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A Numerical Solution for Optimisation Diffraction Issues of Acoustic Waves

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This research is devoted to solving the optimal control problem for the equations of diffraction of acoustic waves by three-dimensional inclusion. It minimises the deviation of the pressure field in the inclusion from a specific interval, due to sound sources changes in the external environment. External boundary value problems for partial differential equations are the mathematical model of the process. In our research, we offer an algorithm for the numerical solution of the problem subject to a limited set of controls. Namely, it imposes a restriction on the power of sources with which one can control the acoustic field. The solution to the problem can be represented as a linear combination of the solutions of direct diffraction problems and unknown coefficients. We use the coordinate descent to find the required coefficients. If we solve direct problems by potential theory, the diffraction problem reduces to a mixed system of a weak singular boundary integral Fredholm equations of the first and second kind on the inclusion surface. The approximation of integral equations by a system of linear algebraic equations is carried out by dividing the unit on the surface, associated with a system of nodal points, and also consistent with the discretisation order of the method of averaging weak singular kernels of integral operators. Multiple integrals arising during discretisation are calculated analytically. This allows one to obtain explicit formulas for approximating boundary integral operators with singularities in kernels and using them to calculate the coefficients of systems of linear algebraic equations. At the same time, preliminary triangulation of the surface is not required. We ran numerical experiments and mathematical modelling of the diffraction process of acoustic waves. When solving a problem on a computer, parallel computations are used.

Key words: *Acoustic waves, diffraction issue, numerical solution of control problems.*

Introduction

The work is devoted to the solution of optimisation diffraction issues of acoustic waves. Such issues are encountered in flaw detection, ocean acoustics and atmosphere, and geophysics. The mathematical model of the process is external boundary value issues for partial differential equations.

We used finite-difference and projection-grid methods of numerical solution to solve it (Scattering, 1971), (Goryunov and Saskovets, 1989), (Zavadsky, 1991), (Ihlenburg, 1998), (Habbal, 1998), (Yanzhao and Stanescu, 2002), (Jahn et al., 2004), (Knabner and Angermann, 2003), (Savenkova, 2007), (Kabanikhin and Shislenin, 2011), (Konyaev and Delicin, 2014), as well as numerical algorithms based on the transformation of the original differential issues to equivalent integral equations (Colton and Kress, 1987), (Voronin, 1978), (Angell and Kirsch, 2004), (Ershov and Smagin, 1993), (Ershov et al., 2010), (Evstegneev and Medvedik, 2014).

Current research is a continuation of articles (Ershov and Smagin, 1993), (Ershov et al., 2010) (Illarionova, 2008) (Illarionova, 2011) devoted to the development of the indirect method of integral equations for the numerical solution of direct and optimisation diffraction issues. The research considers a convex optimisation diffraction issue of stationary acoustic waves in a homogeneous medium with an inclusion.

Objective assignment

Suppose that in a space R^3, filled with a uniform isotropic medium, there is a homogeneous bounded isotropic inclusion Ω_i with a connected boundary S. Put $\Omega_e = R^3 \setminus \overline{\Omega}_i$ and denote it by $\rho_{i(e)}, c_{i(e)}, \gamma_{i(e)}$ the density, propagation velocity of acoustic vibrations, and absorption coefficient in $\Omega_{i(e)}$.
Suppose that there are sound sources in the region Ω_e. Sound waves propagate in space and, reaching inclusion, are scattered on it. As a result, reflected waves appear in the region Ω_e, and transmitted waves appear in Ω_i.

Consider the following problem: by changing the sound sources in Ω_e to minimise the deviation of the pressure field in Ω_i (or on some subset $Q \subset \Omega_i$) from some required. At the same time, the change in sound sources should not be “large”. Mathematically, it can be formulated as follows (Ershov et al., 2010).

Find functions $f: S \to C$ (control) and $\mathrm{F}_i: \overline{\Omega}_i \to C$, $\mathrm{F}_e: \overline{\Omega}_e \to C$ (state) satisfying the following conditions

$$\Delta \mathrm{F}_i + k_i^2 \mathrm{F}_i = 0 \text{ in } \Omega_i, \quad (1)$$

$$\Delta \mathrm{F}_e + k_e^2 \mathrm{F}_e = 0 \text{ in } \Omega_e, \quad (2)$$

$$\mathrm{F}_i - \mathrm{F}_e = g, \quad p_i \frac{\partial \Phi_i}{\partial \boldsymbol{n}} - p_e \frac{\partial \mathrm{F}_e}{\partial \boldsymbol{n}} = f \quad \text{to } S, \quad (3)$$

$$\frac{\partial \mathrm{F}_e}{\partial |x|} - i k_e \mathrm{F}_e = o(|x|^{-1}) \quad \text{as } |x| \to \infty, \quad (4)$$

$$J(\mathrm{F}_i, f) = \frac{1}{2} \int_Q |\mathrm{F}_i - \mathrm{F}_d|^2 \, dx + \frac{\lambda}{2} \int_S |f - f_d|^2 \, ds \to \min, \ f \in K. \quad (5)$$

Here $\boldsymbol{n} = \boldsymbol{n}(\boldsymbol{x})$ is the normal unit vector to the surface S (directed towards Ω_e),

$$k_{i(e)}^2 = \frac{\omega(\omega + i\gamma_{i(e)})}{c_{i(e)}^2}, \quad p_{i(e)} = \frac{1}{\rho_{i(e)} \omega(\omega + i\gamma_{i(e)})},$$

ω — wave circular frequency; g, F_d, f_d —complex-valued functions, K — convex set of functions, defined on S (admissible control set), λ — fixed nonnegative real number.

We propose an algorithm of the numerical solution of problems (1)-(5) on condition that the set K. When solving a problem on a computer, parallel computations are used.

Materials and methods

Theoretical methods.
The research contains methods of functional analysis (theory of generalised functions) and computational mathematics (theoretical foundations of numerical methods for solving systems of differential and algebraic equations, partial differential equations, and potential theory), theory of inverse problems.

Equipment and software.
During the development and research of algorithms, methods of computational mathematics and methods of the theory of optimal control were used.

The software for the numerical solution of problems was created in Fortran-90 and is a console application designed to work on multiprocessor computing systems. The compiler is Intel Fortran Compiler. The software in question runs on the computing cluster of the Far-Eastern Branch of the Russian Academy of Science (http://hpc.febras.net/).

The computing cluster of the Far-Eastern Branch of the Russian Academy of Science is a heterogeneous system with a total capacity of 1520 Gigaflops (consists of 1 control and 17 computing nodes based on Intel Xeon and AMD Opteron processors). A total of 168 computing cores are available. A control network is the InfiniBand network, based on Gigabit Ethernet technology.

The computing subsystem of the cluster is based on three types of nodes. The first type includes 4 Sun x6440 blade servers, each of which is based on four six-core AMD Opteron 8431 processors, is equipped with an InfiniBand controller and has 96GB of RAM. The peak performance of each such node is 230 Gigaflops.

Results
Solution algorithm (1)–(5).

Let $e_1, \dots, e_M$ be a system of functions linearly independent over the field R defined on the surface S.

$$K_M = K \cap \mathrm{Span}\,\{\mathrm{e}_1, \dots, \mathrm{e}_\mathrm{M}\}.$$

Where $\mathrm{Span}\,\{\mathrm{e}_1, \dots, \mathrm{e}_\mathrm{M}\}$ is a linear shell function $e_1, \dots, e_M$ over the field R.
Let U_M be sets of three $(\mathrm{F}_i, \mathrm{F}_e, f)$, comply with (1)–(4), but $f \in K_M$.
Let us look onto the next problem. Find a function $(\mathrm{F}_i^{(M)}, \mathrm{F}_e^{(M)}, f^{(M)}) \in U_M$ such as

$$\forall (\mathrm{F}_i, \mathrm{F}_e, f) \in U_M \quad J(\mathrm{F}_i^{(M)}, f^{(M)}) \leq J(\mathrm{F}_i, f). \tag{6}$$

The correctness of the problem and the convergence of the algorithm are proved in [2].
Functions $\mathrm{F}_{i(e)}^{(M)}$ we will search in the form:

$$\mathrm{F}_{i(e)}^{(M)} = \Psi_{i(e)}^{(0)} + \Psi_{i(e)},$$

Where $\Psi_{i(e)}^{(0)}$ is a solution to the diffraction issue,

$$\begin{aligned} &\Delta\Psi_{i(e)}^{(0)} + k_{i(e)}^2 \Psi_{i(e)}^{(0)} = 0 \text{ in } \Omega_{i(e)}, \\ &\Psi_i^{(0)} - \Psi_e^{(0)} = g, \quad p_i \frac{\partial \Psi_i^{(0)}}{\partial \boldsymbol{n}} - p_e \frac{\partial \Psi_e^{(0)}}{\partial \boldsymbol{n}} = 0 \quad \text{by } S, \\ &\frac{\partial \Psi_e^{(0)}}{\partial |x|} - ik_e \Psi_e^{(0)} = o(|x|^{-1}) \quad \text{by } |x| \to \infty, \end{aligned} \tag{7}$$

And new unknown $\Psi_{i(e)}$ and $f^{(M)}$ must be complied with the formula

$$\Delta\Psi_{i(e)} + k_{i(e)}^2 \Psi_{i(e)} = 0 \text{ in } \Omega_{i(e)}, \quad (8)$$

$$\Psi_i - \Psi_e = 0, \quad p_i \frac{\partial \Psi_i}{\partial \boldsymbol{n}} - p_e \frac{\partial \Psi_e}{\partial \boldsymbol{n}} = f^{(M)} \quad \text{by } S, \quad (9)$$

$$\frac{\partial \Psi_e}{\partial |x|} - ik_e \Psi_e = o(|x|^{-1}) \quad \text{by } |x| \to \infty, \quad (10)$$

$$\frac{1}{2}\int_Q |\Psi_i - \Psi_d|^2 \, dx + \frac{\lambda}{2}\int_S |f^{(M)} - f_d|^2 ds \to \min, \quad f^{(M)} \in K_M. \quad (11)$$

Here $\Psi_d = \mathrm{F}_d - \Psi_i^{(0)}$.

Let functions $\Psi_{k,i}, \Psi_{k,e}$ $(k = \overline{1, M})$ be solutions of the next problem:

$$\begin{gathered} \Delta\Psi_{i(e)}^{(0)} + k_{i(e)}^2 \Psi_{i(e)}^{(0)} = 0 \text{ in } \Omega_{i(e)}, \\ \Psi_i^{(0)} - \Psi_e^{(0)} = 0, \quad p_i \frac{\partial \Psi_{k,i}}{\partial \boldsymbol{n}} - p_e \frac{\partial \Psi_{k,e}}{\partial \boldsymbol{n}} = e_k \quad \text{by } S, \\ \frac{\partial \Psi_k}{\partial |x|} - ik_e \Psi_k = o(|x|^{-1}) \quad \text{by } |x| \to \infty. \end{gathered} \quad (12)$$

Enter $\varphi{:}^M \to R$ according to the formula

$$\varphi(\xi) = \frac{1}{2}\int_Q \left| \sum_{k=1}^{M} \xi_k \Psi_k - \Psi_d \right|^2 dx + \frac{\lambda}{2}\int_S \left| \sum_{k=1}^{M} \xi_k e_k - f_d \right|^2 ds$$

And set $K_\xi = \{\xi = (\xi_1, \dots, \xi_M) \in^M: \quad \sum_{k=1}^{M} \xi_k e_k \in K_M\}$.

Then the solution of (8)–(11) is compiled by the formula:

$$f^{(M)} = \sum_{k=1}^{M} \xi_k e_k, \quad \Psi_i = \sum_{k=1}^{M} \xi_k \Psi_{k,i}, \quad \Psi_e = \sum_{k=1}^{M} \xi_k \Psi_{k,e},$$

Where $(\xi_1, \dots \xi_M) = \arg \min_{\xi \in K_\xi} \varphi(\xi)$.

In the article (Illarionova, 2011) the algorithm described above was numerically implemented for the case when $K = L^2(S)$ (i.e., there are no control restrictions).

In our research, we consider the case when

$$K = \{f \in L^2(S): \quad |f| \le h \text{ by } S\}, \quad (12)$$

Where h is a given positive constant

Hypothesis $|f| \le h$ imposes a limitation on the power of sources with which you can control the acoustic field.

Let functions $\{e_k\}$ comply with the formula

$$\sum_{k=1}^{M} |e_k| = 1, \quad \max_{x \in S} |e_k(x)| = 1 \quad k = \overline{1, M}. \quad (13)$$

It is not difficult to prove, that K_ξ is n -cube:

$$K_\xi = \{\xi \in^n: \quad |\xi_k| \le h \quad k = \overline{1, M}\}.$$

To find unknown coefficient ξ_k we will use the coordinate descent method (Vasiliev, 2017).

Configuration of coordinate descent method

1. Choose an initial point $\xi_0 = (\xi_1^0, \xi_2^0, \dots, \xi_M^0) \in R^M$ and $\varepsilon > 0$ (determines the posterior error estimate).
2. Consider $\xi = \xi_0$.
3. Go down the 1st variable, i.e. exchange ξ_1 to η, where η is a function minimum point $\eta \to \varphi(\eta, \xi_2, \dots, \xi_M)$ under constraints $|\eta| \le h$.
4. Go down the 2nd, 3rd the same way..., M variable. So we get a new approximating $=$ $(\xi_1^1, \xi_2^1, \dots, \xi_M^1)$
5. If $\max\limits_{1 \le j \le M} \left|\xi_j^1 - \xi_j^0\right| \le \varepsilon$, then the algorithm is finished, approximate solution is ξ^1. If not, we consider $\xi^0 = \xi^1$ and move to the step 2.

Conditions of numerical experiments

1. $\Omega_i = Q$ is a ball of unit radius centered at the origin.
2. The set of admissible controls K is determined by the formula (13).
3. To solve the direct diffraction problems, we used the algorithm described in (Ershov et al., 2010). It consists of the following. By methods of potential theory, the diffraction problem reduces to a mixed system of weak singular boundary integral Fredholm equations of the first and second kind on the inclusion surface. The approximation of integral equations by a system of linear algebraic equations is carried out by dividing the unit on the surface, associated with a system of nodal points, and also consistent with the discretisation order of the method of averaging weak singular kernels of integral operators. Multiple integrals arising during discretisation are calculated analytically. This allows one to obtain explicit formulas for approximating boundary integral operators with singularities in kernels and use them to calculate the coefficients of systems of linear algebraic equations. At the same time, preliminary triangulation of the surface is not required. We denote by N the number of sampling points used in solving diffraction problems.

4. Let $M_\phi, M_\theta \in N$,

$$M = M_\phi \cdot M_\theta, \qquad \phi_j = \frac{2\pi j}{M_\phi}, \quad j = \overline{0, M_{\phi-1}}, \quad \theta_m = \frac{\pi m}{M_\theta}, \quad m = \overline{0, M_\theta}.$$

As e_k functions were selected, half of which in spherical coordinates (ρ, ϕ, θ), ($\rho = |x|$, ϕ — longitude, θ — latitude) are determined by formulas (“hat” functions in coordinates (ϕ, θ)):

$$e_k(\phi, \theta) = w\left(\sqrt{|\phi_j - \phi|^2 + |\theta_m - \theta|^2}\right)$$

By $k = jM_\theta + m$, $j = \overline{0, M_\phi - 1}$, $m = \overline{0, M_\theta}$.

Here $w(t) = \begin{cases} \exp\left(t^2/(t^2-\pi^2)\right) & \text{by } |t| < \pi \\ 0 & \text{by } |t| \geq \pi. \end{cases}$

The second half of the e_k functions is obtained from the ones defined above by multiplying by an imaginary unit.

Test calculation and numerical simulation

Define $\delta^* = \frac{1}{2}\int_Q |\,\mathrm{F}_i - \mathrm{F}_d|^2 dx + \frac{\lambda}{2}\int_S |\, f - f_d|^2 ds,$

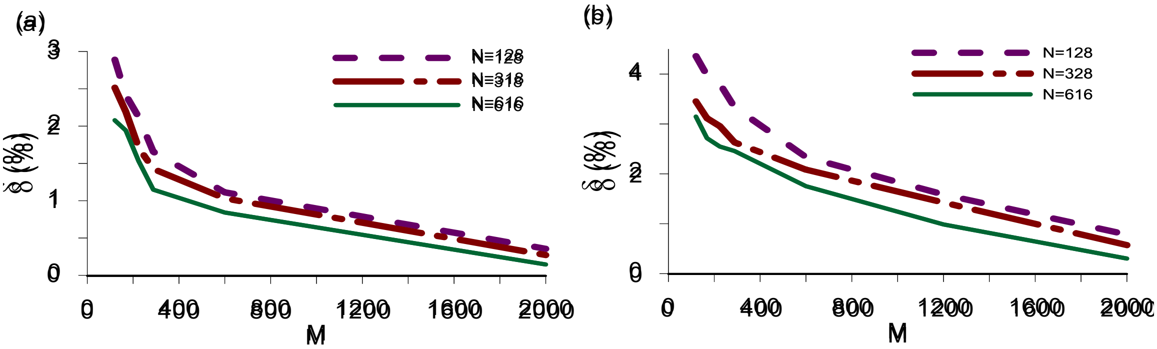
$$\delta = \frac{\delta^*}{{}^1\!/_2 \parallel \Phi_d \parallel^2 + {}^{\lambda}\!/_2 \parallel f_d \parallel^2} \cdot 100,$$

Example 1. Initial data

$$\rho_i = 5,\ c_i = 1,\ \gamma_i = 0.02,\ \rho_e = 3,\ c_e = 0.5,\ \gamma_e = 0.05, h = 5,$$
$$\mathrm{F}_d(x) = \exp\left(ik_i x_3\right), f_d = 0, \lambda = 0.$$

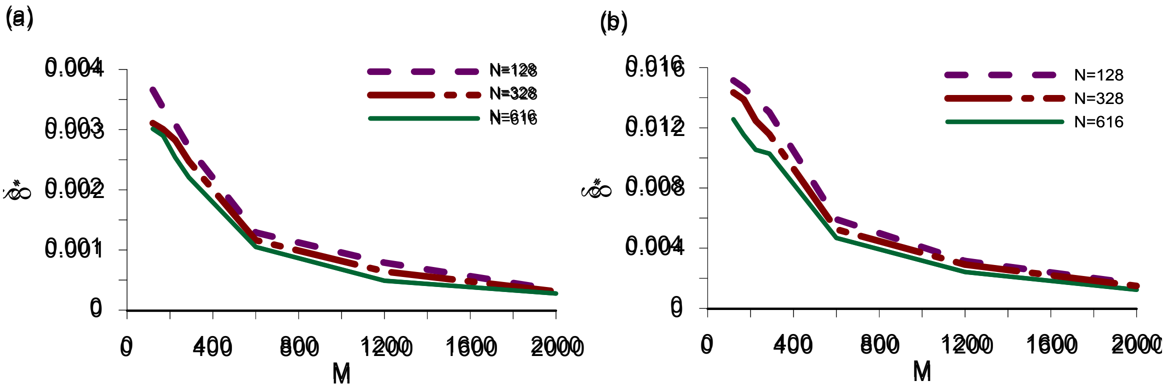
Figure 1 shows graphs of the dependence of δ on the number *M* (the number of functions e_k) for various values of *N* (the number of sampling points of the diffraction problem).

Fig.1. $\mathrm{F}_d = \exp\left(ik_i x_3\right)$, a) $\lambda = 0$, b) $\lambda = 5$



Example 2. In contrast to Example 1, $\mathrm{F}_d(x) = 0$. Figure 2 shows graphs of the dependence δ^* of the number *M* for various values of *N*.

Fig.2. $\mathrm{F}_d = 0$ a) $\lambda = 0$, b) $\lambda = 5$



Figures 1 and 2 show that the error at different values of the parameter differs slightly. The number of basic functions has a greater effect on the error than the number of sampling points of direct diffraction problems.

Example 3. The optimal control problem (1)-(5) is considered with the following initial data

$$\mathrm{F}_d = \sin(\pi x_1), \quad f_d = 0, \quad g = 0,$$
$$\rho_i = 5, \quad c_i = 1, \quad \gamma_i = 0.02, \quad \rho_e = 3, \quad c_e = 0.5, \; \gamma_e = 0.05.$$

The values of the parameters λ and h are indicated below.

Figure 3 shows the projective curves of the functions $|\mathrm{F}_d|$ и $|\mathrm{F}_i|$ for different values of the parameter λ and h on the interval (a) $|x_2| \leq 0.45$, $x_{1,3} = 0.4$; (б) $|x_1| \leq 0.45$, $x_2 = 0.4$, $x_3 = 0$.

Fig.3. Projective function curves $|\mathrm{F}_d|$ и $|\mathrm{F}_i|$ for example 3.

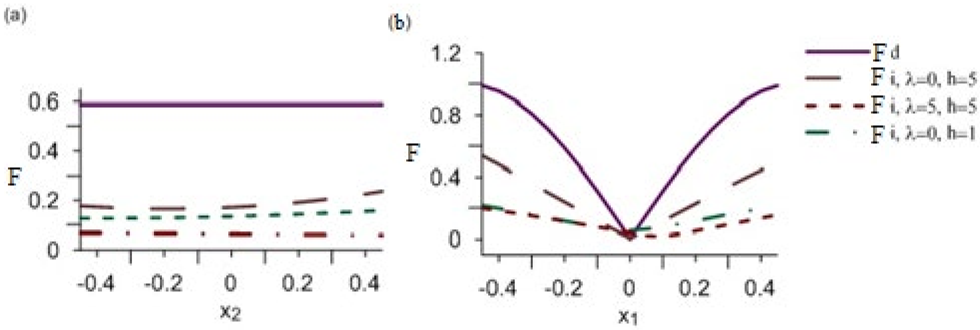
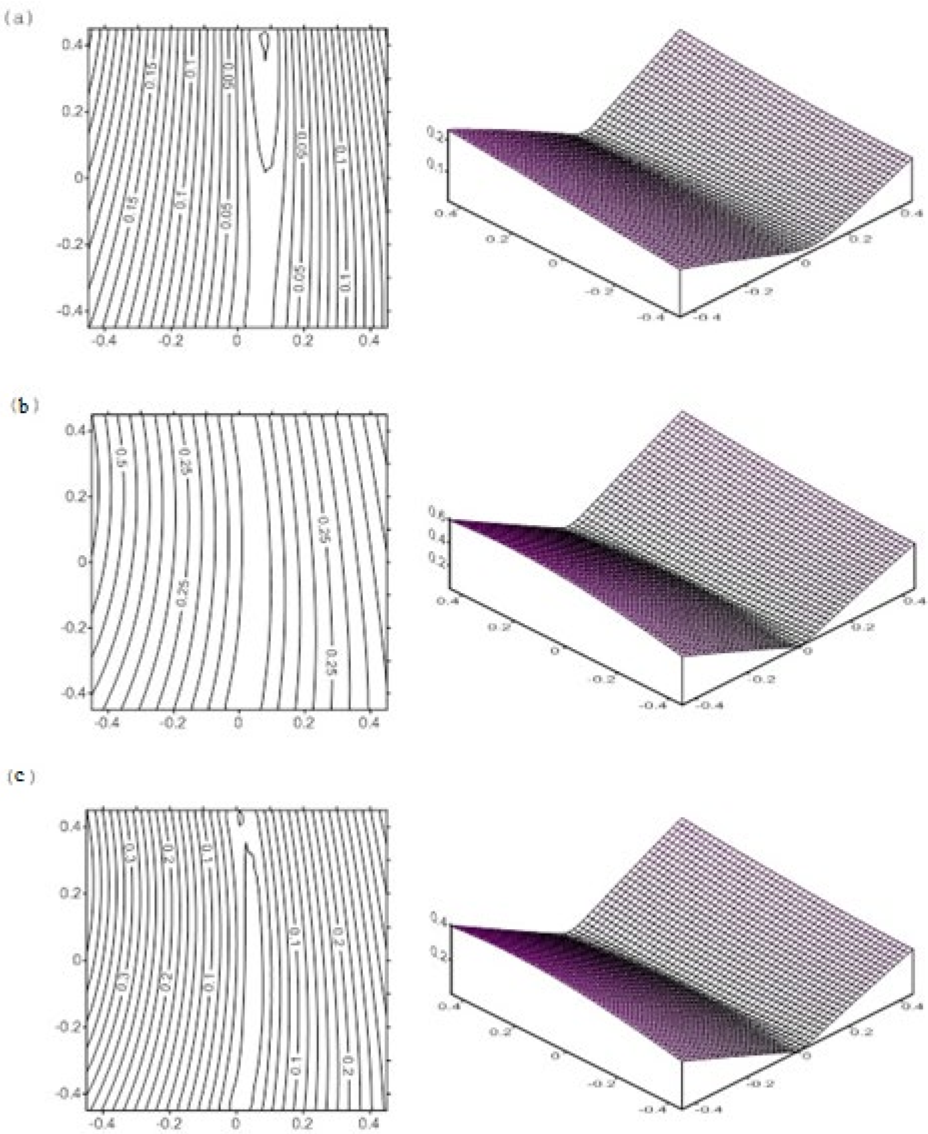


Figure 4 shows level lines and projective function interval $|\mathrm{F}_i|$ on the square $|x_{1,2}| \leq 0.45$, $x_3 = 0$ by (a) $\lambda = 0$, $h = 5$, (b) $\lambda = 5$, $h = 5$, (c) $\lambda = 0$, $h = 1$.

Fig.4. Level lines and projective surface $|\mathrm{F}_i|$ for example 3 by (a) $\lambda = 0$, $h = 5$, (b) $\lambda = 5$, $h = 5$, (c) $\lambda = 0$, $h = 1$.



Figures 3 and 4 allow you to judge the influence of intervals λ and h on the field of optimal pressure. Fields resulting from optimisation are close to the function F_d for small interval values λ and repeats the forms of λ. Decreasing of the interval λ reduces the difference $|F_d - F_i|$. Increasing of the interval h, i.e. the weakening of restrictions on the power of sources leads to a decrease in the deviation of the calculated field from the specified.

Example 4. (Acoustic attenuation task) The optimal control issue (1)-(5) is considered with the following initial data:

$$F_d = 0, \quad f_d = \frac{exp\,(ik_e|x-y|)}{|x-y|}, \quad y = (2,0,0), \quad g = 0,$$
$$\rho_i = 5, \quad c_i = 1, \quad \gamma_i = 0.02, \quad \rho_e = 3, \quad c_e = 0.5, \gamma_e = 0.05.$$

From a physical point of view, this means that it is required to 'extinguish' the sound field in the inclusion created by an external point source.

Figure 5 has projective function curves $|\mathrm{F}_d|$ and $|\mathrm{F}_i|$ at different parameter values λ and h on the interval (a) $|x_2| \leq 0.45$, $x_{1,3} = 0.4$; (b) $|x_1| \leq 0.45$, $x_{2,3} = 0$.

Fig.5. Projective function curves $|\mathrm{F}_d|$ and $|\mathrm{F}_i|$ for example 4.

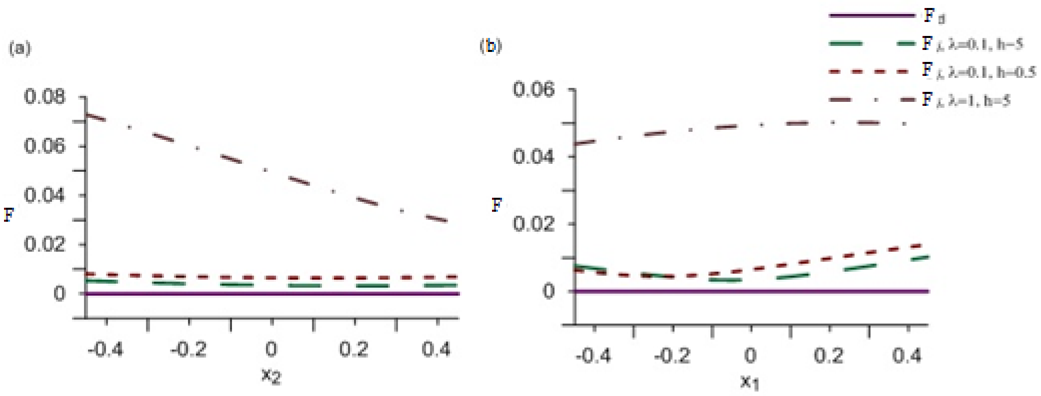
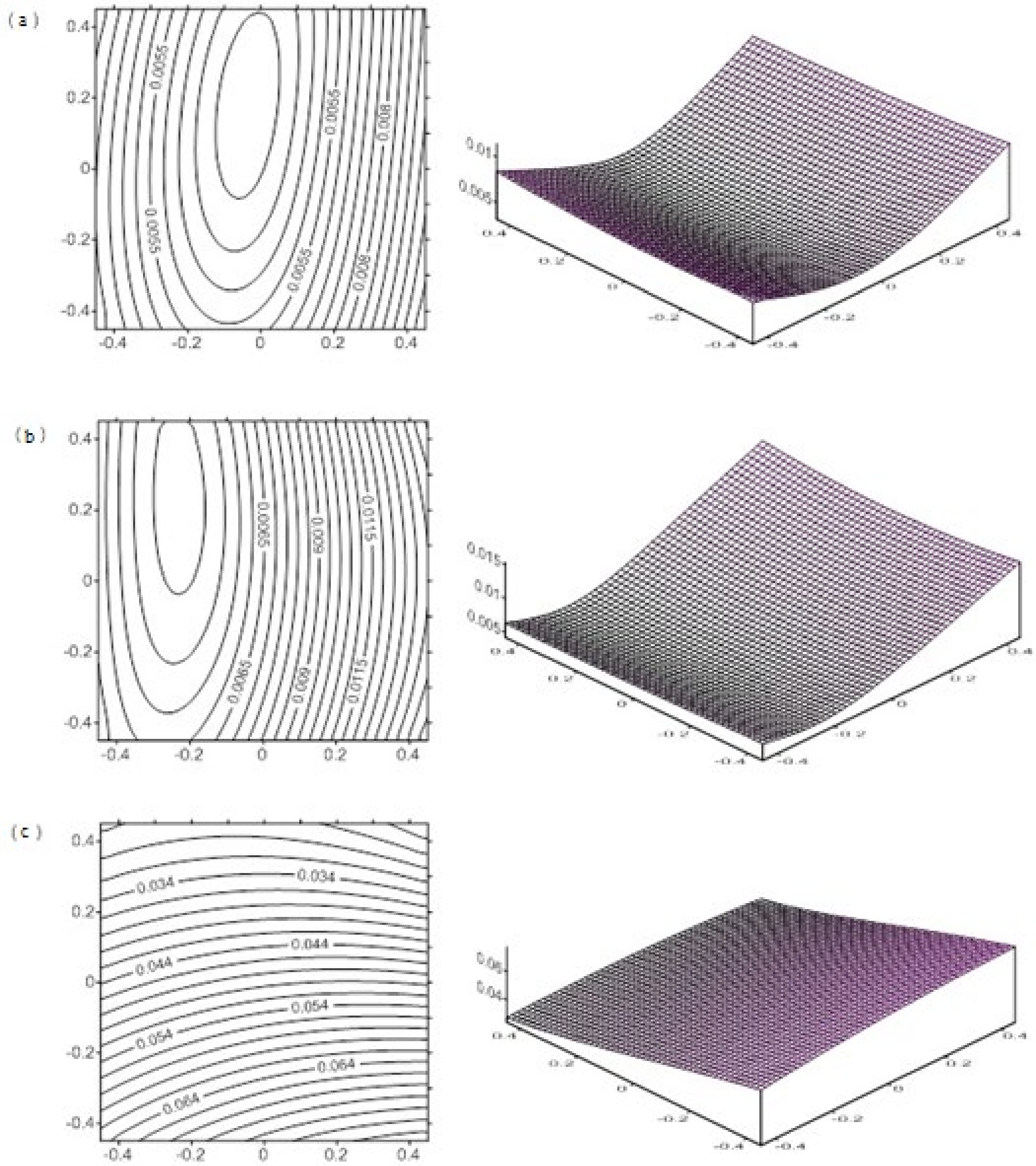


Figure 6 has level lines and projective surface of the function $|\mathrm{F}_i|$ on the square $|x_{1,2}| \leq 0.45$, $x_3 = 0$ by (a) $\lambda = 0.1$, $h = 5$, (b) $\lambda = 0.1$, $h = 0.5$, (c) $\lambda = 1$, $h = 5$.

Fig. 6. Level lines and projective surface $|F_i|$ for example 4 by (a) $\lambda = 0.1$, $h = 5$, (b) $\lambda = 0.1$, $h = 0.5$, (c) $\lambda = 1$, $h = 5$.



Example 5. (Acoustic attenuation task) In spite of the example 3 $F_d = \exp(ik_i x_3)$ (acoustic field is created by a plane wave).

Figure 7 has projective function curves $|F_d|$ and $|F_i|$ at different parameter values λ on the interval (a) $|x_2| \le 0.45$, $x_{1,3} = 0.4$; (b) $|x_1| \le 0.45$, $x_{2,3} = 0$.

Fig.7. Projective function curves $|F_d|$ and $|F_i|$ for example 5.

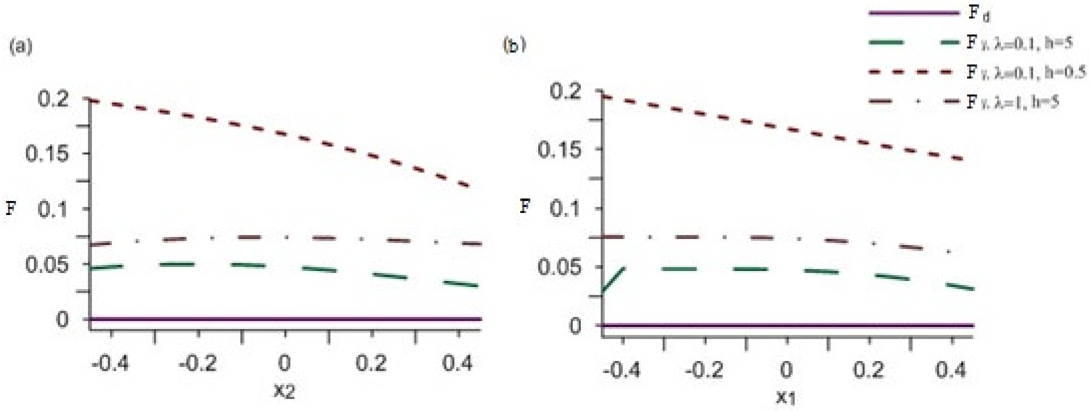
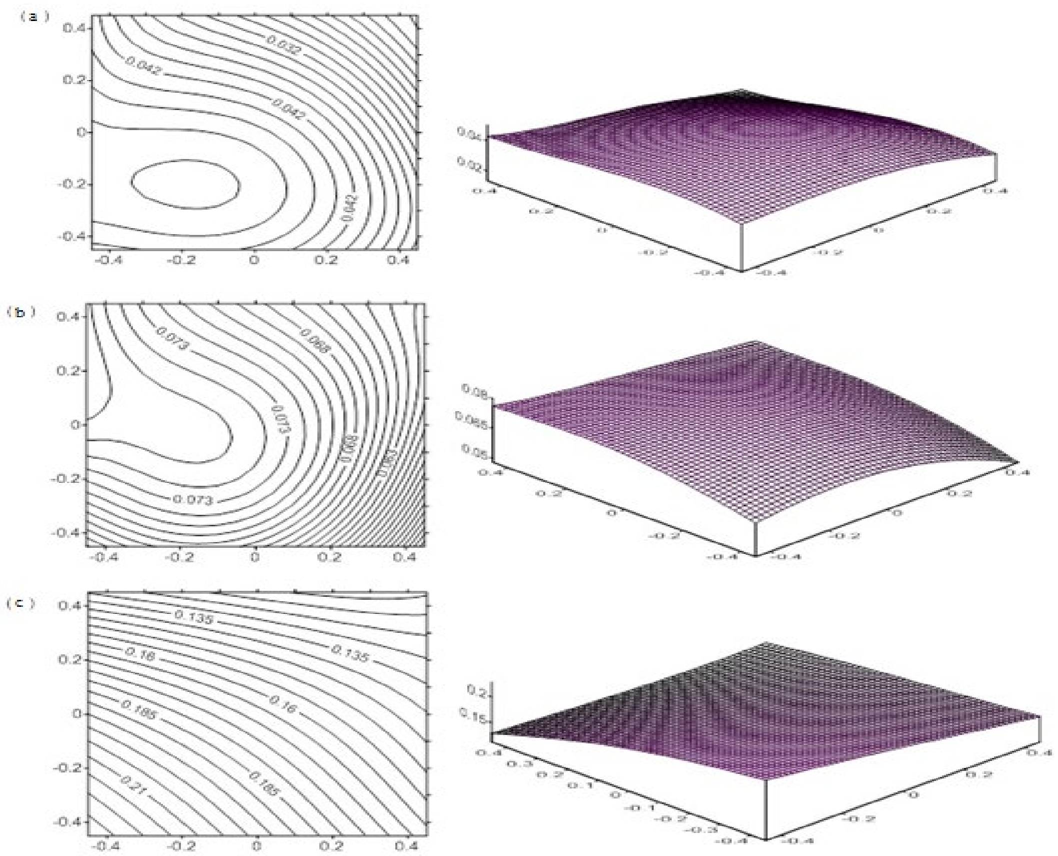


Figure 8 has level lines and projective surface of the function $|F_i|$ on the square $|x_{1,2}| \leq 0.45$, $x_3 = 0$ при (a) $\lambda = 0.1$, $h = 5$, (b) $\lambda = 0.1$, $h = 0.5$, (c) $\lambda = 1$, $h = 5$.

Fig.8. Level lines and projective surface $|F_i|$ for example 5 by (a) $\lambda = 0.1$, $h = 5$, (b) $\lambda = 0.1$, $h = 0.5$, (c) $\lambda = 1$, $h = 5$.



The source of sound waves is located to the right of the inclusion. The figures show that the greatest deviation from the required field occurs in that part of the inclusion, which is closer to the source. The smaller the value of λ and the greater the value of h, that is, the more you can change the field sources in the external environment, the closer the calculated field to zero.

Discussion

The results of test calculations and numerical experiments show that the proposed algorithm converges and can be used to solve problems of diffraction of acoustic waves.

To carry out the calculations, the computing resources of the "Data Center of the Far-Eastern Branch of the Russian Academy of Science" (Sorokin et al., 2017) *were used. This research was supported in through computational resources provided by the Shared Facility Center "Data Center of FEB RAS" (Khabarovsk).*

The bulk of the computation falls on the solution of direct diffraction problems. To speed up the algorithm, the parallelisation method was applied. Since direct tasks are solved independently, it is effective to use the standard Fortran language function for the cycle of solving direct problems. The solution time depends on the number of cluster nuclei involved, while doubling the number of nuclei, the time decreases on average 1.5 times (until the saturation effect is achieved). Without the use of parallel calculations for N=616 and M=288, the error for test cases reached 1%.

The algorithm has linear computational complexity with respect to M and quadratic complexity with respect to N. The results of computational experiments showed that the proposed method accelerates the program up to 5 times. Moreover, for N=616 and M=2000, the error decreased to 0.1%. This means that the method has a second order of accuracy with respect to M. Nevertheless, the usage of parallel computing can effectively solve the issue with greater accuracy.

Conclusion

The main results of the work are as follows:

1. An algorithm for solving the optimisation issue of diffraction of acoustic waves with restrictions was developed and tested.
2. The developed algorithm for solving the issue was tested on model problems of damping the sound inside the inclusion with limited power sources, with which you can control the acoustic fields.

3. Numerical experiments were conducted on a hybrid computing cluster based on the OpenPOWER architecture.

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