International Journal of Innovation, Creativity and Change. www.ijicc.net
Volume 5, Issue 3, 2019 Special Edition: *Science, Applied Science, Teaching and Education*

# Modulating the Multi-Depot Vehicle Routing Problem Model with Time Windows using Occasional Driver

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## *Abstract*

Distribution is the process of distributing goods from producers to consumers. This distribution process requires operational costs and costs when using vehicles to distribute goods. In the process of distribution, all producers must expect to be able to minimize costs. Therefore, the company must be able to find a solution to minimize the costs incurred. One such solution is by using the services of the driver occasionally. Occasional drivers, referred to here, are ordinary people, who are not from shipping companies, who have vehicles that are not fully used in everyday life; these vehicles can be used to deliver goods from producers to consumers. There are several problems that are often faced in the distribution of goods including the determination of routes that can minimize total costs. In determining the route, each producer has different constraints, such as the number of vehicles used, vehicle capacity, consumer demand, distance between consumers, and there are also cases where consumers want to be served according to the time windows they have. The problem of determining the optimal route can be solved by a model in optimization, namely Vehicle Routing Problem (VRP). This VRP is modelling in optimization that aims to optimize the ability to minimize distribution costs. This scientific work aims to formulate the problem of distribution in model multi-depot vehicle routing problem (MDVRP) that has a time limit by using the services of the driver occasionally to minimize total costs. The MDVRP settlement, with occasional drivers, has more than one depot that acts as a company or producer of a product. The results of this model formulation show that this model can be used to minimize distribution costs.

***Keywords:*** Distribution, multi-depot vehicle routing problem, occasional driver, optimization, time windows

## Introduction

The problem of distribution, to a production company, today is an important problem because to serve all existing customers, goods that have been produced and stored in several depots must be distributed well and evenly to the agents of the company in accordance with consumer needs. As a result, the right route must be determined by considering the distance travelled, the number and capacity of the vehicle used, the time needed for the trip, and other factors that can minimize operational costs (Meithania, 2014). In this scientific work, the problem of distributing bread to a number of customers was discussed. In the matter of bread distribution, in this case, the company does not only use company-owned vehicles but also the services of the occasional driver to deliver company property to each customer (Archetti C, Savelsbergh M, 2016). Each vehicle is not only tasked to deliver new bread to customers, but must also consider the time windows for each customer; when the vehicle must arrive at a customer and the time the vehicle must leave the customer. Each vehicle must depart and return to the depot (A., 2009). The VRP Model with the occasional driver that has been created by Archetti et al (Archetti C, Savelsbergh M, 2016) is only for one manufacturer's place, so it needs to be varied for two or more manufacturer's venues. The VRP itself has several variations (Wihartiko, Buono, & Silalahi, 2017). One of them Multi Depot vehicle routing problem (MDVRP), where there are several depot that act as a manufacturer or company as a producer of a product. It is assumed for this research that each depot has enough stock to provide all the products requested by all Consumers(Kumar & Panneerselvam, 2012). The distribution problems are included in Multi Depot MDVRP Vehicle Routing Problem with time windows using occasional driver services. The model aims to minimize the number of vehicles used and be able to minimize the costs incurred by the company with the optimal route. This scientific work is an application part of the previous research on the optimization and VRP Multi Depot that has been done by (Cordeau & Laporte, 2003; Making, Silalahi, & Bukhari, 2018; B P Silalahi & Dewi, 2014; Surekha & Sumathi, 2011; Winston & Goldberg, 2004)

## Review of the Library

### *Linear Programming*

Linear programming concerns maximising or minimizing the linear function by considering the constraints (Archetti & Speranza, 2012). Linear programming in its workmanship fulfils several criteria, among others, as follows: Aiming to maximize or minimize a linear function, this maximised or minimized function, is called an objective function. The values in the decision variables must meet the constraints; each constraint must be a linear equation or linear heracity. Restriction of sign depends on each variable. For any $x_i$, $x_i$ variable must be a not negative variable ($x_i \geq 0$) or an infinite variable.

### *Integer Linear Programming*

Integer linear programming is one of the problems in optimisation or minimizes of a linear function. Integer linear programming is often referred to as Integer programming, if some of its variables or any variable used in this integer programming are the not negative integers. Integer programming is also called pure integer programming, if all of the variables used are integers. Integer programming is also called mixed integer programming, if only a few variables are used or not all of the variables used are integers. While integer programming is also called 0-1 integer programming if all the variables used are variables are 0 or 1 (Archetti & Speranza, 2012).

### *Travelling Salesman Problem*

In the issue of distributing a product, there are several variations of distribution issues that may be possible, ranging from the simplest distribution issue of the Travelling Salesman Problem (TSP) to more complex issues which concern the development of the TSP. TSP is a condition in which a Salesman will depart from one city then visit, exactly once, a number of cities and, at the end of the Salesman's journey, will return to the hometown or depot. The purpose of the TSP is to determine the route that minimizes the distance/travel expenses (Hoffman, Padberg, & Rinaldi, 2013). Multiple Travelling Salesman Problem (M-TSP) is one of the variations of TSP, where there are as many m-salesmen visiting the entire city, but each city can only be visited by exactly one Salesman alone. Each salesman started from a depot and at the end of the journey also had to return to the depot. The purpose of M-TSP is to minimize the total distance of each route. M-TSP problem is also known as Vehicle Routing Problem (VRP), where a city has demand and each salesman or vehicle has a certain capacity. The Total requests in a route must not exceed the capacity of the vehicle through the route (Benavent & Martínez, 2011; Making et al., 2018)

### *Vehicle Routing Problem*

In a distribution of products from a depot/company, to several customers, there are several variations of problems, one of which is the problem of determining the distribution route or Vehicle Routing Problem (VRP). VRP is a matter of determining the vehicle route with the purpose of determining the minimum cost with the provisions, such as which vehicles used will depart from the depot and return to the depot after completion of distribution. Each vehicle will visit the customer exactly once. The total product carried by each vehicle does not exceed the carrying capacity of the vehicle. VRP can simply be described as follows (Bib Paruhum Silalahi, 2014)[20]:

1. The vehicle will depart from the depot to visit all consumers and should return again to the original depot.
2. Each customer can only be visited exactly once by one vehicle.
3. Each vehicle departing must carry the goods and the amount of goods is not more than or equal to the capacity of the vehicle used.
4. Resolve the problem to find a set of routes, in distributing goods from the depot location to the consumer location, at the minimum cost possible.

### *Multi Depot Vehicle Routing Problem*

The Multi Depot Vehicle Routing problem (MDVRP) is one variation of VRP, where there are several depots that act as a distributor of a product, and each depot is sufficient to provide all the goods requested by the consumer. The availability of the depot aims to minimize the distance between the consumer and the depot, so that the consumer can be served by a vehicle that comes from a depot close to it. The purpose of this MDVRP is to minimize the costs incurred by the company in the distribution process (Caric & Gold, 2008; Surekha & Sumathi, 2011)

### *Vehicle Routing Problem with Time Windows*

Vehicle routing problem with time windows (VRPTW) is a problem of determining vehicle routing by minimizing the cost to serve all consumers and meet the constraints of vehicle capacity and to operate within a time windows. Time windows on each consumer are defined as time lapse so that vehicles can start the service after the start time of the consumer and before end time of the consumer. Time Windows are useful to meet the demands of each customer who has their own service time, meaning that each customer has a service deadline in one period

(Kallehauge, Larsen, Madsen, & Solomon, 2005). If the vehicle comes early, to a consumer, then the vehicle must wait until the time the consumer can be served; the vehicle waiting is not subject to additional charges. There are two types of Time Windows on VRPTW, namely hard time windows and soft time windows. On the hard time Windows vehicles must arrive at the consumer before the end of consumer time. On soft time windows, vehicles can come after the consumer end time and incur an additional charge (Kang, Lee, Lee, & Lee, 2008; Kumar & Panneerselvam, 2012).

### *Vehicle Routing Problem with occasional driver*

The Vehicle routing problem with the occasional driver is a vehicle routing problem that uses vehicles belonging to occasional drivers in the distribution process. An 'occasional driver' refers to ordinary people who have vehicles and can be used to deliver goods from producers to consumers. The driver will occasionally be rewarded by the manufacturer when delivering the goods to the consumer. This allows the owner of the vehicle to not fully use his vehicle in everyday life to be able to make money from his vehicles (Archetti C, Savelsbergh M, 2016).

### *Multi Depot Vehicle Routing Problem (MDVRP) with Time Windows using the occasional driver service*

The Multi Depot Vehicle Routing problem (MDVRP) with Time windows using the occasional driver service is a form of development of a vehicle routing problem. In Multi Depot vehicle routing problems with an occasional driver, there is more than one depot that is engaged as a distributor, and the vehicles used in the distribution process consist of two types of vehicles; namely vehicle owned by the companies and vehicles belonging to occasional drivers.

## Materials and Method

## Formulation of the problem

The problem formulation stage is used to formulate the problem of distribution in the MDVRP model with the occasional driver by making objective functions. The objective function of the MDVRP model, with Time Windows using the Occasional Driver Services, is to minimize costs in distributing to two company premises and using two types of vehicles, namely company-owned vehicles and occasional driver-owned vehicles.

## Additional constraints

Additional constraints are used to complete the MDVRP model, with Time Windows using the Occasional Driver Services, aimed at creating optimal distribution routes. Addition of the constraints intended is to assume limitations in the formulation of distribution problems to create an MDVRP model with Time Windows using Occasional Driver Services.

## Completion of the model.

Completion of the MDVRP model, with Time Windows using Occasional Driver Services, is done using an integer linear programming method, which is the method branch and bound, which is one of the exact methods to find the optimal solution. In its completion, this model is divided into 5 cases. Each case uses the value of the costs of different rewards for the occasional driver. It aims to look at the use of occasional driver vehicles for each case.

## Electric consumption

Drawing conclusions are done to answer the problem in the main purpose of writing this scientific paper. Suggestions will be given for the development of writing this scientific paper.

**Results and discussion**

## The problem

Formulations made in this model aim to minimize costs in the distribution process for two depots that act as producers or companies and that use two types of vehicles, namely company-owned vehicles and occasional driver-owned vehicles. Each of these depots is assumed to be sufficient to meet every demand from all consumers.

*Objective function*

The function used in this study is an objective function that aims to minimize the distribution costs. In mathematical form it can be written as follows:

$$\min Z = \sum_{k \in K} P(k) + \sum_{k \in K} R(k) \qquad (1)$$

with

$$P(k) = \sum_{i,j \in I \cup J} c_{ij} x_{ijk} + z_k w_k \qquad (2)$$

representing the distribution costs incurred by the company when using the company's vehicle k, and

$$R(k) = \sum_{i,j \in I \cup J} f_{ij} x_{ijk} \qquad (3)$$

is the distribution cost that the company spends when using the vehicle k, driven by an occasionally driver. Constraints used in working on the model MDVRP with Time Windows using Occasional Driver Services include:

1. Each vehicle will depart from the depot and not all vehicles must be used.

$$\sum_{j\in J} x_{ijk} = w_k, \quad i \in I, \qquad k \in K. \tag{4}$$

2. Vehicles that have been used by a depot may not be used by another depot.

$$\sum_{j\in J} x_{ijk} = 0\ , \quad i \in I, \qquad k \notin K_i, \tag{5}$$

with $K_i$ is the set number of vehicles in depot *i*.

3. Vehicles used to deliver goods to consumers must return to the depot.

$$\sum_{j\in J} x_{jik} = w_k, \quad i \in I, \qquad k \in K. \tag{6}$$

4. Vehicles starting a journey from a depot may not return to different depots when the vehicle is running.

$$\sum_{j\in J} x_{jik} = 0\,, \qquad i \in I, \qquad k \in K_i, \tag{7}$$

with $K_i$ is the set of vehicles in depot *i*.

5. Every consumer will be served right once by a vehicle.

$$\sum_{k\in K} \sum_{i\in I\cup J} x_{ijk} = 1, \qquad j \in J, \tag{8}$$

$$\sum_{k\in K} \sum_{j\in I\cup J} x_{ijk} = 1, \qquad i \in J.$$

6. The number of requests from each consumer in a route that will be passed by a vehicle does not exceed the capacity of the vehicle.

$$\sum_{j\in J} d_j \sum_{i\in I\cup J} x_{ijk} \le Q_k, \qquad k \in K. \tag{9}$$

7. Avoid the occurrence of sub tour that is not feasible, meaning that in a sub tour, if the vehicle that runs from the consumer i goes to the consumer j, then the vehicle may not return to the consumer i.

$$u_{ik} - u_{jk} + Nx_{ijk} \leq N - 1, \quad i, j \in J, \quad k \in K. \tag{10}$$

8. Continuity of the route, meaning that if the vehicle visits a consumer, the vehicle must eventually leave the consumer to go to another consumer.

$$\sum_{\substack{i \in I \cup J \\ l \neq i}} x_{ilk} - \sum_{\substack{j \in I \cup J \\ l \neq j}} x_{ljk} = 0, \ l \in J, \quad k \in K. \tag{11}$$

9. No consumer is visited by a vehicle that is not used by the depot.

$$x_{ijk} \leq w_k, \quad i, j \in I \cup J, \quad k \in K \tag{12}$$

10. There are no vehicles that run from one consumer to the same consumer.

$$x_{ijk} = 0, \quad i = j, \quad i, j \in J, \quad k \in K \tag{13}$$

11. There are no vehicles running from the depot to the depot.

$$x_{iik} = 0, \quad i \in I, \quad k \in K \tag{14}$$

12. Constraints that show the relationship between distance, speed, and travel time of transport vehicles, where the amount of travel time is affected by the distance between consumers and vehicle speed:

$$t_{ijk} = \frac{b_{ij}}{v_k}, i, j, k, i \neq j. \tag{15}$$

13. The service time for a vehicle going to *j* from *i* cannot arrive at *j* before $s_{ik} + t_{ij}$ if $x_{ijk} > 0$ then $y_{ik} + t_{ij} \leq y_{jk}$, or in other words, must be less than or equal to service time at the consumer j. The linear form is:

$$y_{ik} + s_i + t_{ij} - M_{ij}(1 - x_{ijk}) \leq y_{jk}, \forall i, j \in N, \forall k \in K, \tag{16}$$

with $M_{ij}$ is a large constant that is not less than maximum value of

$l_i + t_{ij} - e_i \,; (i,j) \in A.$

Constraints that state the service time in each consumer meets the time windows, meaning that the customer service time is within the time interval determined by the consumer *i*:

$$e_i \leq y_{ik,} \qquad \forall i,k,i \geq 1$$

$$y_{ik} + s_i \leq l_i \quad \forall i,k,i \geq 1,$$

$$y_{ik} \geq 0, \qquad \forall\ i,k,i \neq 1 \qquad (17).$$

The auxiliary variables $u_{ik}$ and $u_{jk}$ are positive

$$u_{ik} \geq 0, \qquad i \in J, \qquad k \in K$$

$$u_{jk} \geq 0, \qquad j \in J, \qquad k \in K$$

Variables decision

$$x_{ijk} = \begin{cases} 1, & \text{if } j \text{ index is visited after index i by vehicle } k \\ 0, & \text{other than that} \end{cases}$$

$$w_k = \begin{cases} 1, \text{if vehicle } k \text{ is used to distribute goods} \\ 0, \text{other than that} \end{cases}$$

with:

$u_{ik}$ = additional variable for constraints on eliminating sub tour for consumers *i*

$u_{jk}$ = additional variable for constraints *j*.

$v_k$ = vehicle speed *k*.

$b_{ij}$ = distance between consumer *i* to consumer *j* (km)

$t_{ijk}$ = travel time from consumer *i* to consumer *j* with vehicle *k*.

$s_i$ = duration of consumer service *i*

$y_{ik}$ = time served by consumers *i*

$[e_i, l_i]$ = *time windows* which shows the fastest time and the longest time in serving consumers $i$

M = positive constant whose value is relatively large (10000)

$I$ = set of all depots.

$J$ = set of all consumers

$K$ = set of vehicle.

$N$ = consumer set and depot.

$d_j$ = demand from consumer $j$.

$Q_k$ = vehicle capacity $k$ (30 *crate)*

Cost of travelling from the index i to index j obtained by multiplying the distance from index i to index j with the fixed cost of distribution (the cost of compensation for the driver of the company-owned vehicle k). Travel costs from index i to index j are obtained by multiplying the distance from index i to index j with the cost of the reward given to the occasional driver.

*Work on the model*

Completion of the MDVRP model, with Time Windows using the Occasional Driver Services, is done by using bread production distribution data (A., 2009). The data can be seen in Table 1 and Table 2.

Table 1 shows the distance between consumers, as well as the distance between consumers and depots. This distance is written in kilometres (km). Index 1 and index 2 are indices for depot 1 and depot 2 while indexes 3 through index 10 are consecutive consumer indices. MDVRP model, with Time Windows, uses formulated Driver Services using the data of distance between consumer locations, number of consumer requests, number of vehicles, length of service and the start and end time of consumers served in the distribution of bread production. Data that has been implemented will form the optimal route and Time Window that applies to each consumer.

In solving this model, the following assumptions are used:

1. Delivery time for each consumer can be done at any time between 07.00-16.00 so that Time Windows for all consumers are [0.560].
2. The number of vehicles used is 8 vehicles. 4 company-owned vehicles denoted by 1 (speed of 0.5 km / minute), 2 (speed of 0.5 km / minute), 5 (speed of 0.7 km / minute) and (speed of 0.7 km / minute), and 4 vehicles belonging to the occasional driver; notified with 3 (speed 0.5 km / minute), 4 (speed 0.7 km / minute), 7 (speed 1.2 km / minute) and 8 (speed 1.3 km / minute) The
3. Capacity of each vehicle is 30 crates.
4. The distribution costs for company-owned vehicles are 5500 units of money/km.
5. The usage fee for the company's driver is 50000 units of monetary value.

Table 2 can be seen as consumer demand, servant time and the Time Windows that applies to each consumer. Table 2, explains that each customer served has a number of requests for goods that must be delivered to their respective destinations. The length of time needed to serve and Time Windows that applies to each consumer is different (A., 2009).

This model is solved by integer linear programming and uses the help of software LINGO 11.0. The completion of this model is carried out in 3 cases, by changing the value of the costs of the rewards given to the occasional driver for each case. While the other assumptions are the same.

***Results of determining vehicle routes and times windows effective***

This section describes the results of implementing the MDVRP model with Time Windows using Occasional Driver Services. The ultimate goal of this research is to find a route that minimizes the distribution costs incurred by the company when the vehicle delivers goods, and where the vehicle can adjust the time when it has to serve consumers. Furthermore, in this problem, three work cases were used to obtain a solution. Completion of the solution was carried out with the company providing a fixed cost of 50000 for vehicle users and a fixed cost of 5500 per trip for company-owned vehicles. These fees apply equally to each case, and the company also rewards the occasional driver whose nominal reward varies in each case of work. In the first case, the company gives a nominal reward of 5500. The second case, the company raises the nominal compensation by 10000 and the third case, the company raises the reward by 20000.

Table 3 describes the results obtained from each case. For example in case 1, out of the 8 available vehicles, 4 optimal vehicles were used, namely vehicles 1, 2, 3 and 4. Route 1 vehicles were 1-3-1. Vehicle routes 2 were 2-8-10-2, vehicle routes 3 were 1-6-5-4-1 and the vehicle route 4 is 2-9-7-2. The optimal number of vehicles used is 2 company-owned vehicles and 2 occasional driver-owned vehicles and these four vehicles carry out distribution activities from depot 1. The total cost incurred by the company is 783760. This explanation applies to cases and routes in each subsequent case.

The results of the MDVRP model with Time Windows using the driver service occasionally indicates that the routes traveled by vehicles belonging to companies and vehicles belonging to the occasional driver are the optimal route, in the sense that the route is a route The most concise distribution process to consumers who can minimize costs in the distribution process. Length of service and time Windows has been known, while the duration of travel and consumer time began to be served can be presented on table 4.

The results of the research that has been obtained can be known when the time the consumer must be served by each vehicle in operation. For example, the explanation in Table 4 for case 1, in vehicle 2 the route formed is 2-8-10-2. From consumer 2 to consumer 8, it takes 36 minutes to travel; while consumers start serving in the 36th minute, the waiting time is 0. Knowing the service time is 37 minutes, then the total time needed to travel from consumer 2 to consumer 8 plus long service to consumers 8 is 73. Then the vehicle leaves for consumer 10 for 7.68 minutes, then the vehicle arrives at 80.68 minutes, and the consumer must be served in the 80.68 minutes then the waiting time for the vehicle before serving the customer 7 is 0 minutes, and service duration for 8 minutes, the total time needed is 88.68 minutes where the total time is obtained from the amount of travel time, service time and length of service in consumer 8 and consumer 10. Furthermore, after doing service at the consumer 8 and consumer 10, the vehicle travels back to the depot which takes 26.88 minutes, then the vehicle arrives at 107.56 minutes. This explanation also applies to routes for all three cases.

The schedule of trips and services produced in the implementation of the model is not continuous, meaning that there is a lag time as the tolerance of the departure of a vehicle. The tolerance time for the departure of the vehicle from the expiration of the service time to the agent until the start of the service time of the agent is further reduced by the length of the trip to the next agent. After finishing distributing goods to agents, the vehicle must return to the depot. When a vehicle arrives at a depot is not specified. But because the depot had time windows, the maximum limit of the vehicle must arrive at the depot in accordance with the deadline of Time Windows. The execution of this model also provides information in addition to the company using its own vehicles in the distribution process. The company also uses a vehicle owned by the occasional driver, once in a while, for some of these distributions in order to minimize the total costs in the distribution process. The concept of collaboration between company-owned vehicles and occasional driver-owned vehicles in the distribution process is very influential in minimizing the total cost of distribution.

Total costs incurred by the company in the model, will be minimal when companies use vehicles owned by the occasional driver with low returns. This is because, where the company distributes with its own vehicles, the company must prepare a fixed cost vehicle. Conversely, if the company uses the occasional driver's vehicle, the company simply prepares the cost of compensation to the occasional driver, which is calculated by distance/km when the driver's vehicle visits the consumer. In addition, driver services are sometimes used when needed, for example when the optimal route is formed that exceeds the number of available vehicles, the company uses the services of the occasional driver even though the actual capacity of the vehicle owned by the company can meet all consumer requests.

The nominal value of the reward is one reason to minimize the total costs incurred by the company in distribution when the rewards offered to drivers are large; the company uses more company-owned vehicles than occasional drivers-owned vehicles. Conversely, if the rewards offered to drivers are low, then the vehicle that is most used in the distribution process is the occasional driver's vehicle.

## Conclusion

The problem of determining the optimal route in distribution can be solved by the model vehicle routing problem, with Time Windows with the occasional driver, and can be applied to distribution problems. A Time Window is the time limit that is owned by each consumer to be served so that, in addition to being able to know the optimal route in distribution, each vehicle can also adjust the time when consumers can be served. In the application of this model three cases of work were carried out by giving different compensation costs in each case. Three completed cases can minimize the total costs incurred by the company in distribution by determining optimal routes.

**Table 1**. Distance between the depot and consumer locations (km)

| **Index** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00 | 9.24 | 11.52 | 18.24 | 14.16 | 12.48 | 13.08 | 18.00 | 13.92 | 13.44 |
| 2 | 9.24 | 0.00 | 8.88 | 9.84 | 9.24 | 10.92 | 6.84 | 6.72 | 2.88 | 2.88 |
| 3 | 11.52 | 8.88 | 0.00 | 6.72 | 2.64 | 1.56 | 22.16 | 6.00 | 4.32 | 4.32 |
| 4 | 18.24 | 9.84 | 6.72 | 0.00 | 4.32 | 8.28 | 8.88 | 10.44 | 6.60 | 6.60 |
| 5 | 14.16 | 9.24 | 2.64 | 4.32 | 0.00 | 1.80 | 1.56 | 4.92 | 4.56 | 4.32 |
| 6 | 12.48 | 10.92 | 1.56 | 8.28 | 1.80 | 0.00 | 0.60 | 4.44 | 2.76 | 2.76 |
| 7 | 13.08 | 6.84 | 2.16 | 8.88 | 1.56 | 0.60 | 0.00 | 3.84 | 2.16 | 2.16 |
| 8 | 18.00 | 6.72 | 6.00 | 10.44 | 4.92 | 4.44 | 3.84 | 0.00 | 4.08 | 3.84 |
| 9 | 13.92 | 2.88 | 4.32 | 6.60 | 4.56 | 2.76 | 2.16 | 4.08 | 0.00 | 0.24 |
| 10 | 13.44 | 2.88 | 4.32 | 6.60 | 4.32 | 2.76 | 2.16 | 3.84 | 0.24 | 0.00 |

**Table 2.** Distance between consumers

| ***Nod e*** | **Number of request *(crate)*** | **Service time (minutes)** | **Start and end times $[e_i,l_i]$** | **timewindows minutes to)** |
|---|---|---|---|---|
| 1 | - | - | [0,0] | - |
| 2 | - | - | [0,0] | - |
| 3 | 17 | 47 | [16,158] | 07.16-09.38 |
| 4 | 3 | 50 | [27,164] | 07.27-09.44 |

| | | | | |
|---|---|---|---|---|
| 5 | 7 | 18 | [40,131] | 07.40-0911 |
| 6 | 17 | 8 | [11,102] | 07.11-08.42 |
| 7 | 8 | 60 | [30,180] | 07.30-10.00 |
| 8 | 12 | 37 | [34,134] | 07.34-09.14 |
| 9 | 19 | 91 | [6,177] | 07.06-09.57 |
| 10 | 17 | 8 | [23,130] | 07.23-09.10 |

**Table 3**. Vehicle, the optimal route formed, the accumulation of distance and cargo and optimum costs obtained in each case with the compensation of each case is 5500, 10000 and 25000.

| **Case 1** | | | | |
|---|---|---|---|---|
| Vehicles | Routes formed | Accumulated mileage (km) | Accumulated distance travel (km) | Accumulated charge (Rp) |
| 1 | 1-3-1 | 11,52 | 17 | 176720 |
| 2 | 2-8-10-2 | 60.5 | 29 | 244040 |
| 3 | 1-6-5-4-1 | 61.8 | 27 | 202620 |
| 4 | 2-9-7-2 | 89.8 | 27 | 160380 |
| | | | | 783760 |
| **Case 2** | | | | |
| Vehicles | Routes formed | Accumulated mileage (km) | Accumulated distance travel (km) | Accumulated charge (Rp) |
| 1 | 1-8-10-1 | 60.5 | 29 | 244040 |
| 2 | 2-6-5-4-2 | 61.8 | 27 | 252620 |
| 3 | 1-3-1 | 11.52 | 17 | 230400 |

| | | | | |
|---|---|---|---|---|
| 4 | 2-9-7-2 | 89.9 | 27 | 291600 |
| Jumlah | | | | 1018660 |

**Case 3**

| Vehicles | Routes formed | Accumulated mileage (km) | Accumulated distance travel (km) | Accumulated charge (Rp) |
|---|---|---|---|---|
| 1 | 1-6-5-4-1 | 61.8 | 27 | 252620 |
| 2 | 2-8-10-2 | 60.5 | 29 | 244040 |
| 3 | 1-3-1 | 11.52 | 17 | 576000 |
| 4 | 2-9-7-2 | 89.8 | 27 | 729000 |
| Jumlah | | | | 1801824 |

**Table 4.** Windows Time that applies to each case

**Case 1**

| Vehicle | Route | Time starts to be served $t_{ijk}$ | Length of travel | Waiting for time (minutes) | Length service $s_{ik}$ (minutes) | *Time Windows* $[e_i, l_i]$ |
|---|---|---|---|---|---|---|
| 1 | 1-3 | 23.04 | 75.64 | 52.6 | 47 | 07.16-09.38 |
| | 3-1 | 23.04 | - | - | - | - |
| 2 | 2-8 | 36.00 | 36.00 | 0 | 37 | 07.34-09.14 |
| | 8-10 | 7.68 | 80.68 | 0 | 8 | 07.23-09.10 |
| | 10-2 | 26.88 | - | - | - | - |
| 3 | 1-6 | 24.96 | 73,32 | 48,36 | 8 | 07.11-08.42 |
| | 6-5 | 3.60 | 87.36 | 2,44 | 18 | 07.40-09.11 |
| | 5-4 | 6.64 | 114 | 2 | 50 | 07.27-09.44 |
| | 4-1 | 36.48 | - | | - | |

| | | | | | | |
|---|---|---|---|---|---|---|
| 4 | 2-9 | 19.89 | 19.89 | 0 | 91 | 07.34-09.14 |
| | 9-7 | 3.08 | 120 | 9.11 | 60 | 07.30-10.00 |
| | 7-1 | 18.68 | - | - | - | - |
| | | | **Case 2** | | | |
| Vehicle | Route | Time starts to be served $t_{ijk}$ | Length of travel | Waiting for time (minutes) | Length service $s_{ik}$ (minutes) | *Time Windows* $[e_i, l_i]$ |
| 1 | 1-8 | 36 | 36 | 0 | 37 | 07.34-07.14 |
| | 8-10 | 7,68 | 80,68 | 0 | 8 | 07.23-09.10 |
| | 10-1 | 26,88 | - | - | - | - |
| 2 | 2-6 | 21,84 | 24,96 | 3,12 | 8 | 07.11-08.42 |
| | 6-5 | 3,60 | 40 | 3,44 | 18 | 07.40-09.11 |
| | 5-4 | 8,64 | 114 | 47,36 | 50 | 07.27-09.44 |
| | 4-2 | 36,48 | - | - | - | - |
| 3 | 1-3 | 23,04 | 111 | 87,96 | 47 | 07.16-09.38 |
| | 3-1 | 23,04 | - | - | - | - |
| 4 | 2-9 | 19,88 | 19.88 | 0 | 91 | 07.34-09.14 |
| | 9-7 | 3,08 | 117,57 | 3,61 | 60 | 07.30-10.00 |
| | 7-1 | 18,68 | - | - | - | - |
| | | | **Case 3** | | | |
| Vehicle | Route | Time starts to be served $t_{ijk}$ | Length of travel | Waiting for time (minutes) | Length service $s_{ik}$ (minutes) | *Time Windows* $[e_i, l_i]$ |
| 1 | 1-6 | 24,96 | 24,96 | 0 | 8 | 07.11-08.42 |
| | 6-5 | 3.60 | 87,36 | 21,8 | 18 | 07.40-09.11 |
| | 5-4 | 8,64 | 114 | 0 | 50 | 07.27-09.44 |

| | | | | | | |
|---|---|---|---|---|---|---|
| | 4-1 | 36,48 | - | - | - | 07.13-09.40 |
| 2 | 2-8 | 36 | 36 | 0 | 37 | 07.40-09.11 |
| | 8-10 | 7,68 | 80,68 | 0 | 8 | 07.23-09.10 |
| | 10-2 | 5,76 | - | - | - | - |
| 3 | 1-3 | 23,04 | 23,04 | 0 | 47 | 07.16-09.38 |
| | 3-1 | 23,04 | - | - | - | - |
| 4 | 2-9 | 19,88 | 19,88 | 0 | 91 | 07.34-09.14 |
| | 9-7 | 3,08 | 120 | 6,04 | 60 | 07.30-10.00 |
| | 7-1 | 18,68 | - | - | - | - |

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