



Mathematical literacy of ninth-grade students in solving PISA mathematics problems

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The purpose of this study is to analyse the mathematical literacy of junior high school students. There are three mathematical processes of mathematical literacy: Formulate, employ, and interpret. The participants of this study were four students in the ninth grade at two junior high schools in Lombok, Indonesia. Data was collected using a test and an interview. There were four Programme for International Student Assessment (PISA) mathematics items used in this study. A meeting was conducted to confirm the students' work in solving mathematical literacy tests. A qualitative descriptive analysis was used to analyse the results, most of the students had difficulties in interpreting, applying, and evaluating mathematical results. A small number of the students had problems formulating situations mathematically.

Keywords: Junior high school, mathematical literacy, mathematical process, Programme for International Student Assessment



Introduction

Mathematical literacy is a term commonly used since the Organization for Economic Co-operation and Development (OECD) initiated a study called the Program for International Student Assessment (PISA) in the 1990s. PISA is one of the international studies that assess mathematical, scientific, and reading literacy for 15-year-old students (Lemke et al., 2001. Tout & Gal, 2015). This study was conducted for both OECD member countries and non-OECD members throughout the world, including Indonesia. Indonesia itself has been following PISA since 2000 consistently.

Mathematical literacy is one of the fundamental competencies for a student. It is needed to solve his/her daily life problems, both inside and outside of school (Ahyan, Turmudi, & Prabawanto, 2019). OECD defines mathematical literacy as an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts (OECD, 2013; OECD 2017). Mathematical literacy is less formal and more intuitive, less abstract, and more contextual, less symbolic, and more concrete (Ewell, 2001). Where mathematical literacy itself is one of the main instructional objectives in schools (Watson, 2002; Steen, Turner, & Burkhardt (2007), although, in the education curriculum in Indonesia, it is not explicitly explained. Mathematics in schools focuses on substantive content, while mathematical literacy focuses on how to use mathematics that has been obtained in class into real life or outside of school (de Lange, 2003; Sumirattana, Makanong, & Thipkong, 2017).

Based on the PISA study from 2000 to 2018, the average score for the mathematical literacy of Indonesian students is still lower than the OECD average (Stacey, 2011; OECD, 2019). For example, the recent study of PISA (PISA 2018) shows that the mathematical literacy of Indonesian student only got 379 points, whereas the OECD average is 489 (OECD, 2019).

The PISA study shows that Indonesian students of average find it more difficult to solve the more complex mathematics problems. Where some 28% of Indonesian students attained level 2 or higher (OECD, 2019), 76% of OECD students averaged at level 2 or higher. Around 1% of Indonesian students performed at level 5 or higher (OECD, 2019). The mathematical literacy of Indonesian students' needs the attention of policymakers, academicians, practitioners, researchers, and others.

Students' mathematical literacy is topical in both Indonesia and overseas. In Indonesia, researchers investigated students solving PISA contextual mathematics problems (by Lutfianto et al.). Researchers interpreted a correlation between student errors and contextual problems. Namely, researchers found the more student errors in PISA mathematics problems (by Ahyan et al.)for Junior high school student, the lesser their ability to solve contextual problems (Lutfianto,



Zulkardi, & Hartono, 2013). Ahyan et al. found that students experienced obstacles in transforming contextual problems or real problems into mathematical problems – such as there was an error in comprehension, transformation, and process skills (Ahyan, Turmudi, & Prabawanto, 2019). To date, no research has analysed the mathematical literacy of ninth-grade students. This article describes how ninth-grade students went about formulating, employing and interpreting mathematical problems.

The history of mathematical literacy

Written information about the term ‘mathematical literacy’ first appeared in 1944 in the United States of America (USA). The National Council of Teachers of Mathematics (NCTM) in Post-War Plans required that schools ensure mathematical literacy for all (Jablonka & Niss, 2014). Furthermore, in 1950, the term was used in the *Canadian Hope Report*.

On the other hand, in addition to the term mathematical literacy, the term ‘numeracy’ appears. The term ‘numeration’ first appeared in 1959 through the *Webster's Collegiate Dictionary*, where ‘numeric’ is presented as the ability to think and express quantitatively (see also Jablonka, 2003) via the *Crowther Report* on numeracy (DES, 1959).

Even though the term numeration appeared in 1959, ‘innumeracy’ and ‘mathematical illiteracy’ became familiar terms only later in 1988 (Jablonka, 2003). The terms innumeracy and mathematical illiteracy were used by John A. Paulos in *Innumeracy, Mathematical Illiteracy, and Its Consequences* (Jablonka, 2003). The book became a best-seller in the USA. In the book, Paulos (1988) presents many authentic examples of innumeracy and develops the concept of numeracy as one's ability to understand better than the quantitative aspects of life. Edward Tufte wrote on how secure quantitative information is in communication and persuasion (Tufte, 1983, 1990, 1997). In 1989, NCTM referred to mathematical literacy and students who were literate mathematically in response to people's precise needs. They issued standards for school mathematics so that students could have access to rich and challenging mathematical concepts (Steen, 2001).

On the other hand, the term ‘quantitative literacy’ has been widely used since Steen (2001) in his book *Mathematics and Democracy: The case for quantitative literacy*. Steen (2001) defined quantitative literacy as a person's capacity to deal with quantitative aspects effectively. Steen's definition refers to this as document literacy following the definition adopted by the National Center for Education Statistics (NCES) in 1993.

The definition of mathematical literacy has not been offered at either the NCTM Standards or the Canadian Hope Report. An explicit description of mathematical literacy was found in the



framework of the initial Organization for Economic and Co-operation Development (OECD) for the 1999 Program for International Student Assessment (Jablonka & Niss, 2014) and developed by Expert Group for PISA mathematics (de Lange, 2014) consisting of eight members, namely, Jan de Lange (Netherlands), Raimondo Bolletta (Italy), Sean Close (Ireland), Maria Luisa Moreno (Spain), Mogens Niss (Denmark), Kyung Mee Park (South Korea), Thomas Romberg (United States), and Peter Schuller (Austria) (OECD, 1999). The definition of mathematical literacy was first put forward by the OECD in 1999 through the PISA Framework book entitled *Measuring Student Knowledge and Skills: A New Framework for Assessment*.

In an interview in *Mathematics and Democracy* (Steen, 2001), Peter T. Ewell was asked about mathematical literacy: Is quantitative literacy something less formal but more intuitive, less abstract but more contextual, less symbolic but more concrete? Peter T. Ewell (2001) said that mathematics and mathematical literacy is significantly different. However, this difference is meaningful and powerful. Mathematical literacy is less formal and more intuitive, less abstract and more contextual, less symbolic and more concrete (National Center for Higher Education Management Systems [NCHEMS], 2001). mathematical literacy focuses on attention and emphasises reasoning, thinking, and interpretation as well as on other mathematical competencies (de Lange, 2014).

In this study, we use the mathematical literacy definition by OECD (2013; 2017). Mathematical literacy is defined as follows:

Mathematical literacy is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged, and reflective citizens.

Mathematical literacy is often also referred to as 'mathematization' (de Lange, 1999). Mathematics, according to de Lange (1987), is defined as organising activities by finding regularities, relations, and structures using fundamental knowledge and skills. PISA uses real world-based questions, so students have a mathematical process to solve real-world problems mathematically (de Lange, 1987; Kaiser, 2005).

The mathematical process in mathematical literacy is categorised into three (OECD, 2017):

1. Formulate the situation mathematically. 'Formulate' in mathematical literacy is defined as the ability of individuals to recognize and identify opportunities to use mathematics and then provide a mathematical structure for problems presented in several contextual forms.



2. Employ the concepts, facts, procedures, and mathematical reasons. Employing mathematical literacy is defined as the ability of individuals to apply concepts, events, methods, and mathematical reasoning to solve problems that are formulated mathematically to obtain accurate conclusions.
3. Interpret, apply, and evaluate mathematical results. Interpreting mathematical literacy is understood as the ability of individuals to reflect numerical solutions, findings, or conclusions and interpret them in real-life contexts.

Methods

Participants

This qualitative study uses a case study approach. Researchers want to know students' mathematical literacy in junior high school (SMP) students, both in and outside the city district. This research was conducted with ninth-grade students in two junior high schools in East Lombok, Indonesia. Four students participated in this study, two students from each school. The selection of participants is based on the results of discussions with mathematics teachers of ninth grade in the two schools.

Data Collection

Data was collected using tests and interviews. The analysis is carried out to determine the students' mathematical literacy by the way students answer four descriptive questions. The questions used in this test were issued by the OECD-PISA and are modified according to the context of students in East Lombok, Indonesia. The interviews were conducted to obtain information verbally from students and their answers were recorded on an answer sheet. The four students were asked for information about their solutions.

Data Analysis

'Qualitative descriptive' was used in this data analysis. This analysis aimed to analyse the research data on the mathematical literacy of junior high school students descriptively. Furthermore, interview data was used to confirm the answers of the students. The data are analysed based on the mathematical process on mathematical literacy issued by OECD-PISA.

Results and Discussion

In this results and discussion section, students' mathematical literacy is explained in three mathematical processes: Formulate, employ, and interpret. See Table 1.

The ability to formulate situations mathematically

Based on the data, 56.25% of students' showed the ability to develop the case mathematically. Only one student could express all the problems given – student B. All students could show the first question, with 50% of students showing responses to the second and third questions, and 25% of students could formulate the fourth question. Most of the students couldn't formulate these problems because they were not careful in translating the contextual issues given into mathematical problems. For example, in the second question, an elementary challenge was presented, namely counting the length of plastic on the Counter Service table. If students noticed, then Pythagorean Theorem could be applied to the mathematical concept. However, only Student D drew this distinction (Figure 2). Student B used the long calculation approach manually, but the calculations were still not accurate (Figure 3). Students A and C were not able to formulate the problem given. They looked for the length of the hypotenuse by multiplying 0.5×3 , where the number 3 is estimated to be the length of the box on the sloping side.

Based on the results of the interview, Student B reported that he forgot how to look for the sloping side. The following is a transcript of the interview with Student B:

Researcher: What about this? How long is the outer edge of the table?

Student B: This, bro, is a bit of a problem, I forgot.

Researcher: What is calculated on this number?

Student B: This one is said to be 0.5 one box, 0.5; 0.5 (while pointing to the upright side), then 0.5; 0.5 (the sambal points to the flat side), then this is divided into half and so.

Researcher: How is it divided?

Student B: This one, like we consider divided only half, is this one second (showing a box on the upright side), one second (showing a box on the flat side) and this is all half (pointing the box on the sloping side)

Researcher: What is the length? (looking at the hypotenuse)

Student B: If it's half the length is 0.25

Researcher: Where did seven get?

Student B: From this (while pointing at the image that was made)

The picture referred to by student B is shown in the following figure 3.

The ability to formulate the problem is essential to solve the mathematical problem. Kilpatrick (1987) describe that 'problem-expressing' is a vital companion for solving the problem. Kamaliyah, Zulkardi, and Darmawijoyo (2013) also said that the ability to formulate the problem is a crucial ability for mathematical literacy. The knowledge is obtained from students' having an

existing accurate understanding of the processes. If students' knowledge of mathematics is weak, their ability to formulate problems will also be.

The ability to employ concepts, facts, procedures, and mathematical reasons

Based on the data, 37.5% of students has the ability to apply concepts, events, methods, and precise ideas in solving the problems. No student could employ the concepts, facts, procedures, and mathematical reasons for all the questions. All students used thoughts, events, methods, and precise ideas just for the first question. 25% of students could employ ideas, facts, procedures, and mathematical ideas for the second, third, and fourth questions. For example, in the third question, the problem given is related to calculating the area of carpet covering the store floor of Ashfiya, except the service area and counter service table. Only Student D employed the concepts, facts, procedures, and mathematical reasons to solve these problems. Student D calculated the area of all store floors and service area and service counter, and then both were reduced (Figure 5). In other content, Student B had formulated the problem, but he was unable to use concepts, facts, procedures, and mathematical reasons to solve these problems. Student B divided the area to be searched into three parts, namely area I (dining area), area II (entrance area), and area III (service counter area). Student B had not been able to calculate the area of each area (Figure 6). Meanwhile, Students A and C could not use concepts, facts, procedures, and mathematical reasons for solving this third question.

Based on the results of the interview, Student B said that he forgot how to find the carpet area needed in area III. The following is a transcript of the interview with Student B:

Researcher: Area intended for number 3, carpet area?

Student B: Look for this one (the designate area I), look for this one (designate an area II), join it together, continue to join this (pointing area III).

Researcher: So how do you find this (pointing to the area I)?

Student B: This one? One, two, three, four, five, six, seven, eight, nine, ten. Wrong means before. Means, 0.5, should be multiplied by eight, plus 0.5 times ten.

Researcher: What about this one? (designate area III)

Student B: If this one is only estimated, 0.5; 0.5; 0.5 is left to add

Researchers: Which 0.5?

Student B: Search for those that are still intact first

Researcher: How about the whole thing?

Student B: It is estimated that the problem is forgotten. It has not been studied for a long time.

This ability requires understanding and knowledge of mathematical concepts well. Students can be productively applied to problem situations if they have school mathematical knowledge (contextual, conceptual, and procedural) (Sáenz, 2009).

The ability to interpret, apply and evaluate mathematical results

Based on the data, 31.25% of students had the ability to understand, use, and assess numerical results. No student could understand, implement, and evaluate numerical results for all questions given. Some 75% of students could understand, implement, and evaluate the mathematical results of the first question. Around 25% of students could interpret, apply, and evaluate the numerical results of the second and third questions. However, no students explained, used it, and assessed the statistical results of the fourth question. For example, in the fourth question, the problem given is related to the dining table set needed by Ashfiya to be placed in his shop. Students A, C, and D did not formulate mathematical situations. They did not employ concepts, facts, procedures, and precise reasons and did not interpret, apply and evaluate numerical results (Figure 6). Only Student B formulated mathematical situations, and showed signs of being able to employ concepts, facts, procedures, and precise reasons, but was not ready to interpret, apply, and evaluate numerical results. Student B achieved the first two mathematical processes but did not achieve success in the process of interpretation. Student B has not been able to interpret the number of table sets needed, he is correct in the calculation process, but he forgets what counts is only half the area of the eating area (Figure 7).

The following is a transcript of the interview with Student B:

Researcher: Do you know what number 4 means?

Student B: No, what is used is drawn this one (sambal refers to the image on the question), same, meaning the length of the width is also the same

Researcher: If number 4 is asked for this one (while pointing to the dining area), to fill the dining area.

Student B: My God, I was wrong. Earlier, less focus had all been counted.

Many students have difficulties solving the problems which need interpretation, application, and evaluation abilities to tend the issues. These abilities are the highest abilities in Bloom's taxonomy (Thompson, 2008). Not many students have these abilities.

Conclusions

The ability of participating students to tend mathematical processes with a high level of mathematical literacy in this study is relatively low. Most of the students have difficulties in interpreting, applying, and evaluating numerical results (68.75%). Besides, a small number of the students have problems in formulating situations mathematically (43.75%). 37.5% of the students can employ the concepts, facts, procedures, and mathematical reasons for solving problems given. Only one of the students can formulate all of the issues presented. Furthermore, only one student can explain about 75% of all the matters in expressing, employing, and interpreting skills.

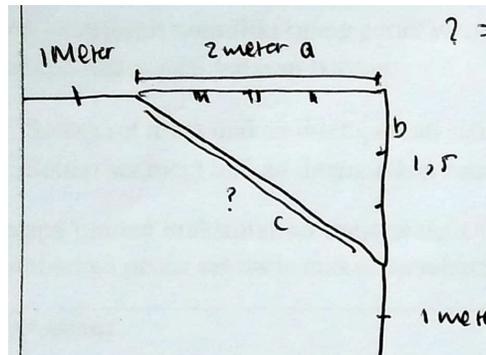


Figure 1. Elementary students use Pythagoras's theorem concept to look for the hypotenuse

Center pelayanan ~~$0,25 + 0,25 + 0$~~
 $= 0,5 + 0,5 + 0,5 + 0,5 + (0,25 \times 7)$

Figure 2. Student B uses ordinary calculations

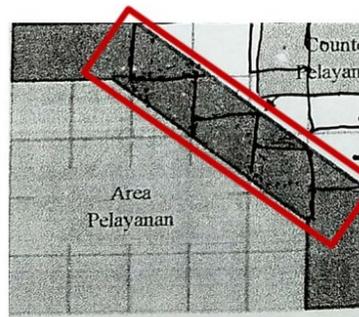


Figure 3. The mistake of Student B in calculating the length of the hypotenuse

$pp = 1 \times 2,5 = 2,5 \text{ m (luas)}$
 $\text{segitiga: } \frac{1,5 \times 2}{2} = \frac{3}{2} = 1,5 \text{ (luas)}$
 $pp = 1 \times 2 = 2 \text{ m (luas)}$

jadi Area pelayanan + meja counter pelayanan
 adalah $2,5 + 1,5 + 2 = 6 \text{ m}$
 maka total luas karpet yang dibutuhkan oleh
 Arwipya adalah: $17 \times 0,5 \text{ m} = 7,5 \text{ m}$
 $10 \times 0,5 \text{ m} = 5 \text{ m}$
 luas keseluruhan = $37,5 \text{ m}$
 jadi total luas karpet yang dibutuhkan $37,5 + 6 \text{ m}$
 $= 31,5 \text{ m}$

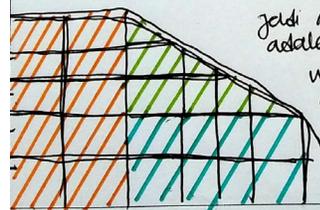


Figure 4. Student D can employ the concepts, facts, procedures, and mathematical reasons

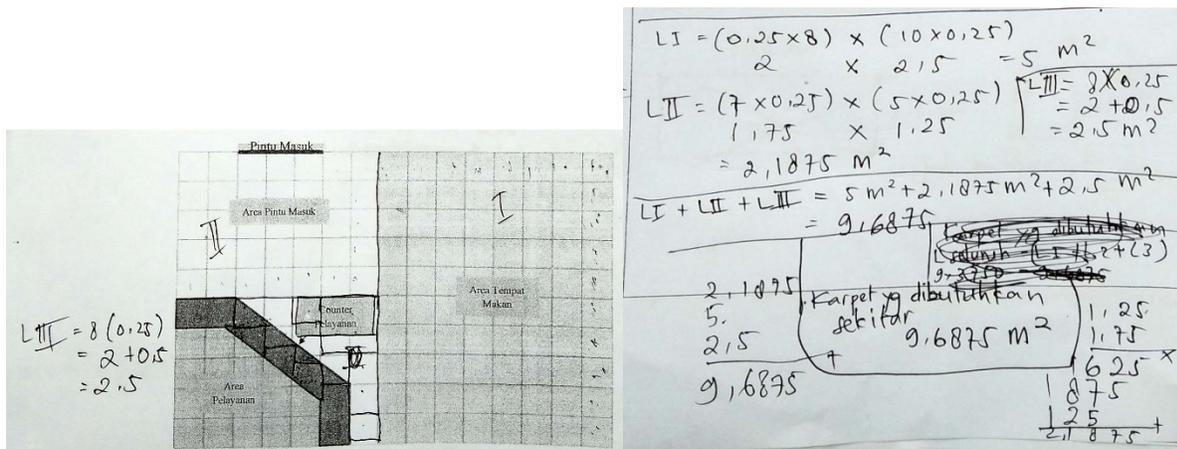
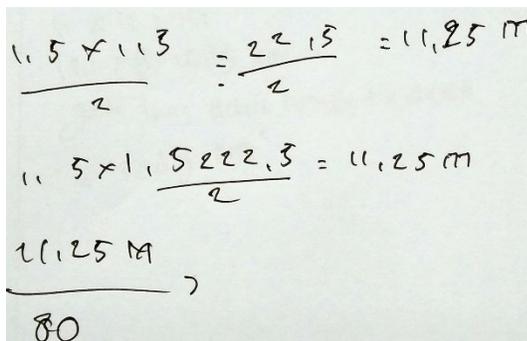


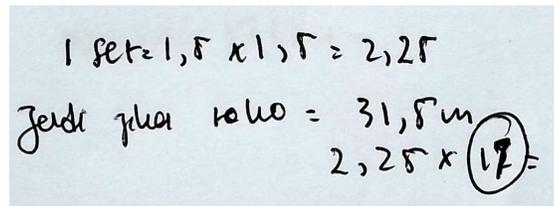
Figure 5. Student B is not able to calculate the area I, II, and III



Student A's answer



Student C's answer



Student D's answer

Figure 6. The incorrect answers of Student A, C and D

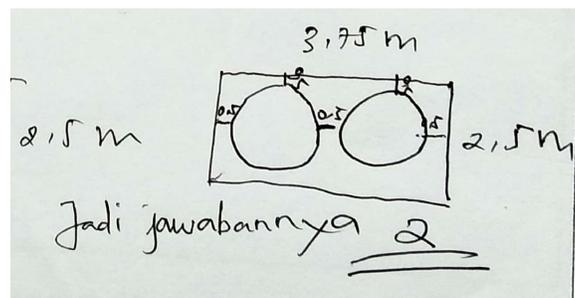


Figure 7. Student D's answer

Table 1. General description of students' mathematical processes

Students' mathematical literacy	Problem 1	Problem 2	Problem 3	Problem 4
Student A	<i>Formulate</i>	+	-	-
	<i>Employ</i>	+	-	-
	<i>Interpret</i>	-	-	-
Student B	<i>Formulate</i>	+	+	+
	<i>Employ</i>	+	-	+
	<i>Interpret</i>	+	-	-
Student C	<i>Formulate</i>	+	-	-
	<i>Employ</i>	+	-	-
	<i>Interpret</i>	+	-	-
Student D	<i>Formulate</i>	+	+	+
	<i>Employ</i>	+	+	-
	<i>Interpret</i>	+	+	-

Description: the sign of plus (+) means capable, and the manifestation of minus (-) means incapable



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