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The Relationship Between Conceptual Knowledge and Procedural Knowledge among Students of the Mathematics Department at the Faculty of Education for Pure Sciences/Ibn Al-Haitham, University of Baghdad

Ban Hassan Majeed [a], [a] The Faculty of Education for Pure Sciences/Ibn Al-Haitham; University of Baghdad.

This research attempts to investigate the relationship between conceptual and procedural knowledge among students of the mathematics department at the Faculty of Education for Pure Sciences/Ibn Al-Haitham, University of Baghdad. The research sample consisted of 82 students, distributed as 38 males and 44 females from a group of second phase students. To measure conceptual and procedural knowledge, the researcher adopted two tests. Firstly, one to measure conceptual knowledge and the second to measure procedural knowledge. The results showed that the students have both conceptual knowledge and procedural knowledge. Also, there is a correlation between them for students as a whole, but no difference based on gender.

Key words: conceptual knowledge, procedural knowledge, mathematics departments.

Introduction

Novelty in mathematics is derived from mathematical structure, which orders mathematical content much like a reliable, connected building. The basic building blocks of this building are mathematical concepts. These are the most important form of mathematical knowledge and the basis of its principles, laws and theories. They offer mathematics flexibility and help in organising it and allowing it to be absorbed. These concepts are important for students and relate to student life. If a student comprehends mathematical concepts, mathematics becomes meaningful and clear (Al-Amin, 2001).

This means that learning mathematical concepts should be a priority for all mathematics teachers and students. It should also be the focus of attention of researchers, and those who develop and advance the field of mathematics (NCTM, 2000). Education should develop processes, perception and understanding that lead to an increase in mathematical comprehension, leading to students' ability to perceive relationships between mathematical knowledge, both conceptual and procedural. They should be able to employ concepts, generalisations and mathematical processes, to realise the potential of these relationships and the links between mathematics and life. It is critical that they understand the relationship between mathematics and other branches of knowledge (Cantlon, 1998).

Although conceptual and procedural knowledge cannot be entirely separated, it is useful to study their relationship, in order to better understand the development of knowledge. This is particularly true since calls from the American National Council for Mathematics Teachers, demanding a balance be found between conceptual and procedural knowledge among mathematics teachers (NCTM, 2000). Baker et al. (2004) indicated that the mathematics presented to students focused on skills and procedures, rather than on understanding mathematics, and confirms that students study a number of concepts in arithmetic, algebra and engineering without understanding. Engelbrecht et al. (2009) believe that mathematics education in most countries of the world focuses on knowing procedures and skills, and that evaluation tasks that teachers usually use are focused on procedural tasks, rather than on conceptual depth, which leads students to have a greater ability to deal with mathematical skills, symbols, and procedures, more than their ability to deal with conceptual knowledge. In this regard Shepherd (2006) noted the importance of students gaining confidence in dealing with conceptual and procedural knowledge, moving from one type to another easily. This is emphasised by Weber (2001), who indicated the importance of providing students with strategic knowledge, which he defined as students' confidence in knowing how and when to use conceptual and/or procedural knowledge. This research aims to determine the conceptual knowledge of second phase students in a mathematics department, an important step, helping improve students' mathematical knowledge in general and, in particular, conceptually. This will develop their skills to extensively use their mathematical knowledge, while also linking it to procedure, which contributes to a strong mathematical structure (Obaida, 2007).

Research Importance

Theoretical Importance

1. The results of this research will provide a better understanding of the nature of the relationship between conceptual and procedural knowledge.
2. The researcher hopes that teachers of mathematics will be aware of the importance of the balance between conceptual and procedural knowledge, and be able to teach students how to balance them (Engelbrecht et al., 2009).

Applied Importance

1. Determine the extent to which students in the second phase at the department of mathematics at the Faculty of Education for Pure Sciences/Ibn Al-Haitham, University of Baghdad, possess conceptual knowledge.
2. Understand the nature of correlation between conceptual and procedural knowledge, according to the gender variable.

Research Objectives:

1. Understand conceptual knowledge.
2. Understand procedural knowledge.
3. Determine the nature and direction of the relationship between conceptual and procedural knowledge.
4. Understand the nature of the difference of this relationship by the gender variable.

Research Hypotheses

The following zero hypotheses have been developed:

1. There is no statistically significant difference, at a level of significance of 0.05, between the average real performance and the average hypothesis performance of second phase students at the department of mathematics at the Faculty of Education for Pure Sciences/Ibn Al Haitham, University of Baghdad, in the conceptual knowledge test (Hiebert and LeFevre, 2016).
2. There is no statistically significant difference, at a level of significance of 0.05, between the average real performance and the average hypothesis performance of second phase students at the department of mathematics at the Faculty of Education for Pure Sciences/Ibn Al-Haitham, University of Baghdad, in the test of procedural knowledge.

3. There is no correlation between conceptual knowledge and procedural knowledge, at a level of significance of 0.05, of second phase students at the department of mathematics at the Faculty of Education for Pure Sciences/Ibn Al-Haitham, University of Baghdad, according to the gender variable.

Search Limits:

1. Second phase students (morning study) in the department of mathematics at the Faculty of Education for Pure Sciences/Ibn Al-Haitham, University of Baghdad.
2. The second semester of the year (2018-2019).

Search Terms

Mathematical power: "The student's abilities to perceive and employ mathematical knowledge in its three dimensions (conceptual, procedural, and problematic) in mathematical discovery, interdependence, and reasoning" (NAEP, 2000).

Mathematical knowledge: Including three types of knowledge and experiences, which are: conceptual knowledge, procedural knowledge, problem solving and post-knowledge. Also includes the student's ability to organize their thinking and guidance, modifying cognitive and intellectual paths, in addition to navigating experiences related to problem-solving (Al-Saeed, 2006).

Conceptual knowledge: "The ability to know mathematical concepts, read, write, classify, distinguish, know their multiple representations, and know the relationship of concepts" (Ubaida, 2007).

The researcher defines this as the ability of sample students to respond to test items focusing on conceptual knowledge, measured by the results they obtain on this test.

Procedural knowledge: "Mastering mathematical skills and defining mathematics components, algorithms and definitions" (Hallett, Nunes and Bryant, 2010).

The researcher defines this as the ability of sample students to respond the test items focusing on procedural knowledge, measured by the results they obtain on this test.

Theoretical Background

Mathematical Power

The National Foundation for the Evaluation of American Educational Progress (NAEP, 2003) has noted that mathematical power describes a student's abilities in mathematical knowledge, across three dimensions – conceptual knowledge, procedural knowledge, and problem-solving. It is used in mathematical reasoning, considering mathematical ideas, and is important when considering interconnection in mathematics. Mathematical power is the central axis of teaching mathematics, which is the ability to use and enjoy mathematics, and it gives us a feeling of control over our decisions and our environment. If we understand mathematics sufficiently, we will be able to use it automatically and will have greater control over ourselves and our decisions in society (Kenschaft, 1997).

Al-Saeed (2003) has also indicated that mathematical power is a general ability that extends beyond mere mathematical abilities, involving cognitive understanding, procedural knowledge, and problem-solving. This includes pupils' abilities to infer mathematical situations and communicate via perceptions and conclusions that can only be derived from a mathematical context. Mathematics is interconnected with decisions in the classroom and in life. The researcher agrees with the NCTM (2000) standards, which state that mathematical power is an overall interaction of mathematical capabilities and operations in the light of content.

Knowledge

Mathematics is the mother of science, as the advancement of any field of knowledge must be linked with broad mathematical knowledge (Sood and Jitendra, 2007). Due to rapid developments in today's world, imposing many challenges on education, a movement for mathematical standards has been established. It came in response to contemporary societal changes, especially technological, scientific, economic and cultural changes, with societal expectations on schools today to provide opportunities for learners who are mathematically educated and flexible. They should be able to learn in new situations and also able to deal with and understand ever-evolving technology.

The National Council of Mathematics Teachers (NCTM, 2000) affirms that contemporary trends in mathematics education and learning emphasise operational standards. These standards represent important goals that educators and learners should all strive to achieve in mathematics education. They help describe the process and outputs of learning, and help illustrate methods to extend, use, and acquire knowledge, as they can be implemented through any mathematical content (Zidan, 2006).

Mathematical Knowledge

Learning mathematical knowledge in general helps students adapt to their environment if there is, in particular, a balance between conceptual and procedural learning. As Karma (1999) indicates, there are three main capabilities in mathematics: conceptual knowledge, procedural knowledge, and problem-solving.

Knowledge of the basic operations of mathematics, and skills in applying that knowledge, should be a main requirement of ordinary citizens, who recognise the importance and necessity of mathematics. Citizens should possess mathematical understanding and skills, such as the knowledge of the four operations, spaces, volumes and amounts, dimensions, and other fundamental areas, as they are of great importance in daily life, as mathematics is deeply inherent in natural phenomena (Secretary, 2001).

Conceptual Knowledge

A concept is an abstract or general idea derived from certain situations (Merriam-Webster's Collegiate Dictionary, 2012). Knowledge of concepts is often referred to as conceptual knowledge (Canobi, 2009). This knowledge is usually not related to specific problem types. The NRC (2001) adopted a simple definition of conceptual knowledge as "an understanding of concepts, processes and mathematical relationships." This form of knowledge is also sometimes called conceptual assimilation or initial knowledge.

Conceptual knowledge includes the relationships connecting all parts of mathematical knowledge, including mathematical facts, generalisations, principles, laws and rules, related to each other in a closely linked network (Groth and Bergner, 2006). It also includes the modelling of concepts and translating them into connotations and ideas, explaining mathematics using symbols, sentences, and relationships necessary for mathematical communication. It also includes an understanding of the complementarity and interdependence between main and subsidiary concepts, defining the principles, laws and rules related to mathematical concepts and explaining the relationships between them (Zulnaidi and Zakaria, 2010).

Conceptual knowledge is clearly distinguished for its richness of relationships. It can be considered as an internet of knowledge, a network which links prominent, discrete pieces of information. These relationships help highlight individual facts and suggestions, so that all parts of information are connected to each other (Hiebrt, 1986).

Baroody, Feil, and Johnson (2007) believe that conceptual knowledge should be defined as knowledge of facts, generalisations and principles. Schneider and Stern (2009) have indicated support for this concept of research on conceptual change, revealing two stages of conceptual knowledge development:

1. Beginner: Conceptual knowledge is often fragmented and requires integration throughout the learning path.
2. Experts: Conceptual knowledge continues to expand and become better organized.

Some mathematics researchers (Star, 2005) note that the term "conceptual knowledge" is not only what is known (as knowledge) about concepts, but also one way concepts can be learned and explored in depth, with rich communication.

Procedural Knowledge

A procedure is a series of steps that have been taken to achieve a goal. Procedure-focused knowledge is often known as procedural knowledge (Canboni, 2009). An example of procedural knowledge is knowing how, or knowing the steps required, to achieve different goals.

Procedures can be:
1. Algorithms: They are a predetermined sequence of procedures that will lead to the correct answer when implemented correctly.
2. Possible measures that must be appropriately sequenced to solve a specific problem, such as equivalent solution steps.

Procedural knowledge is developed through problem-solving training, and it is related to specific problem types. In addition, they make very clear that the nature of sequencing procedures when achieving mathematical solutions (Rittle-Johnson and Schneider, 2015).

Shepherd (2006) stated that procedural knowledge was knowledge that deals with how something is right, and is reached by following a set of specific and sequential steps.

Search Procedures

1. **Research Methodology:** This research adopted a descriptive and analytical research methodology, seeking to collect metadata about the relationship between conceptual and procedural knowledge. It then correlated and interpreted this data to draw conclusions.

2. **Research Society:** The mathematics department at the Faculty of Education for Pure Sciences/Ibn Al-Haitham, University of Baghdad, for the academic year (2018-2019).

3. **Research Sample:** The research sample included 82 male and female students from the second phase (morning study).

4. **Research Tools:** For the purpose of answering research questions, and verifying hypotheses, the researcher adopted two tests. The first to measure the level of conceptual knowledge, and the second to measure the level of procedural knowledge of members of the sample. The two tests were built by the same researcher and applied in other research. Therefore, they were applied directly to the final sample of the research.

5. **Final Application on the Research Sample:** For the purpose of answering the research questions and verifying hypotheses, it was ensured that data was applied under appropriate environmental conditions, ready for statistical analysis.

Presentation and Interpretation of Results

Results Related to Conceptual Knowledge

Table 1: T-test results to measure the level of conceptual knowledge of the research sample

Degree of Freedom	Statistical Significance	T-value Table	T-value Calculated	Hypothetical Mean	Standard Deviation	SMA	Total Sample
81	0.05	1.993	13.018	24	7.54160	34.8415	82

The table above shows the arithmetic mean obtained by the sample students on the conceptual knowledge test, compared with the hypothetical mean, was higher, being in favour of the arithmetic mean. This shows that students possess conceptual knowledge during the academic year 2018-2019.

This tests the validity of the null hypothesis: There is no statistically significant difference, at a significance level of 0.05, between the average real performance and the average hypothesis performance of second phase students at the department of mathematics at the Faculty of Education for Pure Sciences/Ibn Al Haitham, University of Baghdad, in the conceptual knowledge test (Hiebert and LeFevre, 2016).

It is noted that the aforementioned table that calculated T-value 13.018 is greater than the tabular T- value 1.993, rejecting the null hypothesis and suggesting the alternative. That is, there is a difference of statistical significance, at the level of significance of 0.05, between students' average real performance and their mean hypothetical performance (24) on conceptual knowledge testing. This supports the conclusion above, that is, the sample's comprehension of conceptual knowledge.

Results Related to Procedural Knowledge

Despite the diversity of results revealed when correcting, there are questions that offer marks of 0.1 and questions which offer up to 6 marks. As 28 marks is the measure of success, the hypothetical mean was 14.

Table 2: T-test results to measure the level of procedural knowledge of the research sample

Degree of Freedom	Statistical Significance	T-value Table	T-value Calculated	The Average Hypothesis	Standard Deviation	Average
81	0.05	1.993	25.273	14	3.73158	24.4146

After reviewing the mean of the marks obtained the sample students on the procedural knowledge test, we find that all paragraphs have an arithmetic mean greater than the hypothetical average, revealing that the differences are in favour of the arithmetic mean. This proves the sample students possess procedural knowledge.

Now to test the null hypothesis: There is no statistically significant difference, at a level of significance of 0.05, between the average real performance and the average hypothesis performance of second phase students at the department of mathematics at the Faculty of Education for Pure Sciences/Ibn Al-Haitham, University of Baghdad, in the test of procedural knowledge. Note that the calculated T-value of 25.273 is greater than the tabular T-value of 1.993, which rejects the null hypothesis, suggesting the alternative. That is, the performance of students from the research sample on the procedural knowledge test had differences, and this result is normal, as there is no conclusive evidence that supports the superiority of one gender over the other in terms of procedural knowledge.

Results Related to the Correlation Between Conceptual Knowledge and Procedural Knowledge

The validity of the null hypothesis was tested: There is no correlation between conceptual knowledge and procedural knowledge, at a level of significance of 0.05, of second phase students at the department of mathematics at the Faculty of Education for Pure Sciences/Ibn Al-Haitham, University of Baghdad, on the two tests prepared for this purpose.

The Pearson correlation coefficient was used to calculate the correlation coefficient between student marks on the conceptual knowledge test and their marks on the procedural knowledge test. To measure the correlation significance, this study used a T-test for correlation coefficients, testing the validity of the previous hypothesis, with results shown below.

Table 3: Correlation coefficients between conceptual and procedural knowledge – significance of correlation when searching sample

Level of Significance	Degree of Freedom	T-value Table	Calculated T-value for Correlatio Coefficient	Correlation Coefficient	Nunber
0.05	80	1.993	2.990	0.317	82

The calculated correlation coefficient between conceptual knowledge and procedural knowledge of sample scores is +0.317 – a positive correlation coefficient. As the calculated T-value is greater than the tabular T-value at the level of significance of 0.05, the previous zero hypothesis is rejected, in relation to conceptual knowledge and its relationship to procedural knowledge. That is, there is a strong direct correlation between the two variables. This is logical, as students with good performance with conceptual knowledge have the ability to understand facts, generalisations, principles, laws, mathematical rules, and have the ability to control them.

This is confirmed by Rittle-Johnson and Schneider (2015), stating that the relationships between conceptual and procedural knowledge are bidirectional, yet sometimes asymmetric. In some instances, conceptual knowledge is more consistently and strongly supported than procedural knowledge.

Now, to answer the question: Is there a statistically significant difference in the relationship between conceptual and procedural knowledge in the research sample, according to the gender variable?

The Pearson correlation coefficient looked at the relationship between conceptual and procedural knowledge of the sample, looking at the gender variable separately. A Z-test was used to investigate the difference between the two correlation coefficients. It was found that there is no statistically significant difference, as the calculated Z-value was smaller than the tabular Z-value, as shown the following table:

Table 4: Differences in the relationship between conceptual and procedural knowledge, depending on the gender variable

Level of Significance	Degree of Freedom	Z-Valu Table	Z -Value Calculated	Standard Degree for Correlation Coefficient	Correlation Coefficient	Number	Sex
0.05	80	1.960	0.613	0.213	0.208	38	Student Male
				0.354	0.342	44	Student Female

Conclusions

1. Students in the second phase at the department of mathematics in the Faculty of Education for Pure Sciences/Ibn Al-Haitham, University of Baghdad, possess conceptual knowledge.
2. They also possess procedural knowledge.
3. There is a correlation between conceptual and procedural knowledge for students as a whole.
4. There is no significant difference in the relationship between conceptual and procedural knowledge depending on gender.

Recommendations

1. Teachers must focus on promoting conceptual knowledge when teaching mathematics, forming a deep conceptual structure that will allow students to apply mathematical operations in sound ways.
2. Effective educational strategies must be used to improve the structure of mathematical conceptual education for students, with emphasis placed on conceptual knowledge and linked to procedural knowledge, allowing for integration.
3. Modern teaching methods are necessary to strike a balance between conceptual and procedural knowledge when teaching mathematics courses and training teachers.

Suggestions

1. Conduct a similar study, in the same phase at the same department of mathematics, and compare results with the results of this research.

2. Carry out similar studies on the other phases in the mathematics department, as well as the computing department. Then, a comparative study should be conducted between students of the computing department and students of the department of mathematics.

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