International Journal of Innovation, Creativity and Change. www.ijicc.net
Volume 5, Issue 3, 2019 Special Edition: *Science, Applied Science, Teaching and Education*

# Optimal Design of Herringbone Wavy Fin-and-Tube Condenser Based on the Entropy Generation Number

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## *Abstract*

This paper presents a mathematical model which conflates two heat exchanger design approaches – the ε-NTU (effectiveness-Number of Transfer Units) and the EGM (entropy generation minimization) – focusing on heat exchangers with uniform wall temperature for determining optimal design of herringbone wavy fin-and-tube condensers used in refrigeration system. Second law analysis on the herringbone wavy fin and tube condenser was conducted on the basis of correlation proposed by Hermes (2013) and the basis of empirical correlations for heat transfer and flow friction characteristics proposed by Wang, et al. (2002), in which the entropy generation rate was evaluated. An algebraic model which expresses the dimensionless rate of entropy generation as a function of the heat exchanger geometry (number of transfer units), the thermal-hydraulic characteristics (friction factor and Colburn j-factor), and the operating conditions (heat transfer duty, core velocity, surface temperature, and fluid properties) is derived. Results from the mathematical model show that heat transfer with finite temperature difference creates much more effects on entropy generation rate than viscous flow. The maximum design of heat transfer surface of 1.36 $m^2$ and minimum design of heat transfer surface of 0.87 $m^2$.

***Keywords:*** *Condenser, entropy generation number, second law of thermodynamics, number of transfer units*

## Introduction

The fin-and-tube heat exchangers are very widely used in refrigeration and air conditioning applications. The popularity of this type of heat exchangers arises from its low costs, it is quite compact, and it is highly reliability. The herringbone wavy fin surface is one of the most popular since it can lengthen the airflow and allow better mixing of the airflow inside the heat exchanger. Investigation of thermal-hydraulic performance of herringbone corrugated fin, based on commercially available samples, was carried out by Wang et al. (Wang, Fu, & Chang, 1997; Wang, Tsi, & Lu, 1998; Wang, Lin, Lee, & Chang, 1999; Wang, Jang, & Chiou, 1999; Wang, Hwang, & Lin, 2002), Chokeman & Wongwises (Chokeman & Wongwises, 2005), and Wongwises & Chokeman (Wongwises & Chokeman, 2005). All of this aims to systematically examine the effect of fin spacing, the number of tube rows, wave height, fin pattern, and fin thickness. Therefore, this heat exchanger is widely used as a condenser (Wang, et. Al, 2011; Wang, & Liaw, 2012).

The condenser is a heat exchanger with fairly uniform wall temperature and, if well-designed and highly efficient, can help in saving large amount of energy (Sagia & Paignigiannis, 2003). The design processes involves decision making concerning both geometrical (e.g. heat transfer surface and free flow passage) and operational parameters (e.g. flow rates and temperatures of the streams) aimed at accomplishing a certain heat transfer duty, $Q$, at the penalty of pumping power, $W_p$. However, these primary performance indicators ($Q$; $W_p$) respond similarly to changes in design parameters, i.e. heat transfer enhancement (desired effect) is usually followed by pumping power increases (undesirable effect). There are two well-established methods available for the thermal heat exchanger design, the log-mean temperature difference (LMTD) and the effectiveness/number of transfer units (ε-NTU) approach (Kays & London, 1984; Shah & Sekulic, 2003). The second has been preferred to the former for the sake of compact heat exchanger design as the effectiveness (ε), defined as the ratio between the actual heat transfer rate and the maximum amount that can be transferred, provides a 1st-law criterion to rank the heat exchanger performance, whereas the number of transfer units (NTU) compares the thermal size of the heat exchanger with its capacity of heating or cooling fluid. Furthermore, the ε-NTU approach avoids the cumbersome iterative solution required by the LMTD for outlet temperature calculations.

Nonetheless, neither ε-NTU or LMTD approaches are suitable to address the heat transfer or pumping power trade-off, which is the crux for a balanced heat exchanger design. A method devised for this purpose consists of counterbalancing the thermodynamic losses associated with irreversible heat transfer across a finite temperature difference with the irreversibilities associated with viscous fluid flow, in such a way that the heat exchanger thermal and hydraulic characteristics are expressed with regard to the same thermodynamic baseline, i.e. the rate of entropy generation. Such a method, named entropy generation minimization (EGM) (Bejan, 1996), has been widely used as performance evaluation criterion in geometric optimization of various types of heat exchangers (Grazzini & Gori, 1988; Yilmaz, Sara, & Karsli, 2001). However, the application of the EGM technique to heat exchanger design involves complex numerical simulation models (Hermes, Silva, & Castro, 2012; Mishra, Das, & Sarangi, 2009; Saechan & Wongwises, 2008) that do not provide a clear indication of how the heat exchanger design (i.e. number of transfer units) affects its performance (i.e. rate of entropy generation).

Bejan (Bejan, 1977) proposed an algebraic formulation for the optimum $4L/D_h$ ratio in case of nearly balanced counterflow heat exchangers operating with ideal gas streams, and discussed the influence of the remaining irreversibilities due to the unbalanced flow streams in counterflow arrangements. He also investigated the relationship between the dimensionless entropy generation and heat transfer effectiveness in cases of frictionless fluid flow, concluding that the minimum entropy generation takes place when $\varepsilon \rightarrow 1$ (Bejan, 1982). However, such an observation is restricted to cases where the irreversibilities due to pressure drop are negligible in comparison to those due to heat transfer with finite temperature difference. Hermes (Hermes, 2012) proposed an algebraic formulation which expresses the dimensionless rate of entropy generation as a function of the number of transfer units, the fluid properties, the thermal-hydraulic characteristics, and the operating conditions for heat exchanger with uniform temperature. An expression for the optimum heat exchanger effectiveness, based on the working conditions, heat exchanger geometry and fluid properties, was also presented. Hermes (Hermes, 2013) assessment the thermal-hydraulic design approach introduced by Hermes (Hermes, 2012) for designing condensers and evaporators for refrigeration system spanning from household to light commercial applications, which amounts ±10% of electrical energy consumed worldwide. The present paper combines the thermo-hydraulic design approach proposed by Hermes (Hermes, 2012, 2013) and the emperical correlations for heat transfer and flow friction proposed by Wang, et al (Wang, Hwang, & Lin, 2002) for designing herringbone wavy fin and tube condenser for refrigeration system based on the second law of thermodynamics in order to find an optimal design.

## Mathematical Model

In general, condenser is designed to provide a certain heat transfer duty ($Q$) subjected to air flow rate $(\dot{m})$ and face area ($A_f$) constraints. The design is usually aimed at minimizing the heat transfer area ($A_s$) and the pumping power ($W_p$), being both economic issues related to the saving energy (Yilmaz, Sara, & Karsli, 2001).

The mathematical model relies on the assumption that condenser can be regarded as heat exchanger with uniform wall temperature, as depicted in Fig. 1, and the model was formulated based on energy, entropy and momentum balances in the secondary fluid flow (usually air or water) assuming condenser as an even lump, as follows (Bejan, 1996):

$$Q = \dot{m}c_p(T_o - T_i) = \varepsilon\,\dot{m}c_p(T_s - T_i) \tag{1}$$

where $\dot{m}$ is mass flow rate, $T_i$, $T_o$, and $T_s$ are the inlet, outlet and surface temperature, respectively. $Q = hA_s(T_s - T_m)$ is heat transfer rate, $T_m$ is the mean flow temperature over the heat transfer area, $A_s$ and $\varepsilon$ is the heat exchanger effectiveness, calculated from (Kays & London, 1984):

$$\varepsilon = 1 - \exp(-\mathrm{NTU}) = \frac{T_o - T_i}{T_s - T_i} \tag{2}$$

where $\mathrm{NTU} = hA_s / \dot{m}c_p$ is the number of transfer units. The pressure drop can be calculated from (Kays & London, 1984):

$$\Delta P = P_i - P_o = f\,\frac{\rho u_c^2 A_s}{2\,A_c} \tag{3}$$

where $f$ is friction factor, $u_c$ is the velocity in the minimum flow passage, $A_c$, and the subscripts "i" and "o" refer to the heat exchanger inlet and outlet ports, respectively. Equation (1) and (3) can be linked to each other through the Gibbs relation, $Tds = dh - dp/\rho$,

$$T_m(s_o - s_i) = c_p(T_o - T_i) - \frac{(p_o - p_i)}{\rho} \tag{4}$$

where $T_m \approx (T_i + T_o)/2$ and the entropy variation, $s_o - s_i$ is calculated from the second law thermodynamics,

$$\dot{m}(s_o - s_i) = \frac{Q}{T_s} + \dot{S}_g \tag{5}$$

where the first term in the right-hand side account for the reversible entropi transport with heat ($Q/T_s$) whereas $\dot{S}_g$ is the irreversible entropy generation due to both the heat transfer with finite temperature difference and the viscous flow. Substituting equations (1), (3), and (5) into equation (4), it follow that:

$$N_s = \frac{\dot{S}_g}{\dot{m}c_p} = \frac{Q}{\dot{m}c_p}\left(\frac{T_s - T_m}{T_s T_m}\right) + \frac{f\,u_c^2}{2c_p T_m}\frac{A_s}{A_c} \tag{6}$$

where $N_s$ is the dimensionless rate of entropy generation.

Condenser is designed to provide a heat transfer duty subjected to flow rate and face area constraint, and also by Bejan [Bejan, 1997; Bejan, 1982] stated that the compact heat exchanger $T_s T_m \approx T_s^2$ and $A_s / A_c = \mathrm{NTU}\,\mathrm{Pr}^{2/3} / j$, where $j$ is the Colburn $j$-factor. Equation (6) can be re-written as follows [Hermes, 2012]:

$$N_s = \Theta^2 \mathrm{NTU}^{-1} + \frac{f}{2j} U^2 \,\mathrm{Pr}^{2/3}\,\mathrm{NTU} \tag{7}$$

where $U = u_c / \sqrt{c_p T_m}$ is a dimensionless core velocity, and $\Theta = (T_o - T_i)/T_s$ a dimensionless temperature difference with both $T_o$ and $T_i$ known from measurement. The first and second terms of the right-hand side of equation (7) stand for the dimensionless entropy generation rates associated with the heat transfer with finite temperature difference, $N_{s,\Delta T} = \Theta^2 \mathrm{NTU}^{-1}$ and the viscous flow, $N_{s,\Delta P} = (f/2j)\, U^2\, \mathrm{Pr}^{2/3}\, \mathrm{NTU}$, respectively.

The Colburn $j$-factor of air side herringbone wavy fin and tube heat exchangers cross-flow type expressed by Wang, et al [Wang, Hwang, & Lin, 2002] proposed the following correlation:

For $\mathrm{Re}_{Dc} < 1000$

$$j = 0.882\,\mathrm{Re}_{Dc}^{J1}\left(\frac{D_c}{D_h}\right)^{J2}\left(\frac{F_s}{P_t}\right)^{J3}\left(\frac{F_s}{D_c}\right)^{-1.58}(\tan\theta)^{-0.2} \tag{8}$$

where

$$J1 = 0.0045 - 0.491\,\mathrm{Re}_{Dc}^{-0.0316-0.017\ln(N\tan\theta)}\left(\frac{P_l}{P_t}\right)^{-0.109\ln(N\tan\theta)}\left(\frac{D_c}{D_h}\right)^{0.542+0.0471N}\left(\frac{F_s}{D_c}\right)^{0.984}\left(\frac{F_s}{P_t}\right)^{-0.349} \tag{9}$$

$$J2 = -2.72 + 6.84\tan\theta \tag{10}$$

$$J3 = 2.66\tan\theta \tag{11}$$

The correlation for friction factor is given as:

For $\mathrm{Re}_{Dc} < 1000$

$$f = 4.37\,\mathrm{Re}_{Dc}^{F1}\left(\frac{F_s}{D_h}\right)^{F2}\left(\frac{P_l}{P_t}\right)^{F3}\left(\frac{D_c}{D_h}\right)^{0.2054}N^{F4} \tag{12}$$

where

$$F1 = -0.574 - 0.137\left(\ln(\mathrm{Re}_{Dc}) - 5.26\right)^{0.245}\left(\frac{P_t}{D_c}\right)^{-0.765}\left(\frac{D_c}{D_h}\right)^{-0.243}\left(\frac{F_s}{D_h}\right)^{-0.474}(\tan\theta)^{-0.217}N^{0.035} \tag{13}$$

$$F2 = -3.0\tan\theta \tag{14}$$

$$F3 = -0.192N \tag{15}$$

$$F4 = -0.646\tan\theta \tag{16}$$

## Results and Discussion

Analyses were performed using experimental data of vapor compression refrigerator with a maximum heat load of the condenser $Q$ = 3000 W, and the values $\Theta = 0{,}06$ for which the evaporator testing in wet conditions. Mass flow rate and inlet air temperature of the condenser, $\dot{m}_a = 0{,}15$ kg/s and $T_i$ = 302K. The condenser geometry is similar to Kays and London surface 8.0-3/8T (Kays & London, 1984), whose empirical correlations for heat transfer and fluid flow characteristics proposed by Wang, et. al (Wang, Hwang, & Lin, 2002). The arrangement of the tubes is staggered with condenser dimensional geometry is presented in Fig. 1 and Table 1. In this work, effects of the parametric study on the design of condenser on entropy generation rate of the system is investigated.

The dimensionless entropy generation number ($N_s$), NTU and ε as function of the condensation heat load are depicted in Fig. 3. It is seen that $N_s$, NTU and ε are influenced remarkably by condensation heat load, where $N_s$ decreases with decreasing condensation heat load, while the NTU and ε increases with decreasing condensation heat load. Fig. 4 depicts effect of the surface temperature condenser ($T_s$) on $N_s$, NTU and ε. It is seen that $N_s$, NTU and ε are influenced remarkably by surface temperature condenser, where $N_s$ decreases with decreasing surface temperature condenser, while the NTU and ε increases with decreasing surface temperature condenser. The condensation heat load will greatly affect of the surface temperature condenser. In this case the value of the dimensionless entropy generation number associated with the heat transfer withfinite temperature difference is much greater than the dimensionless entropy generation number associated with viscous flow $(N_{s,\Delta T} >> N_{s,\Delta P})$. Fig. 3 and Fig. 4 are depicts that $N_s$ influenced remarkably by $N_{s,\Delta T}$ and not influenced remarkably by $N_{s,\Delta P}$ and also illusrates Eq. (7) with $f = 2.2j$ and $\Pr = 0.7282$, where different entropy generation minimum can be noted, very depending on $\Theta$ and $U$ plays a minor influence on $N_s$. $N_s$ towards a minimum when NTU and ε towards a maximum and and the value of $N_s$ in equation (7) is more influenced by NTU as compared to the friction factor *f* and *j*-Colburn factor.

Fig. 5 depicts the design process through Eqs. (2) and (7) for surface 8.0-3/8T, whose thermal-hydraulic characteristics for f and j through Eqs. (8) and (15), $\sigma = 0.534$ and $D_h = 3.63$ mm. The lines that cross each other represent the individual contributions of $\Delta T_m$ and $\Delta P$ to the global rate of entropy generation. The minimum $N_s$ is observed for values maximum NTU = 1.8326 and ε = 0.84. While the maximum $N_s$ is observed for values maximum NTU = 1,1712 and ε = 0.69.

Fig. 6 describes the optimum $4L/D_h$ and $N_s$ values for different NTU. It should be noted that $4L/D_h$ increase with NTU whereas $N_s$ decreases as fixed face areas, $A_f$, yield lower core velocities, $U$, for a fixed $\sigma$, thus requiring an improved heat transfer surface, $A_s$, to accomplish the constrained heat transfer duty. Since $N_s$ decreases as NTU increases, the design can be further improved by adjusting the face area, $A_f$, to accommodate an enhanced heat transfer surface, $A_s$. The analysis process led to a maximum heat transfer surface of 1.36 $m^2$ and a minimum heat transfer surface of 0.87 $m^2$. This, however, may affect not only the heat exchanger envelope, $A_f L$, but also its manufacturing cost, which represent additional constraints that should be accounted for in the design process.

## 4. Conclusion

From the study for optimal design of condenser used in refrigeration system based on the second law of thermodynamic, it can be concluded that heat transfer with finite temperature difference creates much more effects on entropy generation rate than viscous flow. $N_s$ towards a minimum when NTU and ε towards a maximum and the value of $N_s$ is more influenced by

NTU as compared to the friction factor *f* and *j*-Colburn factor. The maximum design of heat transfer surface of 1.36 $m^2$ and the minimum design of heat transfer surface of 0.87 $m^2$.

**Acknowledgment**

The author thanks the Ministry of Research, Tech., and Higher Education of the Republic of Indonesia and LPPM University of Nusa Cendana for supporting this research and paper through The Grant 2016.

**Table 1**. Detailed parameters geometri of the condensor

| $F_p$ (mm) | $D_c$ (mm) | $P_t$ (mm) | $P_l$ (mm) | $\delta_f$ (mm) | $N$ | $X_f$ (mm) | $P_d$ (mm) | $\theta$ |
|---|---|---|---|---|---|---|---|---|
| 3,5 | 10,5 | 25 | 23 | 0,11 | 2 | 2,5 | 1 | 20° |

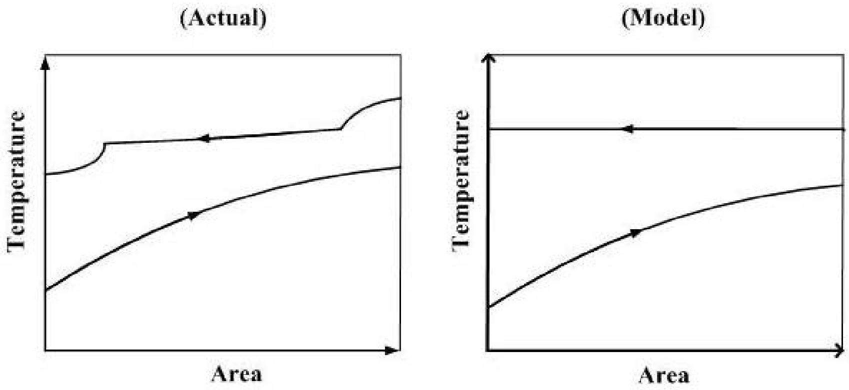


**Figure 1.** Temperature profil in the condenser.

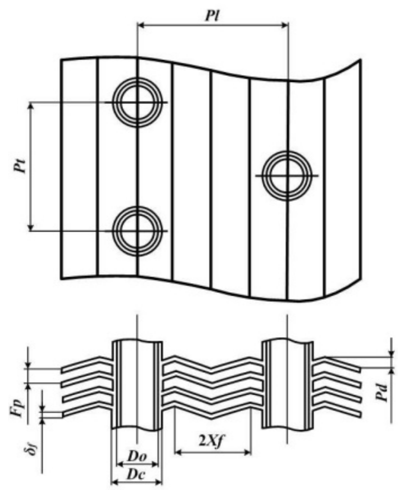


**Figure 2.** Schematic of geometric parameters.

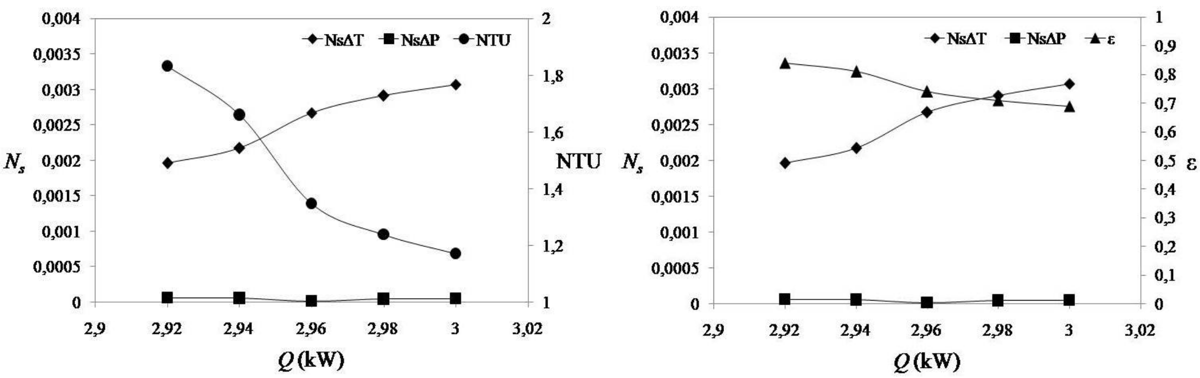


**Figure 3**. Effect of $Q$ on Ns, NTU and ε.

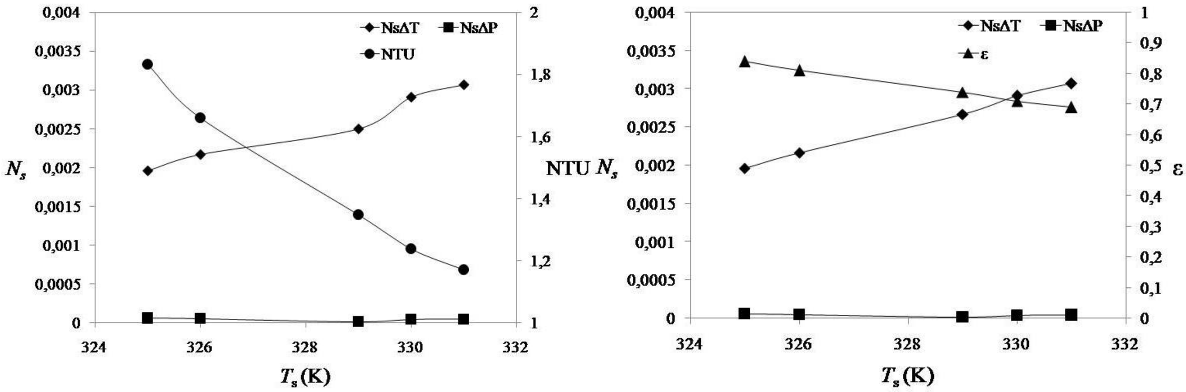


**Figure 4.** Effect of $T_s$ on $N_s$, NTU and ε.

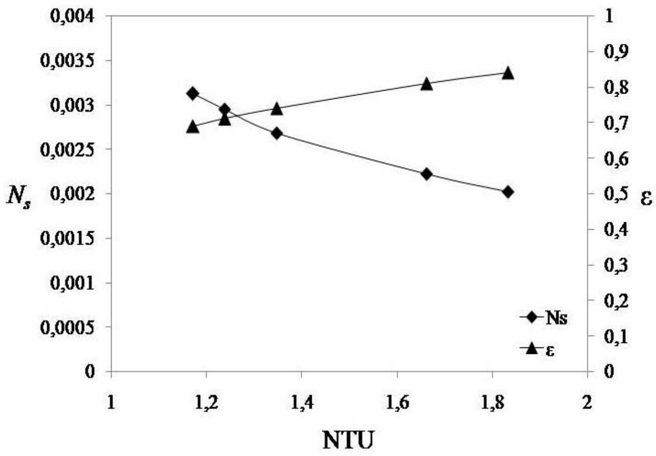


**Figure 5.** Maximum NTU and ε for the condenser.

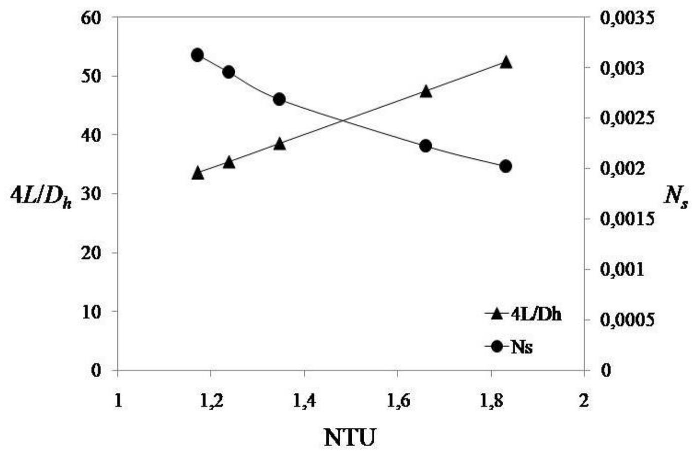


**Figure 6.** Optimum values of $4L/D_h$ and $N_s$ a function of NTU.

## References

Bejan, A. (1977). The concept of irreversibility in heat exchanger design: counterflow heat exchangers for gas-to-gas applications. *Journal of Heat Transfer*, *99*(3), 374–380.

Bejan, A. (1982). *Entropy generation through heat and fluid flow*. (pp. 118–134), John Wiley & Sons. New York.

Bejan, A. (1996). Entropy generation minimization: The new thermodynamics of finite-size devices and finite-time processes. *Journal of Applied Physics*, *79*(3), 1191–1218.

Chokeman, Y., & Wongwises, S. (2005). Effect of fin pattern on the air-side performance of herringbone wavy fin-and-tube heat exchangers. *Heat Mass Transfer*, *41*, 642–650.

Grazzini, G., & Gori, F. (1988), Entropy Parameters for Heat Exchanger Design. *International Journal of Heat and Mass Transfer*, *31*, 2547–2554.

Hermes, C. J. L. (2012). Conflation of ε-Ntu and EGM design methods for heat exchangers with uniform wall temperature. *International Journal of Heat and Mass Transfer*, *55*(13–14), 3812–3817.

Hermes, C. J. L. (2013). Thermodynamic design of condensers and evaporators: Formulation and applications. *International Journal of Refrigeration*, *36*(2), 633–640.

Hermes, C. J. L., e Silva Jr, W. de L., & de Castro, F. A. G. (2012). Thermal-hydraulic design of fan-supplied tube-fin condensers for refrigeration cassettes aimed at minimum entropy generation. *Applied Thermal Engineering*, *36*, 307–313.

Kays, W. M., & London, A. L. (1984). *Compact heat exchangers*.

Mishra, M., Das, P. K., & Sarangi, S. (2009). Second law based optimisation of crossflow plate-fin heat exchanger design using genetic algorithm. *Applied Thermal Engineering*, *29*(14–15), 2983–2989.

Saechan, P., & Wongwises, S. (2008). Optimal configuration of cross flow plate finned tube condenser based on the second law of thermodynamics. *International Journal of Thermal*

*Sciences*, *47*(11), 1473–1481.

Sagia-S. A., & Paignigiannis, N. (2003). Exergy losses in refrigeration systems. A study for performance comparisons in compressor and condenser. *International Journal of Energy Research*, *27*, 1067–1078.

Shah, R. K., & Sekulic, D. P. (2003). *Fundamentals of heat exchanger design*. John Wiley & Sons.

Wang, C-C., Fu, W-L., & Chang, Y-J. (1997). Heat transfer and friction characteristic of typical wavy fin-and-tube heat exchangers. *Exp. Therm. Fluid Sci. 14*, 174–186.

Wang, C-C., Tsi, Y-M., & Lu, D-C. (1998). A comprehensive study of convex-louver and wavy fin-and-tube heat exchangers. AAIA *J. Thermophys. Heat Transfer*, *12*, 423–430.

Wang, C-C., Lin, Y-T., Lee, C-J, & Chang, Y-J. (1999). An investigaation of wavy fin-and-tube heat exchangers; a contribution to databank. *Exp. Heat Transfer. 12*, 73–89.

Wang, C-C., Jang, J-Y., & Chiou, N-F. (1999). A heat transfer and friction correlation for wavy fin-and-tube heat exchangers, *International Journal of Heat and Mass Transfer*, *42*, 1919–1924.

Wang, C-C., Hwang, Y.-M., & Lin, Y.-T. (2002). Empirical correlations for heat transfer and flow friction characteristics of herringbone wavy fin-and-tube heat exchangers. *International Journal of Refrigeration*, *25*(5), 673–680.

Wang, C-C., Liaw, J-S., & Yang, B-C. (2011). Airside performance of herringbone wavy fin-and-tube heat exchangers - data with larger diameter tube. *International Journal of Heat and Mass Transfer*, *54*, 1024–1029.

Wang, C-C., & Liaw, J-S. (2012). Airside performance of herringbone wavy fin-and-tube heat exchangers under dehumidifying condition - data with larger diameter tube. *International Journal of Heat and Mass Transfer*, *55*, 3054–3060.

Wongwises, S., & Chokeman, Y. (2005). Effect of fin pitch and number of tube rows on the air side performance of herringbone wavy fin and tube heat exchangers. *Energ. Convers. Manage.*, *46*, 2216–2231.

Yilmaz, M., Sara, O. N., & Karsli, S. (2001). Performance evaluation criteria for heat exchangers based on second law analysis. *Exergy, an International Journal*, *1*(4), 278–294.