International Journal of Innovation, Creativity and Change. www.ijicc.net
Volume 10, Issue 2, 2019

Mathematical Communication of Prospective Teachers in Mathematics Learning in Senior High School

D Kurniawan[a*], I Yuwono[b], E B Irawan[c], H Susanto[d], Subanji[e], Susiswo[f],
[a]Departemen Pendidikan Matematika, Universitas Siliwangi, Jl. Siliwangi No. 24, Tasikmalaya, 46115, Indonesia, [b,c,d,e,f]Departemen Pendidikan Matematika, Universitas Negeri Malang, Jl. Semarang No 5, Malang, 65145, Indonesia, Email: [a*]diankurniawan@unsil.ac.id

This study aims to describe the mathematical communication of prospective teachers in learning mathematics. The research was conducted in SMAN 1 Pamekasan. The subject of this research is the students of PLP from the Mathematics Education Department of Madura University. The mathematics prospective teacher's learning is using the concept of function limits and the derivative of functions in problem-solving (calculus). This is descriptive qualitative research conducted by interviews, field notes, and video recordings. The results of this study found that prospective teachers were able to represent and analyse the situation and mathematical structure using algebraic symbols and using various symbolic representations, provide understanding of numbers, how to represent numbers, representations between numbers, and number systems, train their students to understand the meaning of operations and how they relate to each other and use good language when explaining the subject matter so as to adapt to students' level of understanding so that symbols, procedures, and strategies can be well understood. Mathematical communication of prospective teachers in learning mathematics can be applied in other aspects of the wider world.

Key words: *Mathematics, Teaching, High School*

Introduction

The knowledge of teachers in mathematics subjects needs to be improved through research, in order to develop the knowledge of mathematics teachers on the aspects of teaching and learning. Research is carried out in support of changes in the conception of teachers, teachers' beliefs about their profession, knowledge of their students, curriculum or teaching strategies used, subjects and information submitted by teachers, so that teachers need to prepare the material beforehand so that the delivery of teaching facts is of an acceptable standard.

Köğce (2015) explains that in high school, there must be substantial growth in students' ability to develop logical thinking chains, express themselves coherently and clearly, listen to other people's ideas, and think of their audience as they write or speak. The relationships that students want to express symbolically and with graphics, as well as notations and representations to express it, must become more sophisticated (Köğce, 2015). For example, it is a generalisation of context-bound thought in context-free thinking (abstract). Griffith (2014) explained it is during this transition period that students face challenges, since there may not be a real and perceived representation directly to advanced mathematical concepts (Griffith and Groulx, 2014).

Students should also be good collaborators and work effectively with others. Ball (1989) points out: "Basic notes and tricks, and definitions do not find meaningful understanding." Basic and secondary education students have difficulty remembering ideas and procedures, so that many do not understand the conceptual concepts of the mathematics they have learned and when delivered (Ball and Diarmid, 1989). Therefore, teachers should master the aspects of mathematical knowledge well; conceptual knowledge, the mastery of definitions, conventions (changes), and procedures finding patterns, conjectures, justifying statements and proving solutions, then seeking generalisations.

Güçler (2014) defines mathematical communication as a discourse that can be distinguished by the use of words, visual mediators, routines, and supported narratives (Güçler, 2014). The purpose of words refers to students who use mathematical words in their discourse. The visual mediator refers to all visible objects that are created and acted upon for mathematical communication. Routines refer to the set of repetitive meta-level rules in the participants' actions as they reinforce their narrative of mathematics. Supported narrations refer to a collection of sayings about mathematical objects and their relationships that are regarded as true participants (e.g. definition, axioms, and theorems) given their use of words, visual mediators, and routines.

NCTM argues that the mathematical communication standard for classes 9-12 is: regulating and consolidating its mathematical thinking through communication; communicating their

mathematical thinking coherently and clearly with peers, teachers, and others; analysing and evaluating other people's math and strategy thinking; using mathematical language to express mathematical ideas appropriately (NCTM, 2000). Canrinus (2011) explains the professional identity of teachers in this regard relating to sustained interpretation and interaction in the context of learning, teachers, commitment to work, changes in confidence and motivation (Canrinus et al., 2011). Learning practices provide useful opportunities to support the growth and development of math or prospective teachers (Amador, 2017). NCTM explains that high school teachers can help students use oral communication to learn and share math by creating a climate in which all students feel safe in expressing opinions, guesses and explanations (Miles et al., 2014). Olteanu (2015) illustrates five teacher approaches to grow and develop as teachers: (1) teachers are meant to develop abilities in the subject matter taught; (2) master learning strategy; (3) teachers are skilled in delivering the subject matter; (4) effective in teaching; and (5) effective in facilitating student learning (Olteanu, 2015).

Learning is undertaken in order to facilitate students to build new knowledge from previous experience and knowledge. Knowledge is built on regular learning; Pourdavood (2015) argues that "when students are given what they are given an opportunity to learn," the transfer of knowledge provided by the teacher, to gain new knowledge through the learning activities undertaken, and delivered either through the assignment, training, and subsequent learning then they will apply prior knowledge (Pourdavood and Wachira, 2015). Ng'eno (2016) affirms that learning is an active process that involves discussion and allows students to reach their own conclusion goals (Ng'eno and Chesimet, 2016). Büşra (2016) suggests a mathematical communication process evaluated according to individual relationships with each other in a classroom environment, observed observational data and consequently three types appear: a communication process conducted by students and teachers, Communication process of students appear each other, and the process of communication that students carry out themselves (Büşra and Ali, 2016).

Mathematical objects are used in written communication, and verbal descriptions of verbal communication. The results of observation of researchers when guiding Field Activity Practices (PLP) for prospective Teachers at the Universitas Siliwangi Tasikmalaya showed one mathematical object in mathematics learning in the form of procedural skills in learning mathematics still has shortcomings, especially because Prospective Teachers often use algorithm procedures with incomplete meaning steps - the process of solving the problem often uses a less raw way quickly. This is in line with the National Assessment of Educational Progress that students are capable of performing routine arithmetic calculations, but many have difficulty with reasonably complex procedures and reasoning (Ball and Diarmid, 1989).

Table 1: Mathematical Communication Indicators, adapted from NCTM (Miles et al., 2014).

No	Mathematical Communication Indicators	Example
1	Understanding numbers, how to represent numbers, relationships between numbers, and number systems.	Avogadro Numbers (6.02×10^{23}), $ 1 billion, $ 1 trillion, 1.05168475E-12 as a very small number given in scientific notation, 6.66666667 as an approximate result of dividing 20 by 3, and ERROR in response to invalid operations.
2	Understand the meaning of operations and how those operations relate to each other.	Given a point with coordinates a, b, c, d, e, f, g, and h, as shown, Which point is closest to ab? To c? to $\frac{1}{f}$? to e? to h? Explain your reasons.
3	Calculate fluently and make a reasonable estimate.	$\sqrt[3]{49} =$ 16.3333333. There is an error because the cube root 49 should be between 3 and 4, and $16 \cdot 16 \cdot 16$ much larger than 49. In order for the calculator to calculate the cube root, the student will need to have entered 49 ^ (1/3) or $49^{\frac{1}{3}}$.
4	Represents and analyses mathematical situations and structures using algebraic symbols	$f(x) = \frac{2x^2 + 11x + 6}{x - 2}$ Can be rewritten as a form $f(x) = 2x + 15 + \frac{36}{x - 2}$

Wood (2012) suggests communication in mathematics learning, one method of which is oral communication, presentation of material; that is how to speak in the form of semi-formal and processing of mathematical idea with a combination of listening and speaking (Wood, 2012). Kline (2008) describes communication as the requirement to express written and oral communication in order to organise and consolidate mathematical thinking through communication; communicating their mathematical thoughts coherently and clearly to peers, teachers, and others; analyse and evaluate the mathematical thinking and strategies of others; and use mathematical language to express mathematical ideas appropriately (Kline and Ishii, 2008). Communication is an integral part of classroom and school processes, and the quality

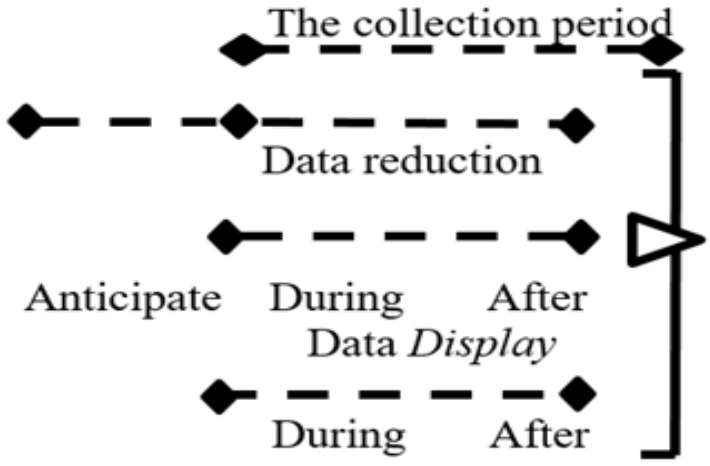
of communication affects the quality of teaching and learning of mathematics (Ball and Diarmid, 1989).

The Competency Standards of Primary and Secondary Education graduates in the Regulation of the Minister of Education and Culture No 20 of 2016 explain that teachers should have factual, conceptual, procedural, and metacognitive knowledge at the technical, specific, detailed and complex levels regarding: science, technology, art, culture, and humanities. Bishop (1991) describes the teaching of mathematics, "teaching of mathematics, many different things have been systematically scrutinized, and for many different purposes, and as the teaching of mathematics is a very complicated matter" (Bishop et al., 1991). Therefore, the study aims to describe the mathematical communication of prospective teachers in learning mathematics.

Method

Qualitative research is an activity that lies in placing observers in the world. It transforms the world into a series of representations, including field notes, interviews, conversations, photographs, recordings, and memos , trying to understand or interpret phenomena in the sense of meaning that people bring to them (Creswell 2009). This research explored mathematical communication phenomenon of prospective mathematics teacher in SMAN 1 Pamekasan, Madura. The research was inductive and the result emphasised the meaning. Figure 1 shows the components in data analysis flow model (Miles et al., 2014).

Figure 1. Components in data analysis flow model.



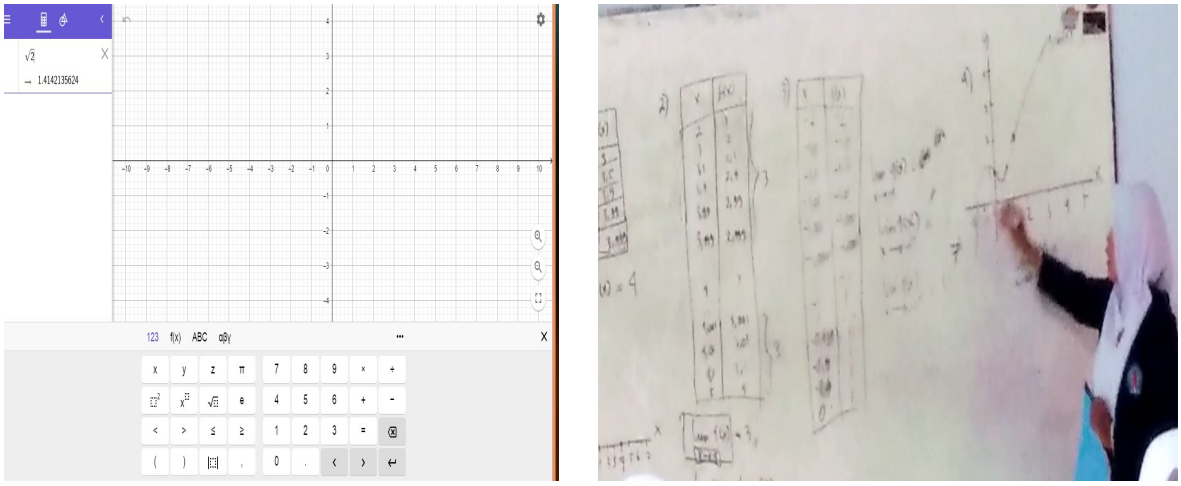
Miles (2018) argued that the data analysis is done as follows: data reduction as a process of selection; data display (presentation of data), the data is organised so that it will be more easily understood; and conclusion drawing or verification (Miles et al., 2014).

Results and discussion

Köğce (2015) reveals that most teachers believe that success in mathematics means the ability to use formulas, rules, and methods simultaneously correctly to make accurate calculations. Prospective teachers should be able to represent and analyse the situation and mathematical structure using algebraic symbols and various symbolic representations (Köğce, 2015).

Prospective teachers appear to have good skills in using formulas, rules, and methods simultaneously correctly to make accurate calculations with competency standards using the concept of function limits and function derivatives in problem-solving (calculus), communicate in writing while working on questions on the board and explain graphics or drawings.

Figure 2. Graphing calculator – GeoGebra and teacher communicate in writing on the board.



Guveli (2015) argues that teachers are people who develop, direct, motivate themselves, develop and practice activities, examine, make students ask and think, listen, work together and evaluate (Guveli, 2015).

Table 2: Dialogue teacher and student.

Teacher: you must find the different form of quadratic equations

Student 1: from $f(x) = x^2 - 2x - 3$

Teacher: the form of quadratic equations one of which is $f(x) = x^2 - 2x - 3$

Teacher: determine the value x that satisfies the quadratic equation, so that the value x is $x = 3$ and $x = -1$.

Student 2: it's factorisation form

Teacher: so, we can find $f(x) = (x-3)(x+1)$

Student 3: another form of equation

Teacher: you should make another form of the equation:

$f(x) = x^2 - 2x - 3$

Student 4: ehmmm…

Teacher: the answer is that there is another form of the above quadratic equation:

$f(x) = (x-1)^2 - 4$

Student 5: Why

Teacher :

$$f(x) = (x-1)(x-1) - 4$$
$$f(x) = x(x-1) - 1(x-1) - 4$$
$$f(x) = x^2 - x - x + 1 - 4$$
$$f(x) = x^2 - 2x - 3$$

Student 6: $f(x) = (x-1)^2 - 4$ is equal to $f(x) = x^2 - 2x - 3$

Prospective teachers provide an understanding of numbers, how to represent numbers, representations between numbers, and number systems.

Student 7: The form $3x = 1$ is equal to $3x \times \frac{1}{3} = 1 \times \frac{1}{3}$, so we can find $x = \frac{1}{3}$

Teacher: How to explain $3x = 1$

Student 8 : $\frac{1}{3}$ is a rational number

Teacher: how about $x^3 = 2$

Student 9: we can find x is equal to the cube root of 2.

Teacher: It's $x = \frac{1}{3}$ is a rational number

Teacher: $\sqrt[3]{2}$ is a real number

Student 10: or $2^{\frac{1}{3}}$

Teacher: why is the result of $x^2 + 4 = 0$ a complex number solution?

Student 11:

$x^2 + 4 = 0$

$x^2 + 4 - 4 = 0 - 4$

$x^2 = -4 = \sqrt{x^2} = \sqrt{-4}$

$x = \sqrt{-4} = \sqrt{(-1)(4)}$

$x = \sqrt{4}\sqrt{-1} = \pm 2\sqrt{-1} = \pm 2i$

Teacher: It's π a repeat number, It's like $\sqrt{2}$

Student 12: π = 3.1428571429…..

Teacher: It isn't repeat

Student 13: $\sqrt{2}$ = 1.4142135624

Teacher: how to find $\sqrt{2}$

Student 14: use graphing calculator – GeoGebra to explain how to find the cube root of 2, so we can find $\sqrt{2}$ = 1.4142135624.

The equation $3x = 1$ has no integer solution but has a rational number solution; the equation $x^3 = 2$ does not have a rational number solution but has a real number solution, and the equation $x^2 + 4 = 0$ has no real number solution but it has complex number solution. Prospective teachers provide students with an understanding of irrational numbers π and $\sqrt{2}$ in real-number systems, and hoped that students' knowledge of rational numbers can be exceeded from

	these two numbers. Students are trained to understand the meaning of operations and how they relate to each other.

Conclusion

Teacher communication plays an important role in learning mathematics especially the ability to use formulas, rules, and methods simultaneously correctly to make accurate calculations. Prospective teachers are able to conduct guidance for their students when implementing mathematics learning and when working on a problem on the blackboard. The language used by the prospective teacher can be adjusted to the level of students' understanding so that the symbols, procedures, and strategies described can be well understood by the students, so that they are able to direct discussion and question and answer activities well. These factors are considered important to be considered in the learning of mathematics in an effort to develop the skills of prospective teachers so that students are accustomed to face similar problems, and apply a mathematical procedure in a newly encountered context. Prospective teachers master communication in learning mathematics. The communication of prospective teachers in mathematics learning can be applied in other wider aspects.

Acknowledgments

Authors would like to thank SMAN 1 Pamekasan, Madura, East Java, Prospective teacher of Madura University, Chancellor of Universitas Siliwangi, Chairman of LP2M Universitas Siliwangi Tasikmalaya, West Java, along with his staff, supervisors and lecturers of Post Graduate, State University of Malang, Malang, East Java, Indonesia.

REFERENCES

Amador, J. M. (2017). Preservice teachers' video simulations and subsequent noticing: a practice-based method to prepare mathematics teachers *Research In Mathematics Education* 217–35

Ball, D. L. and Diarmid, G. W. M. (1989). *The Subject Matter Preparation of Teachers* (East Lansing, Mich. : National Center for Research on Teacher Education)

Bishop, A. J., Olsen, S. M. and Dormolen, J. V. (1991). *Mathematical Knowledge: Its Growth Through Toaching* ed A Bishop, S Mellin-Olsen and J van Dormolen (Netherlands: Springer)

Büşra, S. and Ali, D. (2016). The examination of teacher student communication process in the classroom: mathematical communication process model *SHS Web of Conferences* **26** 01059

Canrinus, E. T., Lorenz, M. H., Beijaard, D., Buitink, J. and Hofman, A. (2011). Profiling teachers' sense of professional identity *Educational Studies* **37(**5) 593–08

Creswell, J. W. (2009). *Research Design Qualitative, Quantitative, and Mixed Method Approaches* 3rd ed (London: SAGE Publications, Inc.)

Griffith, R. and Groulx, J. (2014). Profile for Teacher Decision Making: A Closer Look at Beliefs and Practice *Journal of Research in Education* (Fall and Winter) **24(**2) 103–15

Güçler, B. (2014). The role of symbols in mathematical communication: the case of the limit notation *Research in Mathematics Education* **16**(3) 251–68

Guveli, E. (2015). Prospective Elementary Mathematics Teachers' Problem Posing Skills about Absol*ute* Value *Turkish Journal of Teacher Education* **4**(1) 1–17

Kline, S. L. and Ishii, D. K. (2008). Procedural Explanations in Mathematics Writing : A Framework for Understanding College Students' Effective Communication Practices *Written Communication* **25(**4) 441–61

Köğce, D. (2015). Conceptions and Concept Images of Prospective Mathematics Teachers in a Teacher Training Program Regarding Basic Mathematical Concepts *IJSSE* **5(**4) 678–706

Miles, M. B., Huberman A M, and Saldana, J. (2014). *Qualitative Data Analysis A Methods Source Book 3rd ed* (London: SAGE publications Inc)

NCTM, (2000). *Principles Standards and for School Mathematics The National Council of*

Teachers of Mathematics Inc 1906 Association Drive Reston VA 20191–9988 www.nctm.org

Ng'eno, J, K. and Chesimet, M. C. (2016). Differences in Mathematics Teachers' Perceived Preparedness to Demonstrate Competence in Secondary School Mathematics Content by Teacher Characteristics *Journal of Education and Practice* **7(**18)

Olteanu, L. (2015). Construction of tasks in order to develop and promote classroom communication in mathematics *International Journal of Mathematical Education in Science and Technology* **46(**2) 250–63

Pourdavood, R. G. and Wachira, P. (2015). Importance of Mathematical Communication and Discourse in Secondary Classrooms *Global Journals Inc. USA Type : Double Blind Peer Reviewed International Research Journal* **15**(10)

Wood, L. N. (2012). Practice and conceptions: communicating mathematics in the workplace *Educational Studies in Mathematics* **79(**1) 109–25