



Modelling Volatility in Financial Time Series Using ARCH Models

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Time is the most important factor which maintains success in business, finance and other fields. It is hard to keep up with the pace of time. One such manner, which deals with time based data, is a time series model. Time series model is an adequate model when there are serially correlated data. The field of financial time series has exploded over the last decades. In general, financial time series suffer from the problem of instability in the mean and variance. In such a case, the Autoregressive Conditional Heteroskedasticity (ARCH), is an appropriate approach that clearly models the change in variation over time in a time series. Explicitly, an ARCH approach models the variance at a time step as a function of the residual errors from a mean process. In this study, the ARCH models such as ARCH, GARCH, EGARCH (TGARCH) were used to model the volatility in the financial time series. The best model is the one that has the lowest value of Akaike (AIC) and Schwarz (SIC).

Keywords: *Financial, ARCH, time series*

Introduction

The ARCH is a time-series statistical model used to analyse effects left unexplained by econometric models. ARCH models were firstly proposed by Robert F. Engle in 1982 (Engle, 1982). The ARCH models are commonly used in the financial statements models due to the recent trends in those who want to make profits are not limited to the prediction of shares in financial markets only but became more attention to the element of uncertainty. Therefore we need models dealing with volatility (especially in the financial statements), and the ARCH models are used to deal with these fluctuations in time series data.

It is known in the financial statements, unlike some other time-series data, the variation of the random term is not uniform. Econometric models assume that the variance of this term will

be uniform. This is known as "homoskedasticity." However, in some circumstances, this variance is not uniform, but "heteroskedastic". In this study, we focus on the time series that has many fluctuations during the time. Therefore, the expected values of the random error will be variable over time. So, the ARCH models were suggested for the conditional variance. The research is organised as follows; In Section 2, the methods of study are briefly presented. In Section 3, the time-series data is described, and the results of the study are discussed. Finally, the conclusions and recommendations are given in sections 4 and 5, respectively.

Research Problem

Heteroscedasticity is one of the major problems experienced by the financial statements. So, the Box Jenkins models do not work with this type of data because those models require the stability of the time series (Box, 2013). The research problem is that the time series is not stationary invariance.

Research Aim

This research aims to select the best model for the conditional variance of the time series of the daily closing prices of the Jordanian Stock Exchange.

Methods

Autoregressive Model (AR)

The Autoregressive model (AR) represents the relationship between the current and previous values of the time series. It can be used for the description of a particular phenomenon, either it is natural or economic. The AR model of order p can be represented by the following formula (Falk et al., 2006), (Hall and Yao, 2003), (Raheem, 2017):

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p} + e_t \dots (1)$$

where,

Y_t : the value of time series in time (t)

$\phi_1, \phi_2, \dots, \phi_p$: the finite set of weight parameters that should be estimated and,

e_t is the error term (white noise)

If Y_t is stationary and $\phi_1, \phi_2, \dots, \phi_p$ ($\phi_p \neq 0$) are constant, then, $e_t \sim N(0, \sigma_e^2)$

Moving Average Models (MA)

The moving-average model (MA), also known as the moving-average process, is a standard method for modelling univariate time series. The MA models specify that the output variable depends linearly on the current and various past values of a stochastic [Wikipedia]. The MA model of order q can be formulated as follows (Box, 2013), (Shumway and Stoffer, 2017):

$$Y_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \dots - \theta_q e_{t-q} \quad \dots (2)$$

where,

$\theta_1, \theta_2, \dots, \theta_q (\theta_q \neq 0)$: the finite set of weight parameters that should be estimated and, $e_t \sim N(0, \sigma_e^2)$

Mixed Autoregressive Moving Average Model (ARMA)

In the analysis of time series, (ARMA) models supplies an attribute of a stationary stochastic process for two polynomials models, one for the AR model and the other for the (MA) model. Also, the (ARMA) approach is a tool for comprehension and forecasting future values of time series. The (AR) part includes regressing the variable on its own lagged (i.e., past) values. The (MA) part includes modelling the error term as a linear combination of error terms occurring contemporaneously and at various times in the past. This model is usually called as (ARMA (p, q)), where p represents the instruct of autoregressive and q is the moving average order. The ARMA model (p, q) can be written as follows (Falk et al., 2006; Hamilton, 1994; Raheem, 2017);

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \dots - \theta_q e_{t-q} \quad \dots (3)$$

where; ϕ, θ represent the parameters of AR and MA models, respectively, and $e_t \sim IND(0, \sigma_e^2)$ is distributed as an identically independent normal distribution.

Autoregressive Conditional Heteroscedasticity (ARCH)

The ARCH model is suggested by Engle in 1982 (Engle, 1982). The ARCH models are statistical techniques that describe the variance of the current error term. It is appropriate when the error variance in a time series follows an AR model, and commonly applied in modelling financial time series that exhibit time varying volatility and volatility clustering. It is the first model of conditional autoregressive to heteroscedasticity of error variations. This model is characterised by its average being equal to zero, with constant variance and is conditional in the past. Therefore, consideration must be given, the conditional variance may be influenced by the sequence of squared values of errors for past periods (Sampson, 2001; Shumway & Stoffer, 2017). The ARCH model can be described as the follows.

Assume Y_t is a time series with average equal to zero and variance equal to σ_t^2 , then, the suggestion model for Y_t is given by

$$Y_t = \mu + x_t \quad \dots (4)$$

$$x_t = \sigma_t * \varepsilon_t \quad , \quad \varepsilon_t \sim IND(0,1) \quad \dots (5)$$

$$\sigma_t^2 = \Omega + \alpha_1 x_{t-1}^2 + \alpha_2 x_{t-2}^2 + \dots + \alpha_p x_{t-p}^2 \quad \dots (6)$$

The Equation (6) represents the equation of volatility, that can be rewritten as

$$\sigma_t^2 = \alpha_0 + \alpha_1 x_{t-1}^2 + \alpha_2 x_{t-2}^2 + \dots + \alpha_p x_{t-p}^2 \quad \dots (7)$$

where Ω and $\alpha_0, \alpha_i \geq 0, i = 1, 2, \dots, p$ is the parameters of the model.

Generalised Autoregressive Conditional Heteroscedastic (GARCh)

Generalised Autoregressive Conditional Heteroscedastic (GARCh) model is developed by Bollerslev in 1986 (Bollerslev, 1986). The GARCh model is defined as follows.

When $P \geq 1$ and $q \geq 1$, then,

$$\sigma_t^2 = \Omega + \alpha_1 x_{t-1}^2 + \alpha_2 x_{t-2}^2 + \dots + \alpha_p x_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-2}^2 + \dots + \beta_q \sigma_{t-q}^2 \quad \dots (8)$$

The σ_t^2 is defined as an equation of volatility, that can be rewritten as follows (Falk et al., 2006).

$$\sigma_t^2 = \Omega + \sum_{j=1}^p \alpha_j x_{t-j}^2 + \sum_{i=1}^q \beta_i \sigma_{t-i}^2 \quad \dots (9)$$

Exponential Generalised Autoregressive Conditional Heteroscedastic Models (EGARCh)

The Exponential Generalised Autoregressive Conditional Heteroscedastic (EGARCh) Model is suggested by Nelson in 1991 (Nelson, 1991). In this model, the dependent variable is the conditional variance algorithm. In addition, this model differs from the GARCh model that assumes similar fluctuation around the shock. Moreover, it describes the relationship between the previous random error values and the conditional variance logarithm. The EGARCh model can be formulated according to the following equation (Nelson, 1991).

$$\ln(\sigma_t^2) = \Omega + \sum_{j=1}^q \beta_j \ln(\sigma_{t-j}^2) + \sum_{i=1}^p \alpha_i \left\{ \left| \frac{x_{t-i}}{\sigma_{t-i}} \right| - \sqrt{\frac{2}{\pi}} \right\} + \lambda_i \frac{x_{t-i}}{\sigma_{t-i}} \dots (11)$$

where Ω and β_j , $\alpha_i; j=1,2,\dots,q, i=1,2,\dots,p$ are not require to be positive.

Threshold Generalised Autoregressive Conditional Heteroscedastic Models (TGARCH)

The TGARCH model was proposed in 1991 by Rabemanajara and Zakoian. It is a commonly used model for dealing with financial volatility (Zakoian, 1994).

When,

$$d_{t-i} = \begin{cases} 1 & \text{if } x_{t-i} < 0 & \text{bad news} \\ 0 & \text{if } x_{t-i} \geq 0 & \text{good news} \end{cases}$$

where, d_{t-i} is a dummy variable,

Then, the TGARCH model of class [p, q; p ≥ 1, q ≥ 1] can be defined as following (Sampson, 2001; Zakoian, 1994).

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^p (\alpha_i + \gamma_i d_{t-i}) x_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \dots (12)$$

where, $\alpha_0 > 0$ and $\beta_j \geq 0, \alpha_i \geq 0, i=1,2,\dots,p, j=1,2,\dots,q$

Application Section

The applied Section includes an applied study on the construction and selection of the fluctuation models suitable for the real data related to the research subject.

Data Description and Tool of Study

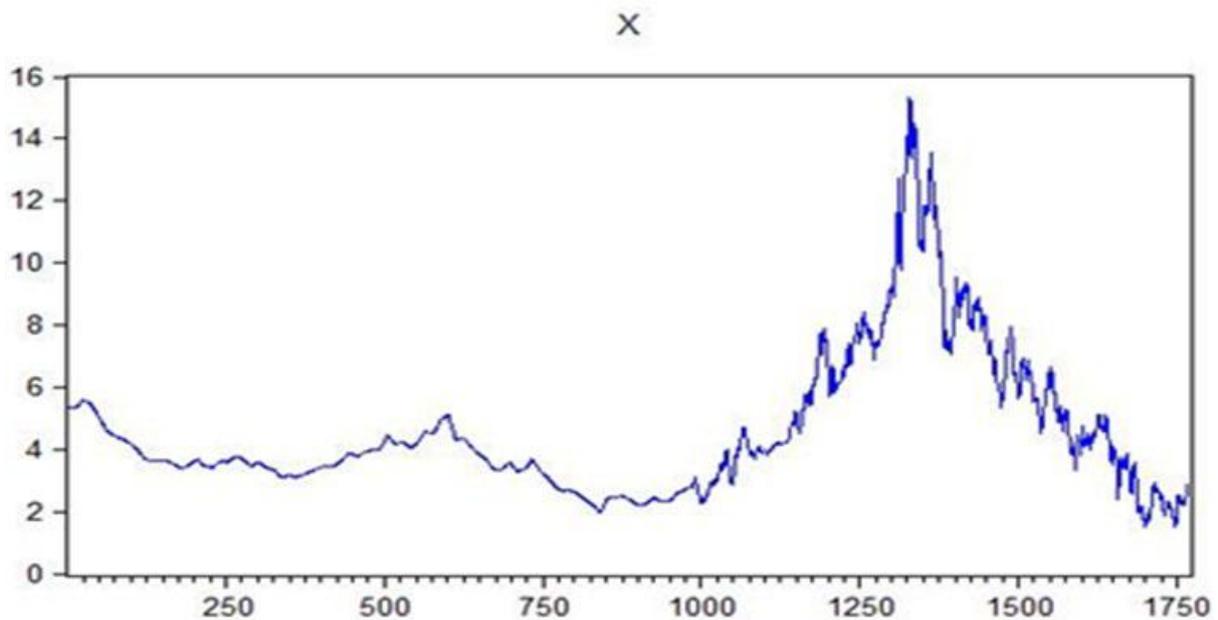
The sample of the study consisted of 1768 observations of the Jordan Stock Exchange for the period (1871-2018) using the Autoregressive Conditional Heteroscedasticity model. The study tools include using the AR model, autocorrelation function (ACF) and partial autocorrelation function (PACF) as follows.

Firstly, the unity root of the time series is examined using KPSS and Philips-Perron tests and then, the normal distribution of the time series is studied using Jarque-Bera. Finally, stability is examined using the BDS test. The analysis of time series data was done by using Eviwes 9 software.

Results and Discussion

To show the features of the particular time series in terms of the stability of the series in the average and variance, we draw the data using the fluctuation chart. Figure 1 shows the instability of the series in the average and variance. Figure 4 shows that the minimum value in the time series is (-1.760000) and the maximum value is (1.610000), and the mean and the standard deviation are (-0.001426) and (0.178321), respectively.

Figure 1. The plot of time series for daily closing prices of Jordan Stock Exchange for the period (1871 -2018)

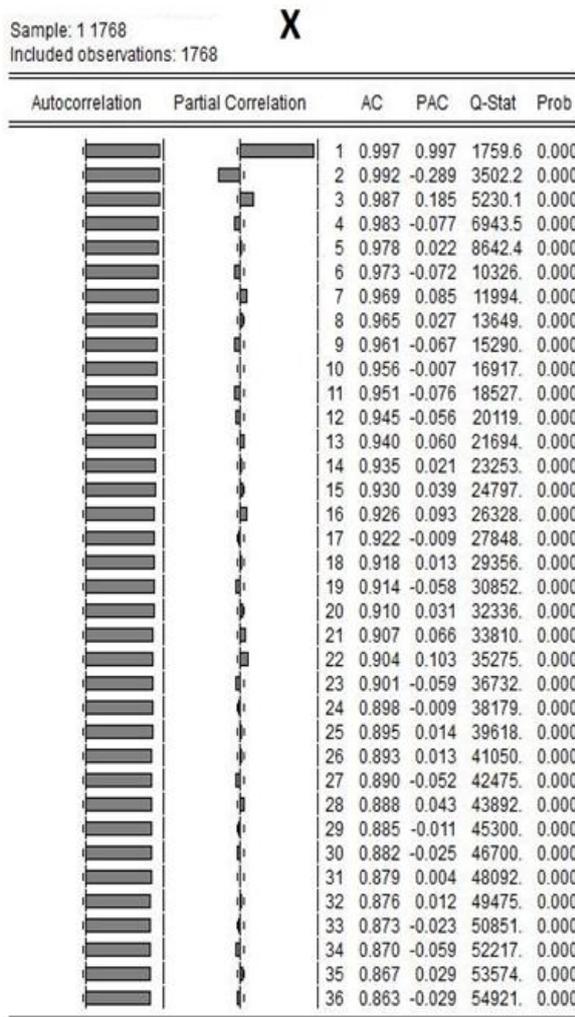


ACF and PACT Tests

The values of ACF and PACF in Figure 2-a shows non-stationary of the time series X due to its significantly different from zero at the ratio of 0.05, i.e., it is outside of the confidence period $[(-1.96) / \sqrt{n}, (+1.96) / \sqrt{n}]$. Here the difference of the first order of the time series must be taken. The values of ACF and PACF after taken the first-order difference to indicate that the time series becomes stationary, as shown in Figure 2b and Figure 3.

Figure 2. The values of ACF and PACF

(a) for the original time series (x)



(b) after taken the first order difference (dx)

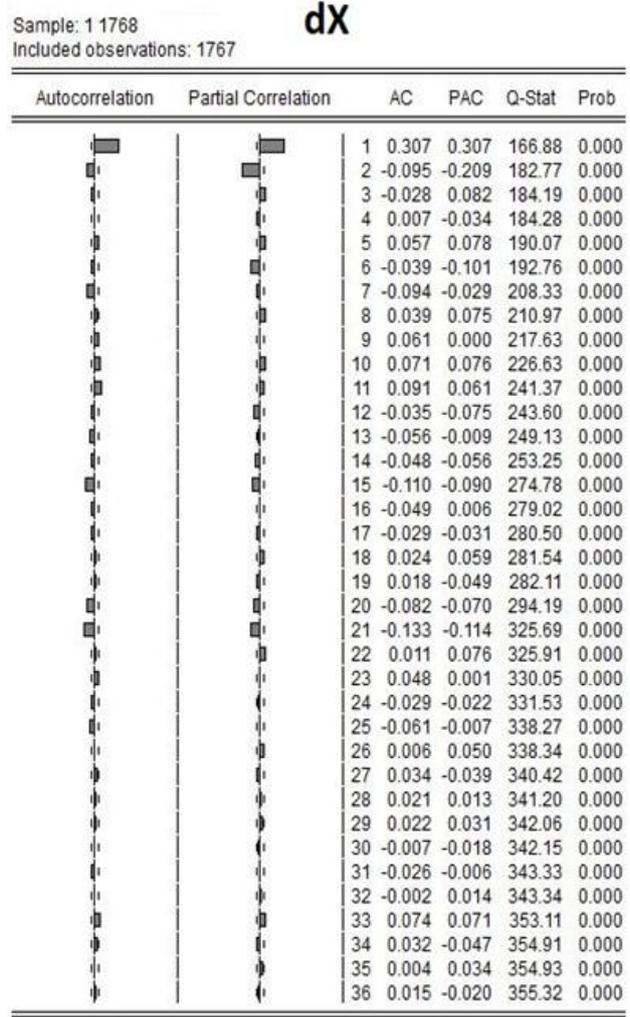


Table 2 shows the results of tests of unit root after taken the first-order difference for the time series data. These results refer to the time series not include a unit root, where, the p-values were smaller than 0.005 that indicate the time series is stationary.

The Unit Root Tests

Table 1 shows the results of the unit root test for the time series. The results indicate the series include the unit root, where, the p-values were greater than 0.005 leads to non-stationary of the original time series.

Figure 3. The plot of time series after taken the first-order difference

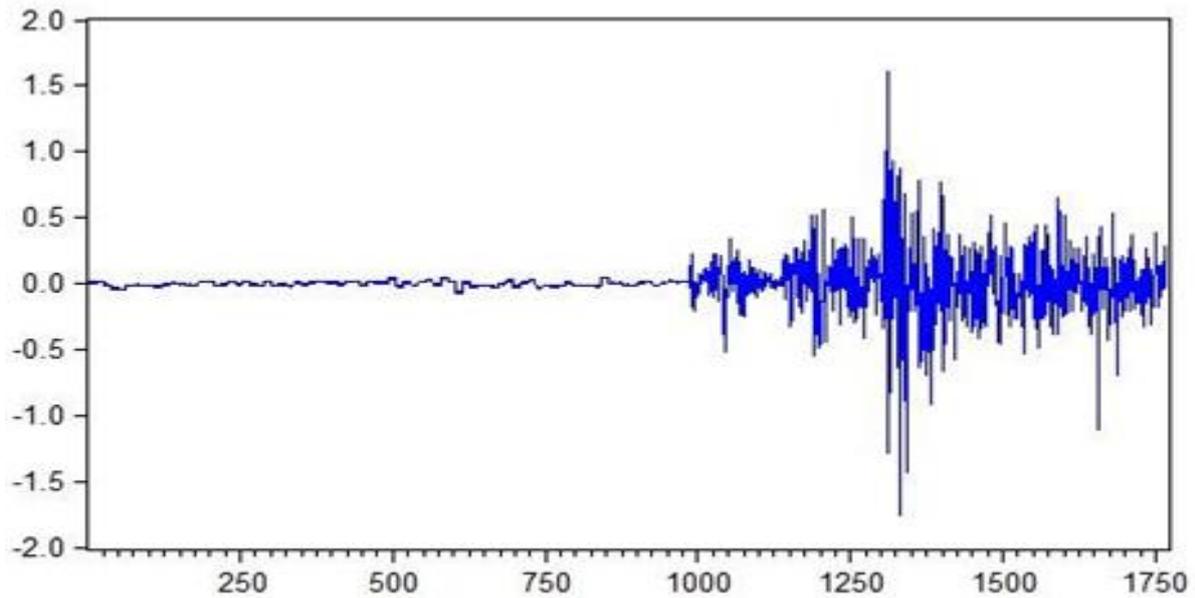


Table 1: The results of the tests for the original time series

Test	Intercept	Trend Intercept	and None
ADF	- 1.806894	- 1.841961	- 1.070795
	- 2.862977	- 3.412363	- 1.941003
P-Value	0.3775	0.6839	0.2575
Philips-Perron	- 1.896860	- 1.936450	- 1.102504
	- 2.862972	- 3.412354	- 1.941003
P-Value	0.3340	0.6348	0.2456
KPSS	1.094638	0.373425	
	0.463000	0.146000	

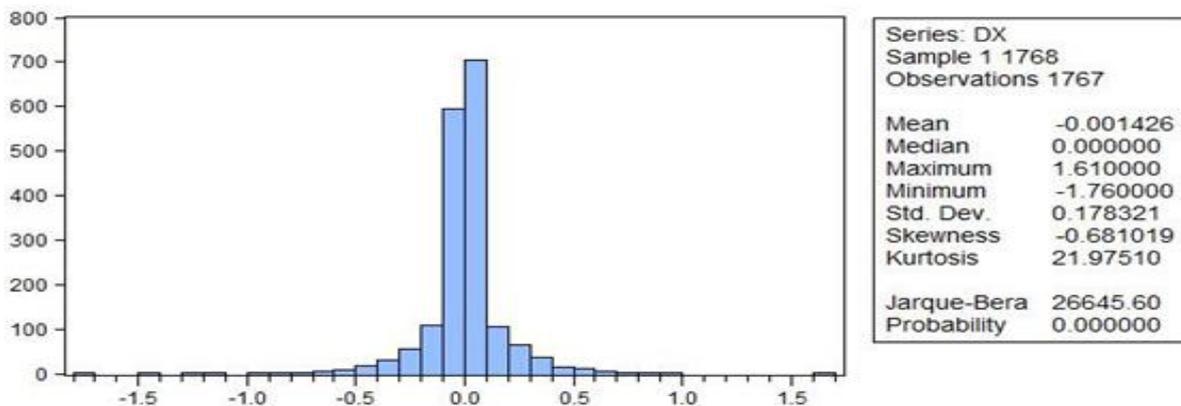
Table 2: The results of the tests after taking the first-order difference

Test	Intercept	Trend and Intercept	None
ADF	-17.53383	-17.53012	-17.53589
P-Value	0.000	0.000	0.000
Philips-Perron	-29.22670	-29.21520	-29.23845
P-Value	0.000	0.000	0.000
KPSS	0.087866	0.085523	
	0.463000	0.146000	

Testing of Normality

From Figure 4, it is easy to note that the value of the skewness coefficient was negative (-0.681019), which indicates that the error distribution is skewed to the left (negative skewed). Also, we can note that the kurtosis coefficient value is (21.97510) which differs from the value of "3" characteristic of the normal distribution, indicates that the time series (dx) is high dispersion and therefore differs from the normal distribution. This result is confirmed by the Jarque-Bera statistics, which indicate the returns time series not follows the normal distribution at a significant level (0.05). This is finding is a general feature of financial time series.

Figure 4. The descriptive statistics of time series after taken the first-order difference



Analysis of ARCH Model

In this part, five methods such as (OLS, ARCH, GARCH, EGARCH, H TARCH) are compared via the SIC and AIC coefficients. The results are presented in tables 3-10.

Table 3: The results of the OLS method for the time series after taken the first-order difference

Dependent Variable: DX
Method: Least Squares
Date: 08/28/19 Time: 23:34
Sample (adjusted): 2 1768
Included observations: 1767 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.001426	0.004242	-0.336186	0.7368
R-squared	0.000000	Mean dependent var		-0.001426
Adjusted R-squared	0.000000	S.D. dependent var		0.178321
S.E. of regression	0.178321	Akaike info criterion		-0.609896
Sum squared resid	56.15601	Schwarz criterion		-0.606796
Log likelihood	539.8430	Hannan-Quinn criter.		-0.608751
Durbin-Watson stat	1.385871			

Testing of ARCH Model

Table 4 demonstrates the results of the ARCH model, and we can see the p-value is less than 0.05 indicate to the time series is influenced by the ARCH model.

Table 4: The results of the ARCH model

ARCH TEST			
F-statistic	2.864367	Prob.F(1,1765)	0.000
Obs*R-squared	2.862967	Prob Chi-Square(1)	0.000

Through the p-value that given in Table 5 is more than 0.05, so there is no influence for the ARCH model on the time series.

Table 5: ARCH test results for residues

ARCH test results for residues			
F-statistic	136.1590	Prob. F(2,1764)	0.07743
Obs*R-squared	236.3020	Prob. Chi-Square(2)	0.07742

Table 6 demonstrates the SIC and AIC values for the methods of study; the ARCH model is the best method due to it has the minimum value of SIC and AIC coefficients.

Table 6: demonstrates the values of SIC and AIC coefficients

	ARCH	GARCH	TGARCH	EGARCH
AIC	-3.13543	-3.16106	-3.16191	-3.1406
SIC	-3.11683	-3.14866	-3.14641	-3.1251

Table 7: The results of the ARCH model

Dependent Variable: DX
 Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
 Date: 08/28/19 Time: 23:24
 Sample (adjusted): 2 1768
 Included observations: 1767 after adjustments
 Convergence achieved after 60 iterations
 Coefficient covariance computed using outer product of gradients
 Presample variance: backcast (parameter = 0.7)
 $Q = C(2) + C(3) * (Q(-1) - C(2)) + C(4) * (RESID(-1)^2 - GARCH(-1))$
 $GARCH = Q + C(5) * (RESID(-1)^2 - Q(-1)) + C(6) * (GARCH(-1) - Q(-1))$

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.003411	0.000329	-10.35750	0.0000
Variance Equation				
C(2)	3.40E-05	3.15E-05	1.077577	0.2812
C(3)	1.019624	0.009551	106.7548	0.0000
C(4)	0.232328	0.084760	2.741010	0.0061
C(5)	0.177394	0.070533	2.515052	0.0119
C(6)	0.790861	0.080591	9.813266	0.0000
R-squared	-0.000124	Mean dependent var		-0.001426
Adjusted R-squared	-0.000124	S.D. dependent var		0.178321
S.E. of regression	0.178332	Akaike info criterion		-3.135430
Sum squared resid	56.16297	Schwarz criterion		-3.116832
Log likelihood	2820.328	Hannan-Quinn criter.		-3.178559
Durbin-Watson stat	1.385700			

Table 8: The results of the GARCH model

Dependent Variable: DX
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 08/28/19 Time: 23:54
Sample (adjusted): 2 1768
Included observations: 1767 after adjustments
Convergence achieved after 33 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.002347	0.000328	-7.156600	0.0000

Variance Equation

	Coefficient	Std. Error	z-Statistic	Prob.
C	5.91E-06	1.15E-06	5.156505	0.0000
RESID(-1)^2	0.305508	0.015720	19.43381	0.0000
GARCH(-1)	0.745525	0.009663	77.15476	0.0000

R-squared	-0.000027	Mean dependent var	-0.001426
Adjusted R-squared	-0.000027	S.D. dependent var	0.178321
S.E. of regression	0.178323	Akaike info criterion	-3.161061
Sum squared resid	56.15750	Schwarz criterion	-3.148662
Log likelihood	2796.797	Hannan-Quinn criter.	-3.156480
Durbin-Watson stat	1.385834		

Table 9: The results of the TGARCH model

Dependent Variable: DX
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 08/28/19 Time: 23:22
Sample (adjusted): 2 1768
Included observations: 1767 after adjustments
Convergence achieved after 33 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-1)^2*(RESID(-1)<0) + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.002118	0.000368	-5.759126	0.0000

Variance Equation

	Coefficient	Std. Error	z-Statistic	Prob.
C	5.85E-06	1.19E-06	4.928421	0.0000
RESID(-1)^2	0.328169	0.020454	16.04448	0.0000
RESID(-1)^2*(RESID(-1)<0)	-0.059307	0.027937	-2.122904	0.0338
GARCH(-1)	0.750066	0.009988	75.09605	0.0000

R-squared	-0.000015	Mean dependent var	-0.001426
Adjusted R-squared	-0.000015	S.D. dependent var	0.178321
S.E. of regression	0.178322	Akaike info criterion	-3.161908
Sum squared resid	56.15685	Schwarz criterion	-3.146409
Log likelihood	2798.545	Hannan-Quinn criter.	-3.156181
Durbin-Watson stat	1.385850		

Table 10: The results of the EGARCH model

Dependent Variable: DX
 Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
 Date: 08/28/19 Time: 23:23
 Sample (adjusted): 2 1768
 Included observations: 1767 after adjustments
 Convergence achieved after 36 iterations
 Coefficient covariance computed using outer product of gradients
 Presample variance: backcast (parameter = 0.7)
 $\text{LOG}(\text{GARCH}) = \text{C}(2) + \text{C}(3) * \text{ABS}(\text{RESID}(-1) / \sqrt{\text{GARCH}(-1)}) + \text{C}(4) * \text{RESID}(-1) / \sqrt{\text{GARCH}(-1)} + \text{C}(5) * \text{LOG}(\text{GARCH}(-1))$

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.003950	0.000275	-14.34217	0.0000
Variance Equation				
C(2)	-0.398018	0.018200	-21.86959	0.0000
C(3)	0.515546	0.019375	26.60879	0.0000
C(4)	0.026214	0.013521	1.938765	0.0525
C(5)	1.002535	0.001689	593.6829	0.0000
R-squared	-0.000200	Mean dependent var	-0.001426	
Adjusted R-squared	-0.000200	S.D. dependent var	0.178321	
S.E. of regression	0.178339	Akaike info criterion	-3.140599	
Sum squared resid	56.16726	Schwarz criterion	-3.125101	
Log likelihood	2779.720	Hannan-Quinn criter.	-3.134873	
Durbin-Watson stat	1.385594			

Conclusions

1. The results of the analysis show that the daily closing prices of the Jordan Stock Exchange are unstable.
2. The Dickey Fuller test shows that the time series data is non-stationary on the mean.
- 3 - The Jarque Bera test showed that the series under study is not following the normal distribution.
4. From the comparison criteria, we conclude that the ARCH model is the best model to represent the time series data under study.

Recommendations

- 1- We recommend adopting the (ARCH 5) model to predict daily closing prices.
- 2- We recommend studying other models to analyse volatile financial chains (NGARCH, IGARCH and GJR- GARCH) and compare them with ARMA and GARCH models by using normal distribution and t-distribution.



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