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The Effect of Pollution on Entropy Estimator for Kumaraswamy Distribution

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Kumaraswamy distribution has many applications especially in determining failure times and estimating the reliability of many experiments. In this research, the theoretical aspects of Kumaraswamy distribution were presented with some estimating methods of its parameters such as Maximum Likelihood Estimation, Moment and Mixed. The practical aspect of the research was to apply 27 unpolluted and 81 polluted simulation experiments with different sample sizes and real distribution parameters. Simulation results showed that the entropy estimator of the distribution was influenced by sample size, contamination rate, estimation method and value of the distribution parameter. Further, other estimation methods, such as white noise and Bayesian can be adopted to compare with the results.

Key words: *Kumaraswamy distribution, Maximum likelihood estimation (MLE), Moment, Mixed, Simulation experiments, Mean square error.*

Introduction

Kumaraswamy distribution can be proposed for many experiments, especially in the field of reliability and to determine the failure times. In this field, many researches have been carried out research of Eldin et al., (2014) estimation results that were compared based on real data representing the water flows of Shasta reservoir in the United States of America. Results show that the estimation methods can provide good estimators for the parameters of Kumaraswamy distribution (Eldin et al., 2014).

The research presented by Simbolon et al., (2016) which present =the (MLE and Bayes) methods to estimate the parameter s for Kumaraswamy distribution, estimators values was compared (precautionary loss function and Square Error Loss Function) through a sufficient number of simulation experiments, results showed the ability of estimation methods to provide optimal estimators (Simbolon et al., 2016).

While the research presented by George and Thobias (2017) in which the (Marshall-Olkin Kumaraswamy) was adopted and MLE method to estimate its parameters research data represent rates of water flow to the river. In the period (1935-1973) in Iowa, United States of America, Lorenz curve was adopted to fit data distribution, results showed the ability of the estimation method to provide the optimal estimators (Simbolon et al., ,2017); (George and Thobias, 2017).

In this paper (MLE, Moment and Mixed) methods for simulate entropy estimator of Kumaraswamy distribution were presented, comparisons results of a set of simulation experiments with different (sample size, parameter values and pollution levels) was introduced (Cordeiro and de Castro, 2011).

Importance of the Research

Pollution in statistical distribution is of varying importance factor in the estimation process of parameters distribution and the ability of get optimal estimators can be effected with pollution rate in the proposed distribution (Jose et al., 2009).

Objective of the Research

The aim of this research is to compare some estimation methods and shows how estimators affected by pollution ratios.

Kumaraswamy distribution (K.D.)

This distribution is one of the most important statistical distributions and has a family of continuous distributions that include a random variable that has a period (Afify et al., 2016). The distribution was first assumed by the Indian researcher (Poondi Kumaraswamy), which has many applications especially in determining reliability and distribution of failure times for various engineering devices (Jose et al., 2009).

Distribution Features

The Probability density function (pdf) for Kumaraswamy distribution can be with the following form:

$$f(x,\alpha,\beta) = \alpha\,\beta\,x^{\alpha-1}(1-x^{\alpha})^{\beta-1} \quad with\;\; 0 \le x \le 1 \quad \ldots(1)$$

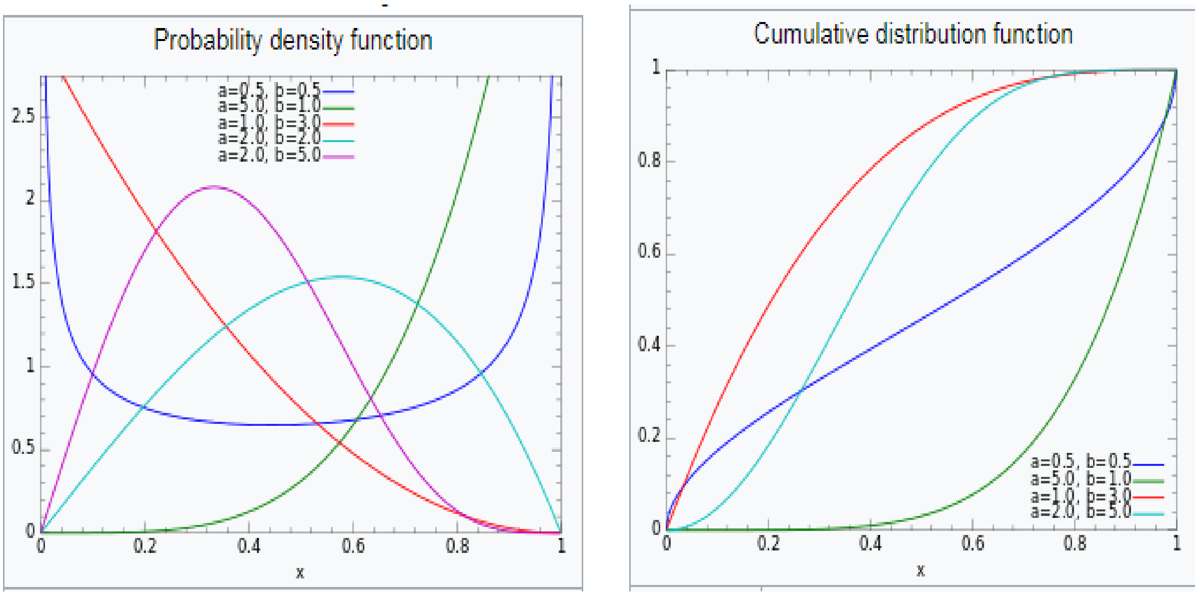
With

(α, β) *represent* (*scale and shape parameters*)

Fig (a-1) represents some (pdf) curves with deferent parameter values
While the Cumulative Distribution (CD) can be

$$F(x, \alpha, \beta) = 1 - (1 - x^{\alpha})^{\beta} \quad \dots (2)$$

Fig (b-1) represents some (cd) curves with deferent parameter values



A B

Fig (1) some (pdf and cd) curves for Kumaraswamy distribution

The entropy function will be

$$H = \left(1 - \frac{1}{\beta}\right) + \left(1 - \frac{1}{\alpha}\right) Hb - \mathrm{Ln}(\alpha\, \beta) \dots (3)$$

With

$$Hb = \sum_{i=1}^{b} \frac{1}{i}$$

Relation with other Distributions

K.D. has a relationship with many other distributions through transformations process that can be made on the random variable of this distribution (Cordeiro and de Castro, 2011).

If (y) a random variable with (uniform distribution) the (x) a random variable with (Kumaraswamy distribution) if

$$y = 1 - \left(1 - x^{\frac{1}{\beta}}\right)^{\frac{1}{\alpha}} \quad \dots (4)$$

If (x) is a random variable with (Kumaraswamy distribution) and (y) is a random variable with (exponential distribution) such that

$$x = -Ln(y) \quad \dots (5)$$

Pollution Distribution

In application experiments the possession of the random variable is a particular distribution is difficult, especially with increasing the size of the sample, because random vocabulary may not be all within the context of the assumption of the distribution function and therefore the pollution ratios that fall within the distribution cannot be completely isolated and thus become a high or little effect depending on many factors including: pollution rate, statistical distribution of pollutant, sample size, extremes or abnormalities of some pollutants.

Derivation of the Differential Entropy For (K.D.)

We can get Entropy function for (K.D.) by the following steps (Eldin et al., 2014)

$$h_x = -\int_0^1 \alpha\,\beta\, x^{\alpha-1}(1-x^{\alpha})^{\beta-1}\, Ln(\alpha\,\beta\, x^{\alpha-1}(1-x^{\alpha})^{\beta-1})dx \quad \dots (6)$$

By taking log we get

$$n(\varphi\,\tau) = Ln(\varphi) + Ln(\tau)$$
$$Ln(\varphi)^{\tau} = \tau\, Ln(\varphi)$$

and

$$h_x = -\int_0^1 \alpha\,\beta\, x^{\alpha-1}(1-x^{\alpha})^{\beta-1}\, Ln(\alpha\,\beta) + (\alpha-1)Ln(x) + (\beta-1)Ln(1-x^{\alpha}))dx \quad \dots (7)$$

$$h_x = -Ln(\alpha\,\beta) - \alpha\,\beta(\alpha-1)\int_0^1 x^{\alpha-1}(1-x^{\alpha})^{\beta-1}Ln(x)dx - \alpha\,\beta(\beta-1)\int_0^1 x^{\alpha-1}(1-x^{\alpha})^{\beta-1}Ln(1-x^{\alpha})dx \dots (8)$$

By taking the process as for the previous integration we get

$$y = 1 - x^{\alpha} \iff dy = -\alpha x^{\alpha-1} dx$$

Then form (8) will be

$$h_y = -Ln(\alpha\,\beta) - \alpha\,\beta(\alpha-1)\frac{1}{\alpha^2}\,\mathrm{B}(1-\beta)\big(\varphi(1)-\varphi(1+\beta)\big)$$
$$-\alpha\,\beta(\beta-1)\int_0^1 \frac{1}{\alpha}\,y^{\beta-1}Ln(y)dy \quad \ldots(9)$$

Since

($\mathrm{B}(1-\beta) = \frac{\Gamma(1)\Gamma(\beta)}{\Gamma(\beta+1)}$) and ($\Gamma(\lambda) = (\lambda-1)!$)

Then

$$\mathrm{B}(1-\beta) = \frac{1}{\beta}$$

Integration can be

$$u = Ln(y) \iff \frac{1}{y}dy, dv = y^{\beta-1}\,dy \iff v = \frac{1}{\beta}y^{\beta}$$

Integration results will be

$$h_y = Ln\left(\frac{1}{\alpha\,\beta}e^{\frac{\beta-1}{\beta}+\frac{\alpha-1}{\alpha}\left(\gamma+\varphi(\beta)+\frac{1}{\beta}\right)}\right) \ldots(10)$$

Such that

(γ) represent *(Euler's constant) and*

$$\varphi(\beta+1) = \varphi(\beta) + \frac{1}{\beta}$$

The Simulation of (K.D.)

Simulation experiments can be proposed with sufficient iterations to obtain the estimators for parameters distribution by formula (2) and the following steps

$R = 1 - (1 - x^{\alpha})^{\beta}$ ... (11)

With (R) representing uniform random variable with [0-1] interval

$$(1 - x^{\alpha})^{\beta} = 1 - R$$
$$(1 - x^{\alpha}) = (1 - R)^{\frac{1}{\alpha}}$$
$$x = (1 - (1 - R)^{\frac{1}{\beta}})^{\frac{1}{\alpha}} \quad ... (12)$$

Estimation Methods

Estimators for parameter distribution can be by many estimation methods such that

Maximum Likelihood Estimation (MLE) Method

If we have random sample with (K.D.) and size (n)
Then the likelihood function will be

$$L(x_1, x_2, \dots, x_n) = \prod_{i=1}^{n} f(x_i, \alpha, \beta) \quad ... (13)$$

By taking form (1) in (13) we get

$$L(x_1, x_2, \dots, x_n) = (\alpha\beta)^n \prod_{i=1}^{n} x_i^{\alpha-1} \prod_{i=1}^{n} (1 - x_i^{\alpha})^{\beta-1} \quad ... (14)$$

Taking log we get

$$Log\big(L(x_1, x_2, \dots, x_n)\big) = nLog(\alpha) + nLog(\beta) + (\alpha - 1)\sum_{i=1}^{n} Log(x_i) + (\beta - 1)\sum_{i=1}^{n} Log(1 - x_i^{\alpha}) \quad ... (15)$$

Taking the partial derivative for (α) and (β) and equalising to zero we get

$$\hat{\beta}_{mle} = \frac{-n}{\sum_{i=1}^{n} Log(1 - x_i^{\hat{\alpha}_{mle}})} \quad ... (16)$$

$$\hat{\alpha}_{mle} = -\frac{n}{\sum_{i=1}^{n} Log(x_i) - (\hat{\beta}_{mle} - 1)\sum_{i=1}^{n} \frac{x_i^{\hat{\alpha}_{mle}} Log(\hat{\alpha}_{mle})}{1 - x_i^{\hat{\alpha}_{mle}}}} \quad ... (17)$$

We can get (MLE) estimators for (α,β) by using (Newton Raphson)

Moment Method

Moment method depends on equalisation processes by the following steps
The (k_{th}) moment for (K.D.) can be

$$E(x^k)=\alpha\beta\frac{\Gamma\left(\frac{k}{\alpha}+1\right)\Gamma(\beta)}{\Gamma\left(\frac{k}{\alpha}+1+\beta\right)}\quad \dots(18)$$

By taking $(k=1)$ we get the first moment with

$$E(x^1)=\alpha\beta\frac{\Gamma\left(\frac{1}{\alpha}+1\right)\Gamma(\beta)}{\Gamma\left(\frac{1}{\alpha}+1+\beta\right)}\quad \dots(19)$$

By taking $(k=2)$ we get the second moment with

$$E(x^2)=\alpha\beta\frac{\Gamma\left(\frac{2}{\alpha}+1\right)\Gamma(\beta)}{\Gamma\left(\frac{2}{\alpha}+1+\beta\right)}\quad \dots(20)$$

The first and second sampling moment will be

$$\tau_1=\frac{\sum_{i=1}^{n}x_i}{n}\quad \dots(21)$$

$$\tau_2=\frac{\sum_{i=1}^{n}{x_i}^2}{n}\quad \dots(22)$$

The equalisation for the (first and second) moment we get

$$\tau_1=\alpha\beta\frac{\Gamma\left(\frac{1}{\alpha}+1\right)\Gamma(\beta)}{\Gamma\left(\frac{1}{\alpha}+1+\beta\right)}\quad \dots(23)$$

$$\alpha\,\tau_1=\beta\frac{\Gamma\left(\frac{1}{\alpha}+1\right)\Gamma(\beta)}{\Gamma\left(\frac{1}{\alpha}+1+\beta\right)}\quad \dots(24)$$

We know that

$$\Gamma(\beta+1)=\beta\Gamma(\beta)$$

Then $(\hat{\alpha}_{\text{mom}})$ will be

$$\hat{\alpha}_{\text{mom}} = \tau_1 \frac{\Gamma\left(\frac{1}{\alpha} + 1 + \beta\right)}{\Gamma\left(\frac{1}{\alpha} + 1\right)\Gamma(\beta + 1)} \quad \ldots (25)$$

And $(\hat{\beta}_{\text{mom}})$ will be

$$\hat{\beta}_{\text{mom}} = \tau_2 \frac{\Gamma\left(\frac{2}{\alpha} + 1 + \beta\right)}{\Gamma\left(\frac{2}{\alpha} + 1\right)\Gamma(\beta)} \quad \ldots (26)$$

We can get Mom estimators for (α, β) by using Newton Raphson

Mixed Method

Mixed method can be by the following forms

$$\hat{\alpha}_{\text{mixed}} = p.\hat{\alpha}_{\text{mle}} + q.\hat{\alpha}_{\text{mom}} \quad \ldots (27)$$
$$\hat{\beta}_{\text{mixed}} = p.\hat{\beta}_{\text{mle}} + q.\hat{\beta}_{\text{mom}} \quad \ldots (28)$$

Such that

$0 \le p \le 1,\ 0 \le q \le 1, p + q = 1$
And if $(p = 1, q = 0)$

We get

$\hat{\alpha}_{\text{mixed}} = \hat{\alpha}_{\text{mle}}$
$\hat{\beta}_{\text{mixed}} = \hat{\beta}_{\text{mle}}$
and if $(p = 1, q = 0)$

We get

$\hat{\alpha}_{\text{mixed}} = \hat{\alpha}_{\text{mom}}$
$\hat{\beta}_{\text{mixed}} = \hat{\beta}_{\text{mom}}$

The value of (p, q) can be found by using the following algorithm

1-start
2-taking initial value for (p) to be smallest as possible, adding value (δ)
3-getting initial $(\hat{\alpha}_{\text{mixed}}, \hat{\beta}_{\text{mixed}})$ by using (27,28) forms , by using (p)
4-getting a new (p_{n}) value by using

$$p_{\text{n}} = p + \delta$$

5-getting a new $(\hat{\alpha}_{\text{mixed}}, \hat{\beta}_{\text{mixed}})$ by using (p_{n})
6-calculate absolute error (ε) between (p, p_{n})
7-if $(\varepsilon < 0.05)$ the step (10)
8- let $(p = p_{\text{n}})$
9-go to (4) step
10-the final $(\hat{\alpha}_{\text{mixed}}, \hat{\beta}_{\text{mixed}})$ will be equal to new $(\hat{\alpha}_{\text{mixed}}, \hat{\beta}_{\text{mixed}})$
11-end

Experimental Results

By applying iterative simulation experiments with the following parameters
The sample size will be $(n_1 = 60, n_2 = 100, n_3 = 200)$
Pollution rate will be $(\varpi_1 = 5, \varpi_2 = 10, \varpi_3 = 15)$

The first parameter will be $(\alpha_1 = 1, \alpha_2 = 1.5, \alpha_3 = 2)$
The second parameter will be $(\beta_1 = 0.25, \beta_2 = 0.5, \beta_3 = 0.75)$

After applying different simulation experiments for (MLE, Mom and Mixed) methods by using (16,17), (25,26) and (27,28) and (3) formulas to obtain the entropy estimators and compute (Mse) by using the following formula

$$Mse = \frac{\sum_{i=1}^{T}\left(\hat{E}_i - E\right)^2}{T} \quad \dots (29)$$

with
(T) represent number of iteration
$(\hat{E}_i)$ represent estimator of entropy
(E) represent true value of entropy

Applying the (27) different un pollution and (81) pollution simulation experiments we get the following results:

Table 1: simulation experimental results for un pollution data

Simulation Parameter			$True_{entropy}$	$Mle_{entropy}$	$Mom_{entropy}$	$Mixed_{entropy}$	Mse_{Mle}	Mse_{Mom}	Mse_{Mixed}	$Best$
α_1	β_1	n_1	-1.61371	-1.52776	-0.17959	-1.52776	0.905221	2.317955	0.905221	0.905221
		n_2	-1.61371	-1.56413	-0.11677	-1.56413	0.882861	2.529457	0.882861	0.882861
		n_3	-1.61371	-1.59034	-3.95E-02	-1.59034	0.867977	2.400418	0.867977	0.867977
	β_2	n_1	-0.30685	-0.28771	0.235725	-0.28856	3.27E-02	0.249715	3.26E-02	0.0326
		n_2	-0.30685	-0.29522	0.296676	-0.29531	3.20E-02	0.216071	3.20E-02	0.032
		n_3	-0.30685	-0.30144	0.375599	-0.30144	3.16E-02	0.181931	3.16E-02	0.0316
	β_3	n_1	-4.57E-02	-3.61E-02	0.366934	-3.62E-02	8.82E-04	6.41E-02	8.78E-04	0.000878
		n_2	-4.57E-02	-0.04135	0.50985	-4.15E-02	7.87E-04	0.122997	7.86E-04	0.000786
		n_3	-4.57E-02	-4.14E-02	0.556392	-4.15E-02	7.48E-04	0.116952	7.45E-04	0.000745
α_2	β_1	n_1	-0.45921	-0.37531	0.399822	-0.37542	8.41E-02	0.454718	8.41E-02	0.0841
		n_2	-0.29004	-0.23483	0.572279	-0.23483	3.45E-02	0.368339	3.45E-02	0.0345
		n_3	-5.98E-02	-3.28E-02	0.863922	-3.28E-02	1.86E-03	0.304289	1.86E-03	0.00186
	β_2	n_1	0.847639	0.865293	0.967567	0.864099	0.508286	0.44135	0.508423	0.44135
		n_2	1.016808	1.0288	1.165374	1.027987	0.335683	0.299644	0.336036	0.299644
		n_3	1.247026	1.252918	1.450341	1.252773	0.231306	0.205026	0.231555	0.205026
	β_3	n_1	1.10884	1.116381	1.199827	1.116143	0.745724	0.678435	0.74578	0.678435
		n_2	1.278009	1.282445	1.429242	1.282444	0.534222	0.482366	0.534223	0.482366
		n_3	1.508227	1.510293	1.66147	1.510223	0.400978	0.370606	0.401054	0.370606
α_3	β_1	n_1	3.31E-02	0.130564	0.211526	0.115545	0.124496	9.66E-02	0.124926	0.0966
		n_2	0.286836	0.345741	0.504413	0.339342	2.25E-02	2.25E-02	2.31E-02	0.0225
		n_3	0.632163	0.666728	0.89133	0.664084	2.31E-03	8.91E-03	1.66E-03	0.00166
	β_2	n_1	1.339935	1.359449	1.246503	1.35745	1.230586	1.248955	1.230777	1.230586

		n_2	1.593689	1.604329	1.552303	1.604013	0.828363	0.853707	0.82852	0.828363
		n_3	1.939015	1.945065	1.911959	1.944864	0.580856	0.630687	0.582009	0.580856
	β_3	n_1	1.601137	1.606513	1.572402	1.606345	1.588087	1.592386	1.588154	1.588087
		n_2	1.85489	1.858795	1.846517	1.858778	1.127433	1.143146	1.127433	1.127433
		n_3	2.200217	2.202338	2.195873	2.202258	0.838926	0.844608	0.839022	0.838926

Numerical results in table (1) showing that the estimation method effected with (sample size and parameter values) and the (Mse) changing from simulation experiment to another. The last column in table (1) show the best method for each random experiment and the number of best times for each method can be represented by fig (a-2). This is showing that (Mle) method represents the best estimation method with (15) times of (27).

Table 2: simulation experimental results for 5 present pollution data

Simulation Parameter			$True_{entropy}$	$Mle_{entropy}$	$Mom_{entropy}$	$Mixed_{entropy}$	Mse_{Mle}	Mse_{Mom}	Mse_{Mixed}	$Best$
α_1	β_1	n_1	-1.61371	-1.58678	0.956204	-1.58678	0.87009	5.417952	0.87009	0.87009
		n_2	-1.61371	-1.60359	1.127636	-1.60359	0.86023	4.87518	0.86023	0.86023
		n_3	-1.61371	-1.61185	1.350412	-1.61185	0.855916	4.464725	0.855916	0.855916
	β_2	n_1	-0.30685	-0.32004	1.051619	-0.32004	3.06E-02	1.294964	0.030038	0.030038
		n_2	-0.30685	-0.31337	1.223889	-0.31337	3.03E-02	0.981239	3.06E-02	0.0303
		n_3	-0.30685	-0.31062	1.485089	-0.31062	3.00E-02	0.791039	3.03E-02	0.03
	β_3	n_1	-4.57E-02	-5.39E-02	1.168763	-0.05427	6.75E-04	0.91522	6.75E-04	0.000675
		n_2	-4.57E-02	-5.26E-02	1.334221	-5.27E-02	5.79E-04	0.661871	5.77E-04	0.000577
		n_3	-4.57E-02	-4.92E-02	1.577179	-4.92E-02	5.78E-04	0.522284	5.77E-04	0.000577
α_2	β_1	n_1	-0.45921	-0.37435	1.038057	-0.37435	8.39E-02	1.060256	8.39E-02	0.0839
		n_2	-0.29004	-0.24327	1.268302	-0.24327	3.30E-02	1.003138	3.30E-02	0.033
		n_3	-5.98E-02	-3.72E-02	1.587408	-3.72E-02	1.76E-03	0.94891	1.76E-03	0.00176
	β_2	n_1	0.847639	0.861197	1.207057	0.858905	0.509123	0.385804	0.509294	0.385804

		n_2	1.016808	1.023201	1.44239	1.02255	0.337223	0.259288	0.337351	0.259288
		n_3	1.247026	1.250272	1.776981	1.249852	0.232077	0.176739	0.232689	0.176739
	β_3	n_1	1.10884	1.113756	1.382758	1.113756	0.746388	0.595247	0.746388	0.595247
		n_2	1.278009	1.280313	1.622865	1.280313	0.535022	0.433511	0.535022	0.433511
		n_3	1.508227	1.509365	1.931998	1.509365	0.401695	0.326406	0.401695	0.326406
α_3	β_1	n_1	3.31E-02	0.12758	0.500406	0.119359	0.125697	0.113573	0.125697	0.113573
		n_2	0.286836	0.340469	0.744165	0.34026	2.28E-02	7.28E-02	2.28E-02	0.0228
		n_3	0.632163	0.661074	1.140866	0.661074	2.36E-03	5.33E-02	1.94E-03	0.00194
	β_2	n_1	1.339935	1.356115	1.057572	1.356015	1.231053	1.354227	1.231053	1.231053
		n_2	1.593689	1.603637	1.383846	1.603637	0.828687	0.960917	0.828687	0.828687
		n_3	1.939015	1.944362	1.77236	1.944362	0.582051	0.724558	0.582174	0.582051
	β_3	n_1	1.601137	1.606745	1.442743	1.606745	1.588113	1.635419	1.588113	1.588113
		n_2	1.85489	1.858391	1.714245	1.858391	1.127518	1.229791	1.127518	1.127518
		n_3	2.200217	2.202193	2.131986	2.202193	0.838435	0.932324	0.838435	0.838435

Table 3: simulation experimental results for 10 present pollution data

Simulation Parameter			$True_{entropy}$	$Mle_{entropy}$	$Mom_{entropy}$	$Mixed_{entropy}$	Mse_{Mle}	Mse_{Mom}	Mse_{Mixed}	$Best$
α_1	β_1	n_1	-1.61371	-1.51677	-0.49661	-1.51742	0.910359	2.029506	0.910042	0.910042
		n_2	-1.61371	-1.55894	-0.44386	-1.55894	0.885931	1.945804	0.885931	0.885931
		n_3	-1.61371	-1.58539	-0.38032	-1.58539	0.870779	1.876019	0.870779	0.870779
	β_2	n_1	-0.30685	-0.2852	6.58E-02	-0.28673	3.34E-02	0.140177	3.32E-02	0.0332
		n_2	-0.30685	-0.29216	0.119968	-0.2925	0.032476	0.138519	3.24E-02	0.0324
		n_3	-0.30685	-0.30062	0.141053	-0.30062	3.16E-02	0.118286	3.16E-02	0.0316
	β_3	n_1	-4.57E-02	-3.25E-02	0.303576	-0.03292	9.90E-04	6.48E-02	9.75E-04	0.000975
		n_2	-4.57E-02	-3.66E-	0.315768	-3.69E-02	8.21E-	4.49E-	8.14E-	0.00081

				02			04	02	04	4
		n_3	-4.57E-02	-0.04162	0.366511	-4.17E-02	7.84E-04	4.23E-02	7.83E-04	0.000783
α_2	β_1	n_1	-0.45921	-0.36434	0.156473	-0.36519	0.087477	0.289353	8.72E-02	0.0872
		n_2	-0.29004	-0.23074	0.368025	-0.23077	3.46E-02	0.234739	3.46E-02	0.0346
		n_3	-5.98E-02	-2.76E-02	0.615932	-2.76E-02	2.18E-03	0.17217	2.18E-03	0.00218
	β_2	n_1	0.847639	0.86667	0.951885	0.865382	0.507939	0.459671	0.507966	0.459671
		n_2	1.016808	1.028374	1.131685	1.027654	0.335426	0.307817	0.335682	0.307817
		n_3	1.247026	1.253488	1.374999	1.25345	0.231168	0.204755	0.231586	0.204755
	β_3	n_1	1.10884	1.115613	1.175867	1.115221	0.745239	0.69989	0.745245	0.69989
		n_2	1.278009	1.282501	1.388351	1.282456	0.534229	0.494337	0.53424	0.494337
		n_3	1.508227	1.511324	1.624471	1.511309	0.400965	0.378318	0.401125	0.378318
α_3	β_1	n_1	3.31E-02	0.133091	5.30E-02	0.117785	2.57E-03	0.105337	0.125839	0.00257
		n_2	0.286836	0.347973	0.462327	0.341624	0.125578	2.12E-02	2.26E-02	0.0212
		n_3	0.632163	0.660967	0.794484	0.659436	0.02234	3.48E-04	1.83E-03	0.000348
	β_2	n_1	1.339935	1.358462	1.28979	1.35725	1.230804	1.241422	1.230957	1.230804
		n_2	1.593689	1.604614	1.556856	1.604238	0.828016	0.853932	0.828161	0.828016
		n_3	1.939015	1.945038	1.93175	1.944723	0.581534	0.602495	0.581966	0.581534
	β_3	n_1	1.601137	1.607396	1.570131	1.60712	1.587461	1.58678	1.587463	1.58678
		n_2	1.85489	1.858945	1.855732	1.858806	1.127503	1.132021	1.127549	1.127503
		n_3	2.200217	2.202913	2.200576	2.202914	0.838505	0.863061	0.83865	0.838505

Table 4: simulation experimental results for 15 present pollution data

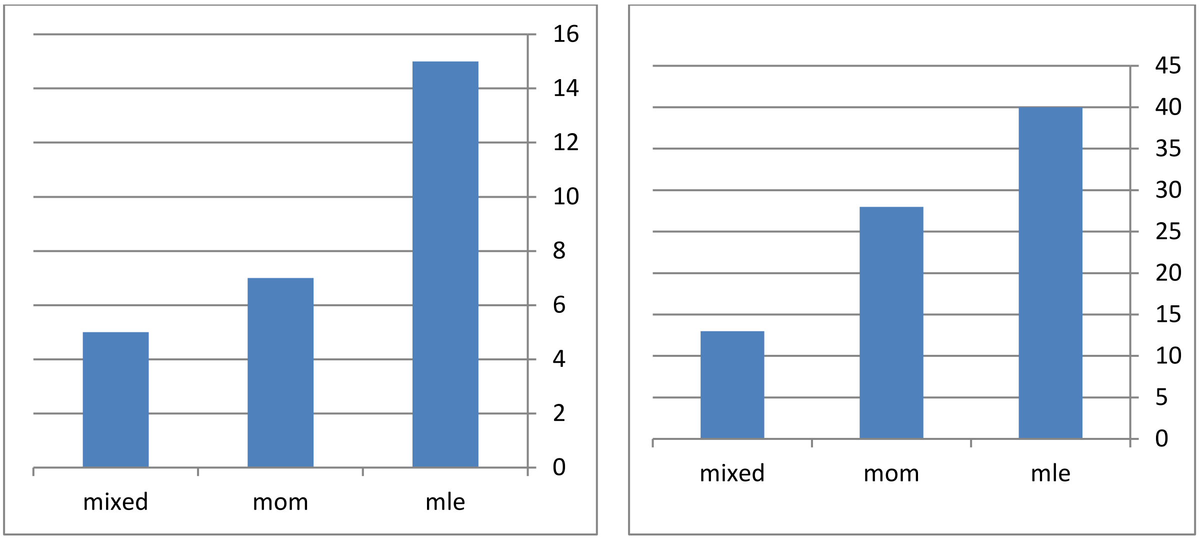
Simulation Parameter			$True_{entropy}$	$Mle_{entropy}$	$Mom_{entropy}$	$Mixed_{entropy}$	Mse_{Mle}	Mse_{Mom}	Mse_{Mixed}	$Best$
α_1	β_1	n_1	-1.61371	-1.4991	-1.80051	-1.62311	0.920968	0.822762	0.914256	0.822762
		n_2	-1.61371	-1.54643	-1.76379	-1.64148	0.893928	0.795912	0.888796	0.795912
		n_3	-1.61371	-1.57986	-1.67154	-1.61987	0.873797	0.741731	0.872583	0.741731
	β_2	n_1	-0.30685	-0.27878	-0.34248	-0.29616	3.37E-02	2.66E-02	3.36E-02	0.0266
		n_2	-0.30685	-0.28731	-0.32811	-0.29918	0.033018	3.22E-02	0.032804	0.0322
		n_3	-0.30685	-0.29654	-0.29848	-0.29428	0.031994	0.030727	3.19E-02	0.030727
	β_3	n_1	-4.57E-02	-2.80E-02	-7.65E-02	-3.82E-02	1.09E-03	1.04E-03	1.06E-03	0.00104
		n_2	-4.57E-02	-3.38E-02	-2.54E-02	-0.01918	9.12E-04	8.53E-04	9.06E-04	0.000853
		n_3	-4.57E-02	-3.89E-02	-3.19E-02	-3.08E-02	7.96E-04	7.77E-04	7.96E-04	0.000777
α_2	β_1	n_1	-0.45921	-0.35863	-0.60703	-0.46215	8.76E-02	5.36E-02	8.54E-02	0.0536
		n_2	-0.29004	-0.23117	-0.41054	-0.30942	3.45E-02	2.18E-02	3.37E-02	0.0218
		n_3	-5.98E-02	-2.85E-02	-0.09746	-5.91E-02	2.14E-03	1.19E-03	2.00E-03	0.00119
	β_2	n_1	0.847639	0.868272	0.791913	0.838153	0.507428	0.529239	0.507681	0.507428
		n_2	1.016808	1.030126	0.980281	1.009547	0.335017	0.347399	0.335246	0.335017
		n_3	1.247026	1.254692	1.214741	1.237172	0.230563	0.244471	0.230868	0.230563
	β_3	n_1	1.10884	1.116132	1.093353	1.111268	0.745134	0.756962	0.745149	0.745134
		n_2	1.278009	1.283563	1.28402	1.28791	0.534321	0.536047	0.534414	0.534321
		n_3	1.508227	1.511365	1.507502	1.511734	0.400966	0.419673	0.400952	0.400952
α_3	β_1	n_1	3.31E-02	0.127667	-0.13251	1.76E-02	0.125153	0.140557	0.125638	0.125153
		n_2	0.286836	0.349941	0.133287	0.254183	2.26E-02	0.046823	2.29E-02	0.0226
		n_3	0.632163	0.664067	0.587775	0.630069	2.25E-03	0.012091	1.82E-03	0.00182
	β_2	n_1	1.339935	1.356927	1.282683	1.327616	1.230498	1.243514	1.230689	1.230498

		n_2	1.593689	1.605592	1.548785	1.582499	0.827493	0.855322	0.827878	0.827493
		n_3	1.939015	1.945189	1.919115	1.934115	0.581917	0.620785	0.582396	0.581917
	β_3	n_1	1.601137	1.607716	1.570156	1.595583	1.587792	1.595005	1.587815	1.587792
		n_2	1.85489	1.858722	1.854498	1.860042	1.12749	1.130324	1.127665	1.12749
		n_3	2.200217	2.20224	2.191239	2.198156	0.83851	0.854783	0.838597	0.83851

The numerical results in tables 2, 3 and 4 showing that the estimation method effected with (sample size parameter values and pollution percentage), the (Mse) changing from simulation experiment to another according to them.

The last column in tables 2, 3 and 4 show that the best method for each random experiment and the number of best times for each method can be represented by fig (b-2) which is showing that (Mle) method represent the best estimation method with (40) times of (81).

Figure 2. Number of best estimation experiments for unpolluted and polluted data



a b

Conclusion and Suggestions

After applying different simulation experiments on unpolluted and polluted data, we got a number of conclusions and suggestions:

1- The entropy estimators of different estimation methods have been affected by (sample size, values of distribution parameters and pollution rates).

2- The mean square error of different estimation methods have been effected by (sample size, values of distribution parameters and pollution rates).
3- Another estimation methods (such as Bayes and Robust) can be adopted for comparison with the proposed estimation methods.
4- Other sample size and values of parameters distribution can be adopted for comparative operations.
5- Estimation of the entropy function of real data with the assumed distribution for the purpose of comparison with simulation results.

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