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A PBL Model to Improve Students' Mathematical Communication Abilities: Self-Regulated Learning

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This study aimed to describe the average improvement and achievement indicators of students' mathematical communication abilities at the Mathematics Education Department at FKIP. It focused on students after they were taught with a Problem Based Learning (PBL) model in terms of Self-Regulated Learning (SRL). This study was an exploratory study using a quantitative-qualitative descriptive approach. The results of this study show students with high SRL achieved an average increase from 34.7 to 84.8. Students with medium SRL achieved an increase from 29.6 to 50.2, and students with low SRL achieved an increase from 24.3 to 35.1. Based on the results, it can be concluded that in three subjects, they can make representations of problems in the form of images, explain concepts using symbols, and solve problems. However, students with medium and low SRL cannot manipulate algebra carefully and express ideas verbally.

Key words: *Mathematical communication ability, Problem Based Learning model and Self-Regulated Learning.*

Introduction

Development of higher order thinking skills is needed in mathematics learning. This is recommended by NCTM (2000): "In mathematics learning, higher level thinking skills must be developed, consisting of mathematical problem-solving abilities, mathematical representation abilities, mathematical communication abilities, mathematical reasoning abilities, mathematical creative thinking abilities and mathematical connection skills."

Mathematical communication abilities, according to Romberg and Chair (Qohar, 2011), involve connecting real objects, images, and diagrams into mathematical ideas. They also involve explaining ideas, situations and mathematical relations verbally or in writing; expressing daily events in mathematical language or symbols; listening, discussing, and writing about mathematics; reading with an understanding of a written mathematical presentation; making conjectures, compiling arguments, formulating definitions and generalisations; and developing and explaining questions about mathematics that have been learned.

Communication skills are abilities that are essential in learning mathematics. This is consistent with what is stated in NCTM (2000): "Communication is an essential element in the teaching and learning of mathematics." Communication is a way in which students share or express ideas to their fellow students or lecturers so that the ideas can be clarified and developed. Communication is essential for new knowledge in mathematics. This is in accordance with the opinion of Vasileiadou (2013): "Communication is essential to building new mathematical knowledge and for the quality of this knowledge. Communication is the impart or exchange of information by speaking, writing, or using some other medium." Furthermore, it is said that communication is a multifaceted mechanism that impacts all aspects of society and has a close relationship with language.

These standards underscore the importance of communication as one of the five process standards emphasising the roles of writing, speaking, and listening in developing mathematical understanding. The Principles and Standards for School Mathematics (NCTM, 2000) asserts, "students gain insights into their thinking when they present their methods for solving problems, when they justify their reasoning to others, or when they formulate a question." The communication standard for pre-kindergarten through grade 12 consists of four goals for students: (1) organising and consolidating their mathematical thinking through communication; (2) communicating their mathematical thinking coherently and clearly to peers, teachers, and others; (3) analysing and evaluating the mathematical thinking and strategies of others; (4) using the language of mathematics to express mathematical ideas precisely (NCTM, 2000).

Lessons learned should receive feedback from students. In this feedback activity, communication will occur between teachers and students in the form of oral and written communication. Communication in the form of oral discourse and students' writing should not be underestimated. Pourdavood & Wachira (2015) explain that the discourse of oral and written communications in the classroom meet three broad and interrelated issues: learning, teaching and assessment purposes. All communication processes must be assessed. Sür & Delice (2016) suggest a mathematical communication process be evaluated according to individual relationships in a classroom environment. This produces observational data, and consequently, the following emerge: (1) a communication process conducted by students and teachers (teacher communication to students); (2) a communication process of students with each other (student to student communication); (3) the process of communication that students do with themselves (internal communication of students).

Mathematical communication abilities are important for candidates to become mathematics teachers. This is because they will be used in teaching mathematical concepts. To find the mathematical communication ability of students, an indicator is needed. NCTM (2000) suggests three indicators of mathematical communication ability: (1) the ability to express mathematical ideas verbally and in writing (to demonstrate and describe them visually); (2) the ability to understand, interpret and assess mathematical ideas verbally and in visual forms; and (3) the ability to use mathematical terms, notations and structures to present ideas, describe relationships and model situations.

According to Van de Walle (2007), communication emphasises the importance of representation, speaking, writing, explaining mathematical concepts and the ability to provide ideas and solve problems. In a study that was conducted in the Department of Mathematics Education at FKIP, the observed aspects of mathematical communication were the ability to represent, speak, write and explain concepts using symbols, and the ability to present ideas and solve problems.

In general, the thing that happens in learning mathematics is that most students are still confused about understanding problems or data that exists in problems. Hence, they have difficulty expressing in mathematical form. In the end, they are not able to determine what concepts or principles should be used to solve problems. Conversely, students also experience difficulties when they have to read or interpret data presented in the form of images, graphs, diagrams or other mathematical symbols. It can be said that these students' mathematical communication abilities are still low. This is a phenomenon that occurs in the Mathematics Education Department at UHO FKIP Kendari. This is related to the test results of *Kapita Selekta* Middle School's Mathematics Education course on problem solving related to mathematical communication. The results show that of 32 student test takers, 8 students who had high SRL obtained an average score of 34.7 out of 100; 11 students who had medium SRL obtained 29.6, and 13 students who had low SRL obtained 24.3. This shows that mathematical communication ability is still a problem in learning mathematics.

To overcome these problems, a learning design is needed that can encourage the improvement of students' mathematical communication abilities. Mathematical communication abilities depend on the ability to collect, organise and explain thoughts and to discover what is known and what is not known. To achieve these aspects, surely an appropriate learning model is needed. One of the learning models that can encourage the ability to express ideas in mathematics is the Problem Based Learning (PBL) model.

PBL is a learner-centred pedagogical approach that affords learners (including prospective and certified teachers) opportunities to engage in goal-directed inquiry (De Simone, 2014). PBL is focused on problems in which students can construct their own knowledge and develop inquiry and thinking skills to a higher level. Students should be able to formulate a temporary answer to problems requiring logical intelligence, courage and active solutions within real situations. Students also need to improve their independence, self-confidence and endurance to solve these problems (Talib and Kailani, 2014).

PBL provides authentic experiences that promote active learning, support knowledge construction, and naturally integrate school learning and real life. They also integrate disciplines (Akcay, 2009). Furthermore, it is mentioned that "in PBL, students are engaged problem solvers, identifying the root problem and the conditions needed for a good solution, pursuing meaning and understanding, and becoming self-directed learners". Teachers use real-world problems and role-playing as they coach learning through probing, questioning, and challenging students' thinking. Teachers are problem-solving colleagues who model interest and enthusiasm for learning and are also cognitive coaches who foster an environment that supports open inquiry (Akcay, 2009).

Studies show that PBL improves students' academic achievements and allows them to work in groups cooperatively and construct their knowledge through social negotiation (compared to traditional teaching methods) (Polanco, Calderon, & Delgado, 2004; Sungur, Tekkaya & Geban, 2006; Goodnough, 2006). As further mentioned, "even though there are many studies to show effectiveness of PBL, there is not enough study to present curriculum materials for teachers to explain how to implement PBL in the actual classroom". The focus of this paper is to describe PBL and identify factors associated with it. More importantly, it is to provide a lesson plan that illustrates a theoretical and conceptual link between science and real-world problems. Hence, teachers can implement these teaching methods in their classrooms to improve their students' academic achievement in science.

The PBL model has specific characteristics, as defined by Barrows (1996), such as "(1) the learning process is student-centred; (2) the learning process is carried out in small groups; (3) the lecturers act as facilitators or supervisors, (4) the problems presented are the stimulus in learning processes, (5) new information is obtained from self-directed learning and (6) problems are rides to develop problem solving skill". Thus, indirectly, the achievement of problem-solving abilities can be developed through the PBL model.

PBL is an instructional approach that enables learners to conduct a study, integrate theory and practice, and apply knowledge and skills in order to develop a solution to a defined problem (Savery, 2006). According to Barrows (2002), the key components of PBL are "(1) unresolved, ill-structured problems that will generate multiple thoughts about the cause and solution, (2) a student-centred approach in which students determine what they need to learn, (3) teachers serve as facilitators and tutors, and (4) problems are authentic and reflect professional practice". PBL is an instructional approach that provides learners with opportunities to identify solutions to ill-structured, real-world problems. Previous study provides evidence to support claims about the positive effects of PBL on cognitive skill development and knowledge retention. PBL is a total approach to education and involves a constructivist approach to learning (Harper-Marinick, 2001). It is a well-known alternative approach to traditional disciplinary-based professional educational programs in higher education. The emphasis of PBL is that students learn through the process of solving so called 'real-world' problems.

PBL also helps students become independent students. In PBL, the roles of the teacher are to ask questions, facilitate students' inquiries, and support students' learning. This is in line with a study conducted by Nufus (2013), which shows that there is an increase in communication and problem-solving skills through the application of mathematical problem-based learning. By applying PBL, student learning is expected to be more meaningful and to make a stronger impression on the students. Students can overcome difficulties in learning mathematics, they can also resolve problems related to everyday life and they are able to develop their creativity.

PBL, compared to other learning models, is a learning model that is constructivist, complex, and provides great opportunities to form independence in learning. PBL is very good when used in developing high-level thinking abilities, such as critical and creative thinking. Complex, contextual, and ill-structured problems give an opportunity to students to develop analytical, evaluative, and reflective thinking abilities, as well as creativity in finding many kinds of information, developing possible solutions, and creating many sources to solve problems that have to be solved (CIDR, 2004). The procedures of implementing the PBL model are mentioned by Alrahlah (2006), "(1) the teacher presents the problem to the students; (2) the students identify the given problem; (3) they seek information from various sources to solve the problem; (4) they choose the most appropriate solution to solve the problem; (5) the teacher evaluates the students' works".

In the 2013 curriculum, it is said that in learning mathematics, hard skills and soft skills need to be developed. One part of soft skills is self-regulated learning (SRL). Students are expected to be able to plan what needs to be done to achieve learning objectives. The ability to plan is one part of SRL.

SRL is defined as the perception of learning as a skill. It is used to analyse learning tasks, implement goals and plan procedures for carrying out those tasks, applying skills and specifically making decisions about how learning will be carried out (Woolfolk, 2009). SRL is also defined as a form of individual learning that depends on students' learning motivation, autonomously develops measurements (cognition, metacognition and behaviour), and monitors learning progress (Baumert, Kurofkina, & Reisch., 1992).

SRL is a combination of abilities and desires. The strategy that students must carry out is to plan, control and evaluate their cognition. Mujiman (2009) said that SRL is an active learning activity. It is driven by the intention and motivation to master a competency in order to overcome a problem that is built on the knowledge or competency possessed. Therefore, intention and motivation are very important things in SRL. The need to plan or conduct self-regulation in mathematics learning is also supported by several study findings, including students who have high SRL. They will be responsible for their own learning, consider learning as a proactive process, be able to motivate themselves, and be able to use appropriate strategies to achieve desirable academic results (Sardarch, Saad & Boroomamand, 2012). Because of the importance of mathematical communication abilities being possessed by a student candidates for mathematics teachers, it is necessary to develop mathematical communication abilities by applying the PBL model with regard to SRL.

This study aims to describe the average improvement and achievement indicators of students' mathematical communication abilities at the Mathematics Education Department at FKIP after being taught with a Problem Based Learning (PBL) model in terms of Self-Regulated Learning (SRL).

Methodology

This study was an exploratory study using a quantitative-qualitative descriptive approach. Respondents in this study were students of the Mathematics Education Department at FKIP (class of 2017/2018) who programmed the *Kapita Selekta* Middle School Mathematics Education course consisting of 32 students. Because the mathematical communication abilities in this study were reviewed in terms of SRL, a preliminary test was done on SRL and the results were obtained: 8 students had high SRL, 11 students had medium SRL, and 13 students had low SRL. To conduct an in-depth search of students' mathematical communication abilities, the subjects in this study were determined: 1 student had high SRL, 1 student had medium SRL, and 1 student had low SRL.

To collect data about mathematical communication abilities, 32 students were given the test. To conduct an in-depth search of the mathematical communication abilities of the three predetermined subjects, test delivery were used techniques, followed by interviews or task-based interviews. To analyse the data in this study, stages of data reduction were followed along with data presentation and the drawing of conclusions.

The mathematics materials in the *Kapita Selekta* Middle School Mathematics Education course in this study involved the application of the concept of derivatives in everyday life. The problem is as follows:

"A square sheet of manila with a size of 20 cm will be made without a lid by cutting a small square at all four corners. What is the size (length, width and height) of the box so that the box is in maximum volume? "

Results and Discussion

Results

After testing the mathematical communication ability of 32 students, a quantitative descriptive analysis was conducted and the results were obtained: students with high SRL received an average of 84.8, an increase of 50.1 from 34.7 in initial test results to 84.8 in test results at the end. Students who have SRL achieved an average of 50.2, which is an increase of 20.6 from 29.6 in initial test results to 50.2 in final test results. Students with a low SRL received an average of 35.1, an increase of 10.8 from 24.3 in preliminary test results to 35.1 in final test results.

Based on the results of the mathematical communication ability test revealed above, it appears there was an increase in mathematical communication abilities after the process of developing mathematical communication abilities through the application of PBL models. However, the increase was not evenly distributed among students who have high, medium and low SRL.

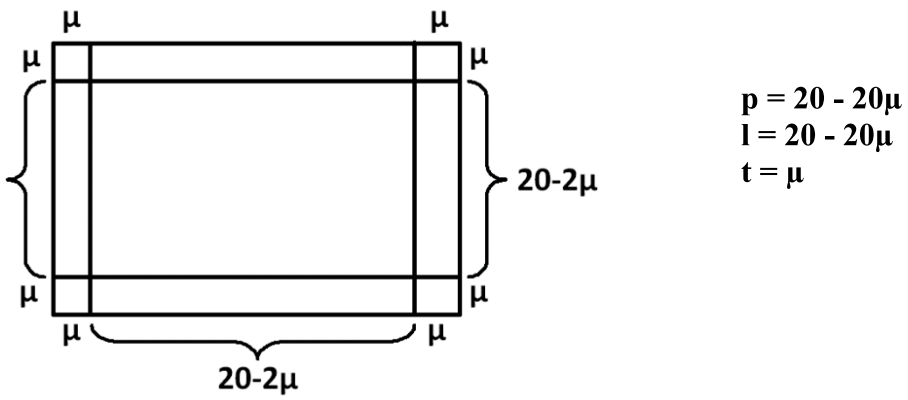
To conduct an in-depth search of the mathematical communication abilities of predetermined subjects, a qualitative descriptive analysis was conducted for each student who had middle, medium and low SRL.

Subject who has high SRL (subject 1)

To answer a given problem, S1 made a visual representation (see Figure 1).

Figure 1: *A visual representation of the given problem by a subject with high SRL (subject 1).*

Note: the side of the square which cut is μ.



Subject 1 considered the size of the square in the form of a unit to be cut with x, so that the size of the box formed had length = p = (20 - 2μ) cm; width = l = (20 - 2μ) cm and height = t = μ cm. In the next step, subject 1 determined the volume formula of the box, then determined the first derivative of V or V^1:

$$V = p \times l \times t = (20 - 2\mu)(20 - \mu)\mu$$
$$V = (400 - 80\mu + 4\mu^2)\mu = 400\mu = 400\mu - 80\mu^2 + 4\mu^3$$

$$V^1 = 400 - 160\mu + 12\mu^2 = 12\mu^2 - 160\mu + 400$$

Furthermore, subject 1 explained that the condition for obtaining maximum volume was the first derivative of V = 0 or $V^1 = 0$ (condition: V max; $V^1 = 0$), as written below.

$$12\mu^2 - 160\mu + 400 = 0$$
$$3\mu^2 - 400\mu + 100 = 0$$
$$(3\mu - 10)(\mu - 10) = 0$$
$$3\mu = 10 \quad or \quad \mu = 10$$
$$\mu = \frac{10}{3} \quad or \quad \mu = 10$$

In the next step, subject 1 explained that to obtain the maximum volume, they firstly determined the μ value for the example by factoring the derivative equation of V, which is $V^1 = 0$. After obtaining the value of μ, subject 1 substituted the value of μ, which was obtained in the volume equation and can be seen where x results in maximum volume as seen below.

$$If\ \mu = \frac{10}{3}, then:$$

$$V = \left(400 \times \frac{10}{3}\right) - \left(80 \times \frac{10^2}{3}\right) + \left(4 \times \frac{10^3}{3}\right) = \frac{4000}{3} - \frac{8000}{9} + \frac{4000}{27}$$
$$= \frac{36000}{27} - \frac{24000}{27} + \frac{4000}{27} = \frac{16000}{27} = 592,6$$

$$If\ \mu = 10, then:$$

$$V = (400 \times 10) - (80 \times 10^2) + (4 \times 10^3) = 4000 - 8000 + 4000 = 0$$

At the time of interviewing, subject 1 could explain that for μ = 10, the volume of the box was zero. The answer of subject 1 was: "if the square manila cardboard has a side size of 20 cm, if it is cut into a small square with a side of 10 cm, then the box cannot be made, so the volume is zero". Finally, subject 1 wrote down the box size, which resulted in maximum box volume as written below.

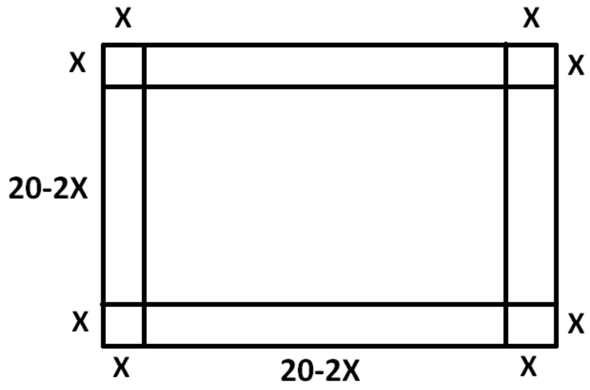
$$p = 20 - 2\left(\frac{10}{3}\right) = \frac{60 - 20}{3} = \frac{40}{3} cm$$
$$l = \frac{40}{3} cm$$
$$t = \frac{10}{3} cm$$

Based on the answers of subject 1 when answering a given problem and at the interview, it can be concluded that subject 1 can make a problem representation in the form of images, can explain ideas verbally or in writing, can explain concepts using symbols, and can solve problems. This shows that student who has high SRL meets all indicators of mathematical communication ability.

Subject who has medium SRL (subject 2)

Subject 2 does the same thing as the subject who had high SRL (subject 1), namely making visual representations as a manifestation of a given problem, as seen below.

Figure 2: *A visual representation of the given problem by a subject with medium SRL (subject 2)*



Next, subject 2 considered the length, width and height of the box to be made and gave reasons that the manila carton was folded left-right and top-bottom on the lines that had been made, so that the size of the box was: length = p = (20 - 2x) cm; width = l = (20 - 2x) cm and height = t = x cm. In the next step, the subject wrote the formula for the volume of the box that was formed as written below.

$$V = p \times l \times t = (20 - 2x)(20 - 2x)x = 400x - 80x^2 + 4x^3$$
$$V = 4x^3 - 80x^2 + 400x$$

In the next step, subject 2 explained that to obtain the maximum box volume, $V^1 = 0$. Subject 2 determined the derivative of the box volume equation, as written below.

$$V^1 = 12x^2 - 160x + 400$$

At first, subject 1 was having difficulty determining the x value of $V^1 = 0$. This is indicated when the subject crossed out what he wrote several times as written below.

$$12x^2 - 160x + 400 = 0$$
$$3x^2 - 40x + 100 = 0$$
$$(3x - 10)(x - 10) = 0$$
$$3x = 10 \quad or \quad x = 10$$
$$x = \frac{10}{3} \quad or \quad x = 10$$

In the next stage, subject 2 substituted the x values obtained in the box volume equation, as written below.

$$If\ x = \frac{10}{3}, then:$$

$$V = \left(4 \times \left(\frac{10}{3}\right)^3\right) - \left(80 \times \left(\frac{10}{3}\right)^2\right) + \left(400 \times \frac{10}{3}\right)$$

$$V = \frac{4000}{27} - \frac{8000}{9} + \frac{4000}{3} = \frac{4000}{27} - \frac{24000}{27} + \frac{36000}{27} = \frac{16000}{27} cm^3$$

$$If\ x = 10, then:$$

$$V = (4 \times 10^3) - (80 \times 10^2) + 4000 = 0$$

Furthermore, subject 2 wrote down the size of the box so that the volume was maximum, as written below.

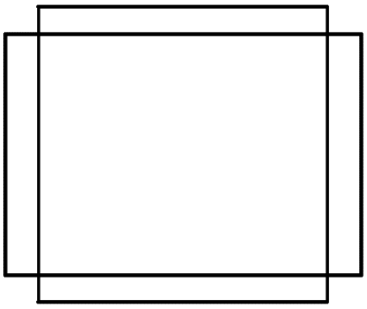
$$p = 20 - \left(2 \times \frac{10}{3}\right) = 20 - \frac{20}{3} = \frac{60}{3} - \frac{20}{3} = \frac{40}{3} cm$$
$$l = p = \frac{40}{3} cm$$
$$t = \frac{10}{3} cm$$

Based on the answers of subject 2 at the time of answering the given problem and at the time of the interview, it can be concluded that the subject could make a representation of the problem in the form of images. They could explain the situation and mathematical relations in writing with pictures and could solve problems, but they could not do algebraic manipulation carefully. They also could not make an argument from what was written. Hence, the subject did not fulfil all indicators of mathematical communication set in this study.

Subjects that have low SRL (subject 3)

A subject that had low SRL made a visual representation of the given problem. After that, subject 3 assumed the size of the box was: length = p = (20 - 2x) cm; width = l = (20 - 2x) cm and height = t = x cm. Furthermore, the subject wrote down the box volume formula and substituted the length, width and height of the box in the box volume formula (see Figure 3).

Figure 3: *A visual representation of the given problem by a subject with low SRL (subject 3)*



The next step taken by subject 3 wrote the volume formula, as written below.

$p = 20 - 2x$; $\quad l = 20 - 2x$; $\quad t = x$

$$V = p \times l \times t = (20 - 2x)(20 - 2x)(x) = (400 - 80x + 4x^2)x$$
$$V = 400x - 80x^2 + 4x^3 = 4x^3 - 80x^2 + 400x$$

Furthermore, subject 3 wrote the first derivative of V or V^1 and wrote a condition for maximum V but could not explain the reason why $V^1 = 0$, as written below.

$$V^1 = 12x^2 - 160x + 400$$
$$12x^2 - 160x + 400 = 0$$
$$3x^2 - 40x + 100 = 0$$
$$(x - 10)(3x - 10) = 0$$
$$x = 10 \quad or \quad x = \frac{10}{3}$$

Subject 3 made mistakes in determining the x value after determining $V^1 = 0$. This is indicated by the fact that some of the answers are crossed out. In the next step, the subject wrote down the volume box, as written below.

$$If\ x = \frac{10}{3}, then:$$

$$V = \left(4 \times \left(\frac{10}{3}\right)^3\right) - \left(80 \times \left(\frac{10}{3}\right)^2\right) + \left(400 \times \frac{10}{3}\right)$$
$$V = \frac{4000}{27} - \frac{8000}{9} + \frac{4000}{3} = \frac{4000}{27} - \frac{24000}{27} + \frac{36000}{27} = \frac{16000}{27} cm^3$$

$$If\ x = 10, then:$$

$$V = (4 \times 10^3) - (80 \times 10^2) + 4000 = 0$$

Subject 3 could not explain why x = 10 resulting in the box volume equalling zero. In the final step, subject 3 wrote down the size of the box, resulting in the maximum volume of the box, as written below.

$$p = 20 - 2\left(\frac{10}{3}\right) = 20 - \frac{20}{3} = \frac{60}{3} - \frac{20}{3} = \frac{40}{3} cm$$
$$l = p = \frac{10}{3} cm$$
$$t = \frac{10}{3} cm$$

Based on the answers given, it can be concluded that the subject with low SRL could make visual representations of the given problem, could connect real objects and images into mathematical ideas. They could not make algebraic manipulations accurately and precisely, can solve problems, and could express ideas in an oral manner. This shows that the subject with low SRL did not meet all mathematical communication indicators when answering the given problem.

Discussion

The results of this quantitative descriptive analysis showed an average increase in mathematical communication abilities for each level of SRL after the process of developing mathematical communication abilities through the PBL model. Through the PBL model, students can get many benefits. This is according to the opinion of Barell (2010): "PBL gives many benefits to students to develop high-levelled thinking abilities such as critical thinking, communication abilities, finding and using learning sources, developing cooperative working abilities, and lifelong learning." Through cooperative learning, students can work together to solve a given mathematical problem, namely by discussing and helping each other. This is supported by the opinion of Djamilah (2011): “through PBL, college students in groups will discuss intensively, therefore, orally, they will be asking each other, answering, criticising, correcting, and clarifying every concept or mathematical argument arisen throughout the discussion. In such discussion, it will also develop the ability of the students to create, refine, and explore conjectures, so it will consolidate their understanding about the mathematical concept they are learning, or toward the solved mathematical problems and finally, the students also have to be able to communicate their ideas, either orally or in writing, to solve the problem given”.

There are several advantages in implementing the PBL model in teaching mathematics. This is in accordance with Barrows' opinion (1996): "the implementation of PBL in learning process provides benefits, such as: (1) preparing the students is better to implement their learning into real-world situations; (2) allowing the students to become knowledge producers rather than just being consumers; and (3) helping the students to develop communication, reasoning, and critical thinking skills". Therefore, the PBL model can improve the mathematical communication abilities of students at the Mathematics Education Department of FKIP UHO.

PBL has other advantages in the learning process. Ibrahim and Nur (2000) state, "in PBL situations, students can be involved in high-levelled thinking and problem solving in a situation oriented to the problem in real life, including learning how to learn". Furthermore, Ibrahim dan Nur (2000) also state that "students are expected to have complete understanding of material formulated in a problem, mastery of positive attitudes, and skills gradually and sustainably; students' mental activities are required to understand a concept, principle, and skill through a situation or problem presented at the beginning of a lesson; students understand the concept and principle of material through working and learning about the situation and problem given through investigation, inquiry, and problem solving. Students build concepts or principles using their own abilities, which integrates skills and knowledge which have been understood previously."

A person's mathematical communication ability can be influenced by how good a person's SRL is. SRL will be formed if students have a sense of responsibility related to their learning needs. SRL will also be formed if students have high intention and motivation. As Mujiman (2009) says: "SRL is an active learning activity, which is driven by the intention and motivation to master a competency to overcome a problem that is built on the knowledge or competency possessed." Therefore, intention and motivation are very important things in SRL. In the learning process, students should be motivated continuously so that their SRL becomes better. If students' SRL becomes better, then students will be able to plan, control and evaluate their cognition. Planning, controlling and evaluating cognition properly are the characteristics of students who have high SRL.

Students who have high SRL will be able to plan and conduct self-regulation well in learning (Sardarch, et al., 2012). The findings in this study indicate that students who have high SRL also have better mathematical communication abilities than students who have medium and low SRL. This is indicated by an increase in the average mathematical communication ability of students who have high SRL and a better than the average increase in the mathematical communication ability of students who have medium and low SRL. The findings of this study also show that students who have high SRL are able to use all indicators of mathematical communication ability well, while students who have medium and low SRL are not. This is supported by the opinion of Hargis (2000): "individuals who have high SRL, tend to learn better, be able to monitor, evaluate and manage their learning effectively, save time in completing their work to obtain a high score". SRL and communication skills have a positive relationship because students who have a good SRL will also have good mathematical communication abilities.

Conclusion

Based on the results and discussion, the following can be concluded: (1) Before and after being taught with the PBL model, students who have high SRL achieve an average increase from 34.7 to 84.8, students who have medium SRL achieve an average increase from 29.6 to 50.2, and students with low SRL achieve an average increase from 24.3 to 35.1. (2) Students who have high SRL can make representations of problems in the form of images, can communicate ideas verbally or in writing, can explain concepts using symbols, and can solve problems. Student who have medium SRL can make representations of problems in the form of drawings, can explain situations and mathematical relations in writing, and can solve problems but cannot manipulate algebra carefully and cannot make arguments from what is written. Student who have low SRL can make visual representations of a given problem and can connect real objects and drawings into mathematical ideas but cannot do algebraic manipulations carefully and precisely. They can solve problems but cannot express ideas verbally.

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