional goals in all three textbooks. Also, it represents the range of each textbook and variation of ratings across instructional criteria. Table 3 shows that NSM2 had a mean rating of 1.875 on the rating scale. The NSM2 includes 62.50 % tasks that meet the indicators while the textbooks ‘M8 and ‘CD’ had a mean rating of 1(33.33%) and 0.25(8.3%) respectively. For the goal 2, NSM2 provided potential and substantial opportunities for students to develop important concepts and skills. Table 3 reveals that NSM2 and M8 provided 58.33% (1.75) and 29.17% (0.87) quality instructional support respectively. In contrast, the table shows that CD contained no evidence that assist students to develop conceptual understanding of mathematics. Figures 1 provides a picture of how NSM2 prompted the introduction of the topic.

Table 3

Three Textbooks Series Showing Average Score with Percentage on each of the Learning Goals Across all three Learning Standards Used for Analysis

Standards	Learning Goals	Average Ratings		
		NSM2	M8	CD
CU	1 The subject matter develops mathematical concepts in students.	1.87(62.50)	1(33.33)	0.25(8.33)
	2 The textbook material assists students to understand the major mathematical concepts which are useful in particular contexts.	1.75(58.33)	0.87(29.16)	0(0)
PF	3 The textbook contains and encourage to use variety of procedures to develop fluencies and procedural skills in students.	2.25(75)	0.5(16.67)	0.25(8.33)
SC	4 The textbook provides explorative content and guide teachers to engage students in mathematical investigation.	1.75(58.33)	1.12(37.50)	0.25(8.33)
	5 Subject matter contains making connections within and across the subject to develop strong mathematical understanding in students.	1.25(41.67)	0.87(29.17)	0(0)
	Overall	1.77(59)	0.87(29)	0.15(5)

Note. CU= conceptual understanding, PF= procedural fluency, SC=strategic competence

The textbook introduced some graphics of cones from daily life before formal introduction of the topic. In Fig 1 (a), there is an image of an ice-cream cone; a cone-shaped party hat, and the picture of Wheelock Place, Singapore. Research also supports such warm-up examples to assess students’ prior knowledge and to guide the instruction of the unit of study (Crowley, 1987). The subject matter also prompt teachers to make an inquiry about students by asking questions like: What do these shapes look like? What are some common features of these cones? (P.208). Then each lesson begins with defining and describing cone and sphere and its components. There are some other activities which are the orientation of topic. For example, Fig 1(b), which implicitly suggest teachers to engage students in drawing the nets of the cone

(Boyd C. J., Burrill, Cummins, Kanold, & Malloy, 1998). In Fig 1(b), a cone and its net (semi-circle) is given and students are asked: How do you find the curved surface area of the cone (P.214)? This activity clearly implies that the concept is intentionally connected to the area of the circle which had already explained in the previous grade. Van & M (1986) claims that students use their previous knowledge to observe, manipulate, and represent physical objects to understand the nature of objects, their relationships and be able to classify and develop abstract definitions.

Fig 1(c) presents varied kinds of cones which provide opportunities for students to recognise misconceptions about the topic so that irrelevant features do not become important. (Many students, for example, just think that the right circular cones are really cones or that oblique cones are no longer cones). Similarly, in Figure 1(d), the concept of the cone is constructed from the pyramid. The other features in NSM2 are the new words and conceptual terms which were highlighted in the text (although there is no evidence of any separate activity that enhances the practice of mathematical terms). The material frequently uses pictorial models for key mathematical concepts and interrelate them with a multiple representation of concepts (Van & M, 1986). We found no evidence in NSM2 about how to engage or address struggling student's difficulties in conceptual understanding. However, the material includes a substantial number of the range of simple and rich problems that explicitly help students make sense of mathematics and develop connections between concepts (Van & M, 1986).

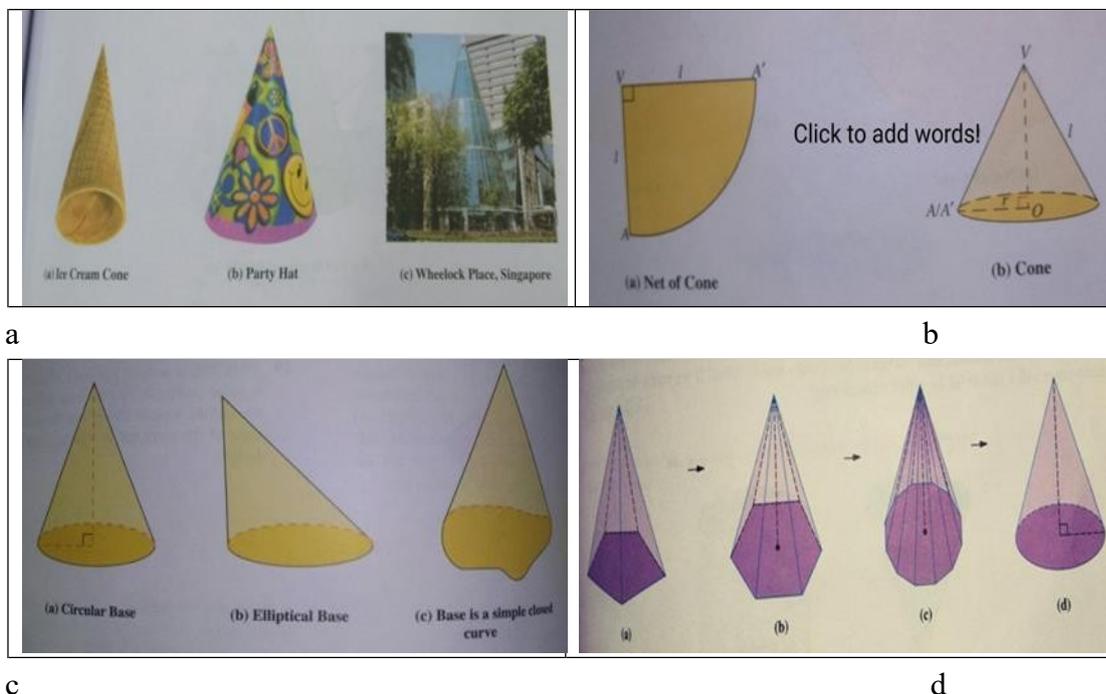


Figure 1. Some examples; a, b, c, and d as evidence that show how material in NSM2 link previous knowledge and current topic using manipulatives to develop relationships and to build conceptual understanding (Teh et al, 2009, pp.208-214).

In contrast, M8 and CD reported rare reference to address prerequisite ideas or skills. The lessons often attempted to direct orientation of new concepts or procedure without reference to earlier work. M8 and CD often introduced terms and object components before students have experienced with the concepts. The material in 'M8' attempted a certain degree to clarify student's misconceptions about the concept of the cone and sphere. A figure of an oblique cone (similar to NSM2) was given in M8 with the heading 'Interesting Information' (p.154). Students were asked 'can you tell why it is not a right circular cone? Although this activity is helpful to build conceptual thinking of students and allow teachers identify students' ideas; it would be ideal if there were examples of hands-on experiences and manipulatives for the initial mastery of concept that would help students build new knowledge from their initial ideas (NCTM, 2000).

Procedural Fluency

Table 2 depicts that there is a large difference among the three textbooks in the design of strategies for students to attain the fluencies and procedural skills. The textbook "NSM2" included the highest number of the variety of problems and tasks (75%) based on procedural skills. Also, NSM2 contained an adequate number of problems for each topic that reinforce the concepts, skills, and processes taught. In contrast, the other two textbooks "M8" and "CD" presented very limited tasks (16.6% and 8.33% respectively). The textbooks "M8" and "CD" focused merely on limited worked examples and few exercise questions, and rarely embedded justifiable, flexible and coherent procedures to different problems. In NSM2, the exercises and examples have been purposefully arranged to aid students in progressing within and beyond each level. Some exercise questions have been marked with a '*' that require more thinking and involve more calculations (p.234).

There was a difference in quality and number of tasks among the textbooks. Regarding the quality and number of tasks, NSM2 provided the highest number that includes exercise questions, review exercises, and extension questions. Among other textbooks CD included very traditional and procedural problems at the end of chapter exercise without connections to mathematics sense-making, application, and procedural skills. As for as the variety of complexity of tasks is concerned, 'NSM2' provided well-designed problems that progressively assess students' concepts, build fluency and focused on the variety of skills, applications, and representations that lead to gain mathematical proficiency. Mary & Heller (1983) found that students face difficulties in solving complex problems due to the less incorporation of range of complex problems in textbooks. NCTM (2000) emphasised that mathematics textbooks must provide varied kind of problems to facilitate students' conceptual understanding and procedural fluency.

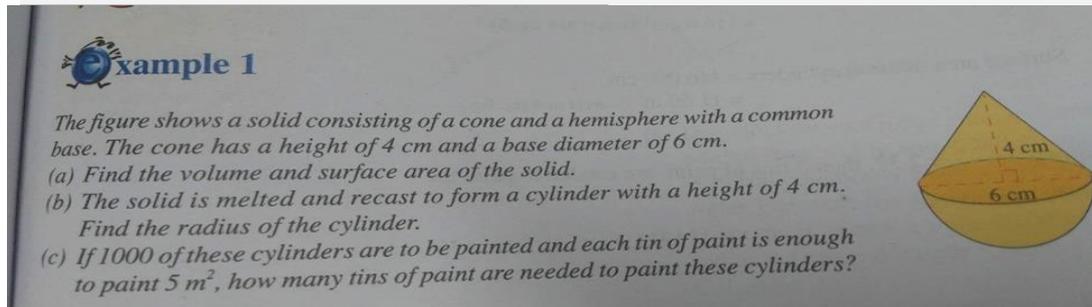


Figure 2. A sample example (verbal plus visual) from 'NSM2' that require students to use computational strategies on the basis of their conceptual understanding (Teh, Loh, Joesph, & Ivy, 2009, pp.234)

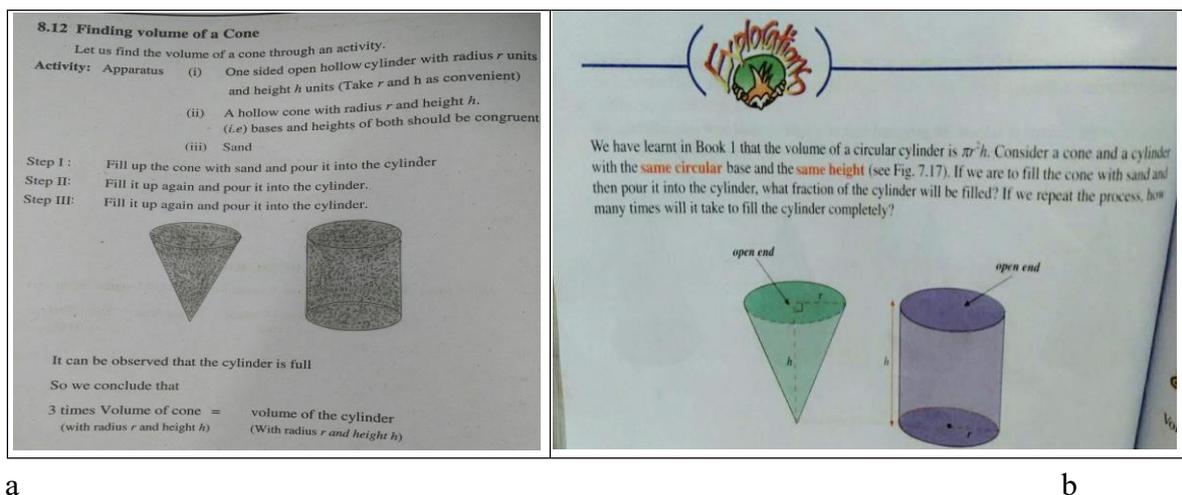
Figure 2 illustrates the close inter-relationship between procedural fluency and conceptual understanding. For example, the students can demonstrate high level of computational fluency, and at the same time, they exhibit a deep understanding of the relationship among different measures in geometry. Also, this problem entails students to use efficient mental computational strategies to carry out the problem, which in turn helps support the development of efficient written strategies (Bass, 2003). Whereas, examples and exercise questions in the textbooks "M8" and "CD" rely on just written computations that require following procedural steps without conceptual understanding. For such procedures, Hiebert & Lefevre (1986) claimed that "procedures learned with meaning are actually procedures and these procedures must be connected to conceptual knowledge" (p. 8). There was varied emphasis placed by all textbook series on verbal and visual problems. The NMS2 had equal focus on verbal, visual, and combined form (verbal plus visual) whereas M2 placed more emphasis on problems of verbal form and less focused on visual form of problems; by contrast, the textbook "CD" contained only verbal forms of problems.

Strategic Competence

Table 2 shows that NSM2 contained numerous activities which provided 58.33% alignment with indicators for the first goal of strategic competence. The textbook "M8" included 37.50% effective activities and strategies that support student's strategic competence in Geometry. Whereas, in the textbook "CD", we evidenced only 8.33% such tasks which can be helpful for students to achieve the criterion for the learning goal. Likewise, for the second goal, NSM2 fulfilled 41.67% criteria to addresses the key ideas of strategic competence. M8 contained 29.17% strategies that provided quality instructional support to meet the indicators. No evidence was found in the textbook "CD" that adhere indicators for the development of strategic competence in students.

NSM2 and M8 provided several activities that address learning goals and benchmark ideas of National curriculum about geometry. Also, NSM2 and M8 provided a variety of context and manipulative activities including explorative tasks, hand-on-activities and verbal statements.

Figure 3 shows two tasks from M8 and NSM2. The common purpose of these explorative activities in both textbooks is to involve students in investigation to find the volume of the cone but there is a considerable difference between tasks with regard to the development of strategic competence in students. In Fig 3(a) students have been given guided instruction to fill up the cone three times with sand and asked to pour it into the cylinder and then a conclusion is also drawn that three times volume of a cone is equal to the volume of the cylinder of equal base and height. This investigation confined the level of students' thinking and students are less likely to use their own problem-solving strategies to reach the conclusion. On the other hand, the exploration activity given in NSM2 created an inquiring and challenging environment for students. Two questions have been asked from students: 1) if we are to fill the cone with sand and pour it into the cylinder, what fraction of the cylinder will be filled? 2) If we repeat the process, how many times it will take to fill the cylinder completely? (p. 211). This is a hands-on material activity that may get students to think about the relationship between a cone and cylinder.



a b
Figure 3. Activity prompts in NSM2 and M8 used to engage students in mathematics investigation (p. 160a, 211b)

This is in line with the Montague (2005) findings that in such cases students can start thinking about the exploration task in groups and decide on how to set up the problem. Students can also experience hands-on exploration and may have a lot of fun filling these shapes up with sand and comparing. Then they proceed to analyse and generalise their finding until they can apply, explain and draw conclusions about the concept (Van & M, 1986). Council (2013) claimed that in such types of tasks students use integrated process skills such as writing hypotheses, designing and carrying out mathematical investigations, constructing data tables and graphs, and analysing relationships between geometric concepts.

It is worth noting that there was the greater difference that existed between routine and non-routine tasks in the selected textbook series. NSM2 provided considerably more tasks in exercises whose solutions required thinking skills and problem-solving heuristics and then

followed by M8 whereas, it was surprising that not a single non-routine problem was found in CD. Likewise, NSM2 provided students with more opportunities to experience problems with a variety of complexities, contextual tasks and open-ended problems as compared to other series of textbooks. Stein & Smith (1998) argued that open-ended problems and contextual tasks help students solve mathematical tasks having implicit procedures.

Summary and Conclusion

Textbooks serve as an important instructional tool for implementing curriculum goals for many teachers. Mathematics textbooks with quality instructional activities can play a central role in helping teachers develop mathematical proficiency in students. For many teachers and textbook reviewers, maybe because of the lack of research material on proficiency-based instructional analysis, it is challenging to identify the effectiveness of textbooks on students' achievement in mathematics learning goals. Also, the data from many international assessments like PISA and TIMSS reveal that the mathematics curriculum requires revision in its instructional approach. This procedure of analysis was used to identify to what extent the selected textbooks provided quality instructional strategies that address the important concept of geometry and most likely to develop geometric proficiency in students. In this study the quality of instructional activities of a three textbook series for middle-grade students were compared. Overall, from this analysis, NSM2 appeared to stand out over other textbooks. As table 2 indicates, the material in NSM2 rated 1.77 (59%) on overall average. According to performance grading criteria, NSM2 falls in the "Medium potential" category. Similarly, M8 scored an average 0.87(29%) and fall between the cumulative percentage of 20 to 50 and hence in the performance grading criteria M8 fall in "Low potential" category. CD rated only 0.15(5%) on average ratings on 0 to 3 scale. According to the performance grading criteria, CD falls in "No potential" category. When we looked at how instructional strategies in textbooks addressed and provided instructional support across three learning standards, we evidenced a significant gap among textbooks with regards to quality tasks and instructional approach. The textbook 'NSM2' placed the highest emphasis on conceptual understanding and procedural fluency (65.27%). The textbook 'M8' contained somehow a medium instruction for all learning standards. On the other hand, the textbook 'CD' did not seem to provide justifiable evidence for conceptual understanding, procedural fluency, and strategic competence. It was really surprising that the textbook "CD" failed to offer any single evidence of effective strategy for the learning standard "strategic competence". The result calls for urgent consideration from textbook authors to the issue of the wider instructional gap between textbook and the national curriculum.

NSM2 put emphasis not only procedural routines and practices; it also provided procedures with connections to inculcate underlying meaning of the mathematical idea. Moreover, NSM2 used colorful pictorial, visual, concrete and symbolic models as instructional representations to present important mathematical concepts. The national curriculum lays much emphasis on problem- solving strategies which we termed here as "Strategic Competence". The study



reveals that in all the textbooks especially in CD there needs to be much improvement to fill the gap between curriculum standards and the instructional approach embedded in the text. Finally, the textbook “NSM2” provided comparatively better instructional support in geometry and M8 provided a satisfactory job in some instructional areas in developing mathematical ideas. While the textbooks “CD” were identified as inconsistent and weak in their instructional support for students and teachers.



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