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# Pedagogical Content Knowledge (PCK) Among Mathematics Teachers on Function Materials through Lesson Study in Junior High School

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Pedagogical Content Knowledge (PCK) is the teacher's knowledge of aspects such as pedagogy, students, subject matter and curriculum. The PCK for mathematics teachers is very important for the success of students' learning process, as well as their interest. Mentoring is defined as a model of guidance and assistance through collaboration between lecturers and teachers. In addition, the collaboration between lecturers and teachers in preparing plans, implementing learning processes and reflection, can improve the quality of learning itself. This research was conducted in developing a mathematics teacher mentoring model based on PCK and lesson study. The focus was not on the process of developing a teacher mentoring model, but rather, to describe the teacher's PCK during the learning process via observation. Subsequently, data was collected through observation while learning, lesson plan analysis and interviews. The participants of this study consisted of five mathematics lecturers and five mathematics teachers. One teacher was appointed as a model teacher who prepared a lesson plan, while other teachers and lecturers were the observers. Furthermore, the observation in learning was carried out before and after the mentoring session through a lesson study, which began from plan, do and see. The data obtained during plan, do and see were then analysed through a qualitative approach. The studied PCK components were knowledge of the subject matter (KSM), knowledge of pedagogy (KP) and knowledge of students (KS). Knowledge of the subject matter (KSM) is the teacher's understanding of the concepts and procedures of the function materials. Knowledge of pedagogy (KP) is the teacher's knowledge about planning and organising learning. Meanwhile, knowledge of students (KS) is the teacher's knowledge of students' mistakes and misconceptions in the function materials. The PCK of mathematics teachers in junior high school on the function materials was implemented after mentoring in learning

practices, which includes KSM (a combination of conceptual and procedural knowledge), KP (exploration of the pre-requisite knowledge by asking questions to students, using examples as a form of representation and applying student-centred strategies), and KS (overcoming students' mistakes and misconceptions by identifying the types of errors and addressing it based on the type of errors and misconceptions of the function materials). Understanding the function materials that are integrated with students' knowledge has an impact on a teacher's knowledge of pedagogy. Meanwhile, understanding the function materials that are integrated with pedagogical knowledge has an impact on the teacher's knowledge about students. To conclude, the PCK of mathematics teachers on function materials in junior high school can be developed through teacher's mentoring and lesson study.

**Key words:** Pedagogical Content Knowledge, functions, teachers, lesson study.

## Introduction

The process and results of mathematics learning in class are largely determined by the teacher's knowledge of mathematics itself. Inadequate mathematical understanding of the teacher will result in a flaw when the learning process is carried out in the classroom. According to Shulman (in Ma'rufi, 2017), Pedagogical Content Knowledge (PCK) is knowledge of specialised skills for the teaching profession. Likewise, Ball and Bass (2000) defined that PCK is a type of knowledge that integrates teacher mathematical knowledge with knowledge about students, learning and pedagogy. Being a professional mathematics teacher, just mastering the mathematics material is not enough. It is also crucial to have a specific type of knowledge, namely PCK. PCK, in mathematics learning, is the knowledge that integrates the knowledge of mathematical content, pedagogy and students' thinking.

According to An, Kulm and Wu (2004), learning can be viewed as a divergent and convergent process. A divergent process of learning is based on content knowledge and curriculum, but it overlooks students' mathematical reasoning. Divergent processes are in line with learning as knowledge or learning to know (learning as knowing). A teacher who believes in learning as knowledge (learning to know) assumes that mathematics is learned and understood if a concept or skill is being taught. Whereas, a convergent process of learning focuses on knowing a student's thinking, which consists of four aspects, namely: (1) building students' mathematical ideas, (2) overcoming student misconceptions, (3) involving students in mathematics learning, and (4) advancing mathematical thinking students. The four aspects of convergent learning are learning ideas as a process of understanding and are necessary for efficient learning (Carpenter, Fennema & Franke, 1996). A teacher who believes learning as understanding or learning to understand realises that knowing is not

enough, but it must be understood, and there is an internalisation process of knowledge by connecting fundamental knowledge through convergent processes.

The existence and improvement of teacher professionalism are vital to be developed and nurtured continuously through teacher mentoring. PCK is a distinct type of knowledge for teachers, intended for effective learning that will be studied and developed through lesson study with the collaboration from lecturers of mathematics education programs and mathematics teachers.

To ensure the teacher PCK exploration process runs smoothly, it certainly needs to be supported by the right selection of materials. For this reason, a study on the materials is needed. It includes not only the knowledge of procedures but also conceptual understanding, and teachers must have both, including the knowledge and understanding about how to teach the material to students. There is something unique and exciting about 'function' materials. There are instructions that focus on shifting numbers, traits and counting operations, the introduction of symbols and solving equations.

Panasuk (2010) explained that students who can recognise the structure of relationships in algebra show a conceptual understanding. These students can interpret, connect and translate a concept into various forms of representation, and they may have the ability to solve similar problems (functions) which are presented in the form of words, diagrams and variables. Therefore, researchers consider function materials as the initial transitional materials in algebra. Transitional is defined as the transition from simple algebraic forms (symbol recognition) to abstract algebra (relationships between symbols). It is abstract in terms of determining the algebraic formula of a special relationship with certain mapping rules and representing them with specific symbols. In the introduction of algebraic forms, students are required to determine the nature and operation of it. Therefore, in further function materials, students are asked to present the relationship between variables in the form of algebra based on certain association rules. This relationship underlies the definition of the concept of function.

The problems in learning function materials lie in numerical abstraction into variable forms. Gyamfi (2015) explained that students are vulnerable to difficulties when conducting linguistic-to-algebraic translations. The difficulties are not easily overcome by simple learning designs. Furthermore, Bossé (2011) explained that the difficulties of learning algebra can be overcome by utilising the skills of teachers in the types of knowledge (PCK) to design instructional designs (*plan*) by considering the cognitive framework of students. The learning process (*do*) runs by focusing observations on aspects of analysing student activities, defining the types of mistakes students make and recognising the frequency of student interactions in learning. It is hoped that in the end, the teacher can conduct a meta-analysis (*see*) about

student thinking, actions and patterns of errors in understanding mathematical representation. Overall, it is hoped that this will lead to an instructional process that helps students learn to avoid another mistake (the concept of Lesson Study). Furthermore, Doig (2011) explained that the lesson study allows teachers to improve learning and understanding. As reported by Lewis et al. (2009), teachers ultimately have the initiative to meet and revise their teaching programs to enable co-workers to observe lessons through a medium in the form of research activities.

This study examined the development of Pedagogical Content Knowledge (PCK) of mathematics teachers' mentoring and lesson study mentoring models. The focus of this study was the PCK of a junior high school mathematics teacher on the function materials developed through lesson study. The formulation of the problem in this study were: (1) what was the description of the teacher's knowledge about content, pedagogical knowledge and the teacher's knowledge about students in junior high school on the function materials; and (2) what was the description of the junior high school mathematics teacher's pedagogical content on the function materials?

The purposes of this study were to describe: (1) knowledge of the subject matter, knowledge of pedagogy and knowledge of students in junior high school on the function materials, and (2) mathematics teachers' pedagogical content knowledge on the function materials in junior high school.

According to Shulman (Ma'rufi, 2016), PCK is a way of knowing how to represent topics effectively, to encourage students understanding and learning, and to reduce misconceptions and difficulties on the materials. Furthermore, Grossman (Rohaan, Taconis & Jochems, 2009) positioned PCK as a distinctive and central domain for a teacher's knowledge of teaching. PCK is seen as a synthesis of pedagogy and content knowledge to understand better how a particular topic, problem or issue can be compiled, represented and adapted according to students' interests and abilities in their learning practices. Furthermore, Grossman and Schoenfeld (in Guzel, 2010) divided PCK into four components, as follows:

1. Teacher's knowledge of the objectives required to teach a subject.
2. Knowledge of students' understandings and possible misunderstandings related to that subject.
3. Knowledge of programs and related materials.
4. Knowledge of representations and strategies for teaching a particular topic.

PCK is a specialised knowledge that integrates mathematical knowledge with student, learning and pedagogical knowledge. Knowledge of pedagogical content is essential for teaching because this can help teachers anticipate learning difficulties and ready them to provide alternative models or explanations to overcome those difficulties. According to

Shulman (in Ma'rufi, 2017), seven components of PCK are knowledge of teaching materials, knowledge of pedagogical content, general pedagogical knowledge, knowledge of students, knowledge of curriculum, knowledge of educational context and knowledge of educational goals. Moreover, Tamir (in Ma'rufi, 2017) categorised the knowledge of pedagogical content into four components: (1) knowledge of student understanding, (2) knowledge of methods, strategies and teaching techniques, (3) knowledge of measurement and evaluation, and (4) knowledge of the curriculum.

In this study, Pedagogical Content Knowledge (PCK) incorporated mathematical knowledge, pedagogy and knowledge about students. The PCK components in this study were divided into three categories: knowledge of the subject matter, knowledge of pedagogy, and knowledge of students.

**Knowledge of the Subject Matter (KSM)**

A teacher should master the subject matter. Besides, he or she must understand the mathematics materials, including facts, concepts, theories and procedures. Knowledge of content (An et al., 2004) or knowledge of the subject matter (Kilic, 2011), is the knowledge about mathematics subject matter or mathematical content. According to Kilic (2011), knowledge of the subject matter includes knowledge of mathematical facts, mathematical concepts and the relationship between the two. Knowledge of the subject matter, for teachers, is very significant in learning (Mishra & Koehler, 2009), as it focuses more on the framework for investigating teacher knowledge. Mishra et al. (2009) argued that the subject matter knowledge is knowledge that must be taught and learned. Knowledge of facts, theories, concepts, procedures, topics of materials, and explanatory frameworks organises and connects the ideas, while knowledge of rules shows evidence and proof.

Knowledge of the subject matter is the teacher's understanding of mathematics material including facts, concepts, principles, procedures, relationships between concepts and principles, terms, the uses of mathematical formulas, and reasons for using certain procedures. Teacher's knowledge of the subject matter is necessary for learning so that learning objectives can be achieved. A teacher can explain mathematical concepts if he or she has a complete understanding of the mathematical concepts being taught. The knowledge of the subject matter in this study is the teacher's understanding of conceptual knowledge and procedural knowledge on the function materials.

**Knowledge of Pedagogy (KP)**

The teacher's knowledge of pedagogy in mathematics is the teachers understanding of learning principles in teaching mathematics subject matter or certain mathematical content,

organisation of learning material, understanding of student thinking and assessment of student knowledge. According to Kilic (2011), knowledge about pedagogy is knowledge about planning organisational learning and learning strategies. Ma'rufi (2016a, 2016b) explained that knowledge about pedagogy is related to planning and organising learning, including the implementation of learning plans, representations to be used, selection of examples and assignments, planning methods and assessment techniques. Meanwhile, the application of learning strategies includes apperception and learning objectives, the use of representations, examples and assignments, learning methods, and assessment techniques.

Knowledge about pedagogy is essential and fundamental for teachers as part of pedagogical competence. In the Government Regulation of the Republic of Indonesia Number 19 of 2005, the pedagogical competence is the ability to manage learners' learning, which includes understanding the students, designing and implementing learning, evaluating learning outcomes, and developing students to actualise their potential. In this study, knowledge of pedagogy includes apperception, representation and strategies which encourage student activity in learning on the function materials.

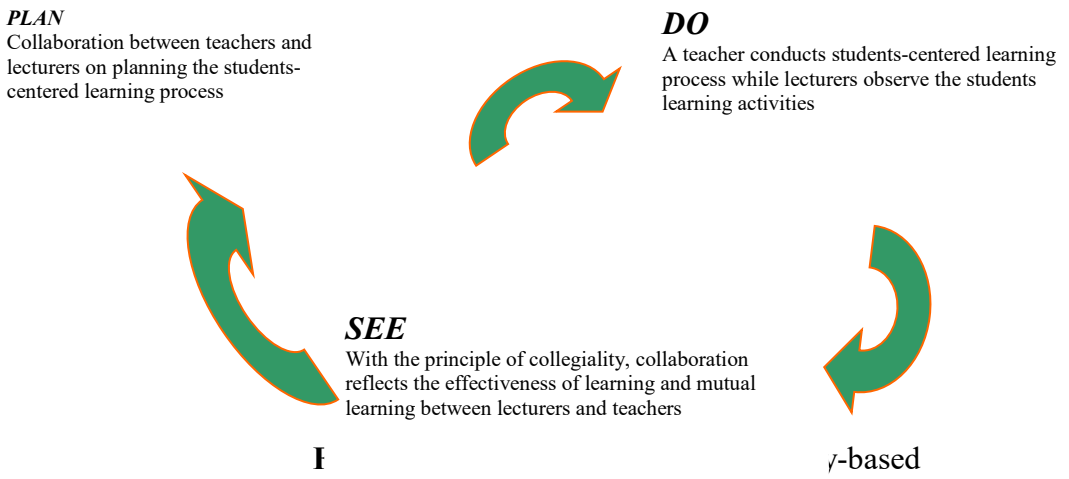
**Knowledge of Students (KS)**

Teachers should know the knowledge acquired by students when studying certain materials or subjects, both conceptual and procedural, as well as errors related to the materials. According to Kilic (2011), knowledge of students includes knowledge about students' general difficulties, errors and misconceptions. Ma'rufi (2016a, 2016b) mentioned that the knowledge about students includes teacher's knowledge about the types of students' errors and misconceptions, causes of students' errors and misconceptions, and ways to overcome students' mistakes and misconceptions. Furthermore, according to Fennema and Franke (in Kilic, 2011), Knowledge of Students (KS) is generally defined as a teacher's knowledge about the characteristics of students or certain groups of students in creating and planning an appropriate classroom environment to meet the students' needs for learning. In this study, the teacher's knowledge of students' errors and misconceptions includes the types, causes, and how to deal with students' errors and misconceptions on the function materials.

**Pedagogical Content Knowledge and Lesson Study**

Lesson Study activities consist of three main stages; *plan*, *do* and *see* (the basic competencies of a teacher or lecturer). In the planning stage, a group of teachers or lecturers will select the learning topic, conduct a study on the teaching material, and focus on it. Lewis (2002) and Iverson (2002) described how this can happen by discussing the five paths of Lesson Study: (1) bringing the educational goals and standards of the real world into the classroom; (2) promoting improvements with databases; (3) targeting the achievement of various students'

qualities that affect learning activities; (4) creating fundamental demands for the need to increase learning; and (5) upholding the value of teachers. The lesson study is carried out in three stages; continuous *plan*, *do* and *see* (Figure 1). In other words, lesson study is a way to improve the quality of education for continuous improvement.




According to Lewis (2002), there are eight opportunities that can be obtained by teachers when implementing lesson study, namely: (1) think carefully about the learning objectives, subject matter and field of study; (2) study and develop the best learning method; (3) deepen the knowledge of the subject matter being taught; (4) think deeply about the long-term goals to be achieved by focusing on students; (5) design collaborative learning; (6) carefully examine the ways and processes of learning, and students' behaviour; (7) develop strong pedagogical knowledge; and (8) see the results of learning through students and colleagues.

Collaboration between lecturers and teachers in preparing plans, implementing learning practices and conducting reflection, can improve the quality of learning. Besides, it can also produce scientific work as a result of a continuous learning assessment through lesson study. The lecturer's mentoring on PCK-based teachers can improve professional and pedagogical competence. The teacher is guided about mathematical content and how to teach it, so that the subject matter is easily understood by students. Teacher's mastery of the subject matter or mathematical content is important for success in teaching. A teacher cannot be expected to explain mathematical concepts if the teacher does not have a comprehensive understanding of the mathematical concepts being taught (Yeo, 2008). On the other hand, mastering the materials to be taught is not enough to achieve the desired goals for students (Isiskal & Cakirogu, 2010). Thus, teachers who are expected to teach effectively, are teachers who master the materials to be taught, and they are able to explain about it to make the students understand.

## Method

### *Participants*

The participants involved in this study were five lecturers of the mathematics education program at the Faculty of Education, Universitas Cokroaminoto Palopo, and five junior high school mathematics teachers from five different schools. The selected lecturers and teachers were those who had attended lesson study socialisations and workshops. The subjects or participants were junior high school mathematics teachers, three (3) students from Palopo Public High School, four (4) students from Palopo Public Middle School, and three (3) students from Palopo Cokroaminoto Middle School. Of the ten (10) participants, one (1) teacher was appointed as the model teacher who prepared the lesson plan, while the other lecturers and teachers acted as observers. These observers shared the main tasks and did not help the students.

### *Instrument*

In this study, some instruments were used to obtain the data, namely the teacher's PCK observation sheet for learning and the students' observation sheet for the learning process. This observation sheet was developed by the research team through discussions with the lecturers. Furthermore, the instrument was first validated by two experts in PCK and lesson study before it was used.

**Table 1:** PCK teacher observation sheet in learning

| PCK components and observed aspects |
|---|
| *Knowledge of the Subject Matter (KSM)* |
| • What is the accuracy of facts, definitions and mathematical symbols related to the materials presented in learning?<br>• What is the accuracy of the concepts and relationship between mathematical concepts related to the materials presented in learning?<br>• What is the accuracy of mathematical formulas related to the materials presented in learning?<br>• What is the accuracy of mathematical procedures related to the materials presented in learning? |
| *Knowledge of Pedagogy* (KP) |
| • What is the form of representation used to explain concepts in learning?<br>• How do you communicate the used representations to explain concepts in learning?<br>• What are the examples given to students in learning?<br>• What are the tasks given to students in learning?<br>• How do you involve students in learning? |

| |
|---|
| • How do you encourage student activity and creativity in learning?<br>• How do you apply the learning strategies or methods? |
| *Knowledge of Students(KS)* |
| • How do you know the possibility of students' difficulties, mistakes and misconceptions in learning?<br>• How do you know the source of the difficulties, mistakes and misconceptions of students in learning?<br>• How do you overcome students' difficulties, mistakes and misconceptions in learning? |

For this study, the teacher PCK observation sheet contained aspects observed for three components. Namely, Knowledge of the Subject Matter (KSM), Knowledge of Pedagogy (KP), and Knowledge of Students (KS). CBOs were concerned with the understanding of concepts and the relationship between concepts or mathematical procedures on the function materials. KP included the use of apperception, representation and strategies that encouraged student activities in learning and in completing tasks, both individually and in groups. Knowledge of Students (KS) included teachers understanding of types, causes and ways to overcome students' mistakes and misconceptions on the function materials in junior high school.

**Data Analysis**

Data analysis was conducted with the aim to develop a Mathematics Teacher Mentoring Model based on the Pedagogical Content Knowledge and Lesson Study. Besides, it described the observations of the development of the PCK teacher during the implementation of learning for two cycles. Data obtained during the *plan-see* stage was then analysed via a qualitative approach.

**Research Results**

The collaboration between lecturers and teachers through lesson study started from planning, implementation and reflection, in which each stage was integrated with PCK. At the planning stage, the lecturers and teachers who were the models, alternately conducted peer teaching. In the first and second cycle, the teacher who acted as a model carried out the learning. During the *see* stage, the lecturers and teachers discussed the learning process. The materials in the first cycle were used to determine the value of the function, while materials for the second cycle were to draw a function graph.

**The First Cycle of Lesson Study Activities**

***a. Planning Stage (Plan)***

This stage began with the lecturer and the teacher sitting together, and discussing the lesson plan in the form of teaching materials, worksheets and plans for the learning process in the classroom. Lecturers and teachers collaboratively shared ideas to develop learning designs to produce ways of organising teaching materials, learning processes and preparing learning aids. Before implementing it in class, the learning design prepared was then simulated first.

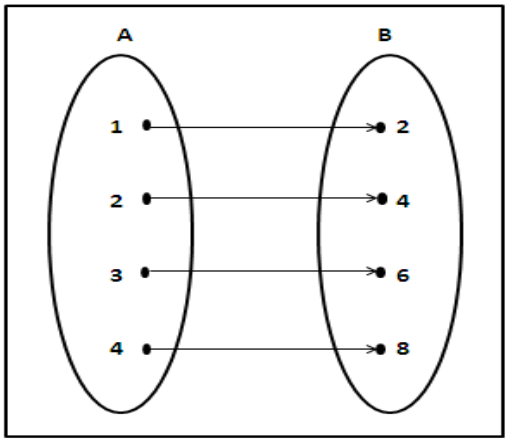
The *Plan* was carried out in the classroom by dividing students into small groups and then giving them examples of teaching materials simultaneously with worksheets that were used in this first cycle. Furthermore, other lecturers and teachers provided input regarding the groups that must be heterogeneous. Besides, each group consisted of one student who was an expert.

The goal of this cycle was for students to determine the function values. For educators, it is important to improve learning in a more professional manner. Thus, lecturers and teachers were required to have the courage to carry out self-reflection at the end of the learning process.

***b. Implementation Stage (Do)***

At this stage, a teacher acted as a model for practising the design that had been discussed with the lecturers and other teachers involved in this research. Other lecturers and teachers observed the learning process in the classroom to see the activities of the students. They were not allowed to intervene with the students during observations. The materials discussed in the first cycle were utilised to determine the function values.

In the initial stage, the teacher divided students into small groups consisting of four to five students, so that six groups were formed. The teacher gave an apperception to students by describing the Venn diagram with set A to set B, which was mapped by f, where set A = {1,2,3,4}, B = {2,4,6,8} as follows:




Teacher: Who can write a set of sequential pairs resulting from the mapping f from set A to set B!

Ana: Wrote the answer on the board {(1,2), (2,4), (3,6), (4,8)}
Teacher: Good, the answer is correct! (1,2) we can say f maps 1 to 2 or it is written f (1) = 2. How to (2.4)?
Adi: f (2) = 4!
Teacher: Good, and so on, so f (1) = 2, f (2) = 4, f (3) = 6 and f (4) = 8. So it can be written f (x) = 2x. So that f (3) = 6 means the function value f (x) = 6 for x = 3. Do you understand?

Student: Yes ma'am!
Teacher: Next look at the Worksheet that has been distributed!

In the main stage, the teacher provided worksheets to each group to discuss and then the teacher helped the group that did not understand the process of solving the problem. The answer of one group who experienced an error in answering one of the questions is described below:

Question:
A linear function is defined as f (x) = ax-b. If known f (1) = 1 and f (-2) = - 5. Determine the values of a and b and write the shape of the function.
Answer:
f (x) = ax-b
f (1) = a (1) -b and f (1) = 1

1 = a-b
a-b = -1 (1)

f (-2) = a (-2) -b
f (1) = - 2a-b and f (-2) = - 5
-5 = -2a-b
2a-b = 5 (2)
From (1) and (2),
a-b = -1
2a-b = 5 (it gives line here mam with "-" mark on its tip)
-a = 4
a = -4 then 4-b = -1 b = -1-4 b = -5
So the function is f (x) = ax-b = -4x-5

The teacher appointed a representative from one group, whose answer was correct, to write the answer on the board. Afterwards, the teacher directed the group whose answer was incorrect to compare it with the correct answer. Then, the teacher explained as follows:

Teacher: 1 = a-b, this is the same as a-b = 1, so is -5 = -2a-b equals -2a-b = -5?
Ika: The mark changed because of moving the section
Teacher: The mark has not changed; it is still the same 1 = a-b with a-b = 1
Student: Yes ma'am
Teacher: Where is the next mistake?
Ardi: -1-5 = -6 not 4 so all the answers are incorrect ma'am
Teacher: Yes ..., addition and subtraction of integers still needs a lot of practice

During the closing stage, the teacher provided an evaluation by giving chances to students to express what they understood about the materials given.

***c. Reflection Stage (See)***

The *see* stage was conducted after the learning process was completed in which the model teacher and observers discussed all the observations been carried out. The activity began with a reflection from the model teacher and was followed by the observers. The results from the discussion agreed that at the *do* stage, there were still things which required attention, such as pre-requisite knowledge. There were still students who were not directly involved in group work. Therefore, the researchers suggested that the second cycle should retain activities that create progress or developments in the first cycle. Particularly, the PCK teachers were advised to explore students' pre-requisite knowledge, especially in addition and subtraction of integers, and to be aware of the types and causes of student errors so they could overcome those while solving the problems.

The materials used for reflection were the notes that the observers had and the recording that the documentation team recorded. The notes were used as a consideration in preparing the next learning plan.

**The Second Cycle of Lesson Study Activities**

***a. Planning Stage (Plan)***

The planning stage in the second cycle was similar to the plan stage in the first cycle. The lecturers and teachers discussed the learning design that was made together. At this stage, the learning design was designed based on the results of reflection in the first cycle. The learning design had several improvements, especially in the knowledge of pre-requisites about drawing function graphics, how to overcome errors and student misconceptions and class management. This improvement was expected so that all students could be active in the learning process.

***b. Implementation Stage (Do)***

The model teacher began the learning activities by opening and conveying the learning objectives and dividing students into several groups. The teacher invited students to recall the materials they have learned to explore students' knowledge of the previous materials. The students recalled the materials to determine the value of the function as it could be used in learning to draw graphs of function, specifically drawing graphs of the linear function. The teacher emphasised on pre-requisite knowledge by writing down some forms of algebraic operations:

Teacher: How to determine the value of a from -a-3 = 5

Nunu: -a = 5 + 3 bu,: -a = 8 or: a = -8
Teacher: Why does the mark: -3 and: -a change?
Sati: Because you moved the section, ma'am

Teacher: If you use a move segment that is what usually causes the answer to be incorrect or incorrect. So the truth is: -a-3 = 5, the first step of the two fields is added to 3 (why?)
Sinta: -a-3 + 3 = 5 + 3, -a = 8
Teacher: Then the two sections are divided by: -1 (why?)
Aris: (-a) / (- 1) = 8 / (- 1) then a = -8
Teacher: Good, this will be used later in drawing graph functions

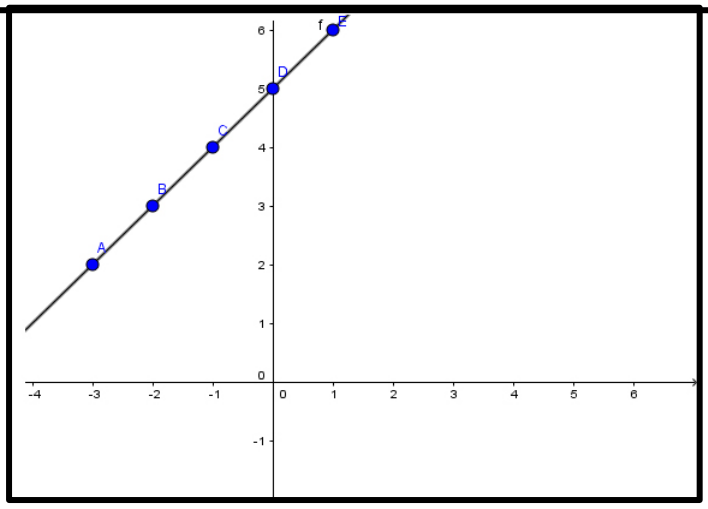
In the main stage, the teacher gave an explanation of the materials and an example to draw a function graph. She or he then distributed student worksheets to each group. Furthermore,

students worked on worksheets related to drawing function graphs, while the teacher was scaffolding the students. The answers to one of the groups experiencing errors were presented below:

Question:
Make a table of functions and graphs, if a function is expressed with f (x) = - x + 5 with the area of origin {x | -3≤x≤3, x∈R}
Answer:
f (x) = - x + 5
f (-3) = - 3 + 5 f (-2) = - 2 + 5 f (-1) = - 1 + 5 f (0) = 0 + 5
f (-3) = 2 f (-2) = 3 f (-1) = 4 f (0) = 5

f (1) = 1 + 5 f (2) = 2 + 5 f (3) = 3 + 5
f (1) = 6 f (2) = 7 f (3) = 8

| $x$ | $-3$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $f(x) = y$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 |




After that, the teacher approached the group with an incorrect answer while providing scaffolding to ensure students could find the correct answer. The teacher recalled the example given at the beginning of the meeting. The teacher emphasised that f (x) = - x + 5, then wrote on the student worksheet; f (...) = - (...) +5. Next, the students were welcomed to fill in the points in parentheses with the value; x = -3, x = -2 and so on, until the students found the right answer. Afterwards, the teacher gave an opportunity to the group whose answer was correct to write and present the results, while allowing other groups to ask a few questions.

As the closing activity, the teacher directed students to write a summary. She or he then asked questions to determine the extent to which students understood the materials on function graphs.

***c. Reflection Stage (See)***

Discussion in the *see* phase of learning was carried out after the *do* phase was completed. The results of the discussion indicated that the teacher's PCK on drawing function graph materials had resulted in positive changes. For instance, in overcoming student errors, the teacher first identified the type of student error before addressing it.

**Discussion**

The PCK components in this study were divided into three; Knowledge of the Subject Matter (KSM), Knowledge of Pedagogy (KP) and Knowledge of Students (KS).

**Knowledge of the Subject Matter (KSM)**

Knowledge of mathematical content, according to Black (2008), is divided into two categories. Namely, conceptual knowledge and procedural knowledge, which includes mathematical processes for mathematical use. Knowledge of the Subject Matter (KSM) is the teachers understanding of conceptual and procedural knowledge on function materials. Teacher's understanding is related to conceptual knowledge shown by the teacher to understand the concepts and relationships between concepts in functions. The teacher's understanding of the concept of functions is indicated by providing an explanation of the functions value in the first cycle and drawing graphs of functions in the second cycle. The teacher associates the concept of function with a system of linear equations when explaining how to determine the value of functions. Teacher's procedural knowledge is shown while explaining the procedure for solving the problem of determining the shape of the function and drawing a graph of the function. Therefore, the teacher conceptually understands the function materials; is able to explain the concepts and the relationships between concepts appropriately, and understands the procedure for solving the problem of the function and the reasons underlying each step of completion.

Procedural knowledge is the understanding to carry out certain procedures to produce solutions. In this study, procedural knowledge consists of the understanding of using methods or techniques and outlining procedures in accordance with the correct mathematical rules. Procedural knowledge contains algorithms, techniques and methods and is generally known

as the procedure. An understanding of different methods and solutions supports the teacher in mastering and using mathematics in learning.

Based on the above explanation, some findings were obtained including: (1) a teacher's knowledge of facts, definitions, symbols and concepts of a function is used to build students' understanding in constructing function formulas, equations and graphs; (2) procedural knowledge (arithmetic operations) are used to assist students in solving related problems by determining the value of functions; and (3) the teacher also has relational knowledge that makes it easier for students to represent the form of functions to linear equations. From the findings, it can be concluded that the aspect of PCK teachers, in terms of KSMs, was instrumental in building students' overall conceptual and procedural abilities. The conceptual knowledge of the teacher allows each student to trace the relationship between the concept of a function and the equation and the presentation of functions (variables), while the procedural knowledge of the teacher allows each student to be involved in determining the value of a function using algebraic counting operations.

**Knowledge of Pedagogy (KP)**

Knowledge about pedagogy, according to Kilic (2011), is the knowledge of planning and organising lessons as well as the teaching strategies. Teachers who have good pedagogical knowledge, have lists of teaching activities. The teacher determines the function value and graphs in the first cycle, while the linear function in the second cycle is indicated by the teacher giving apperception and learning objectives, as stated in the lesson plan. The teacher begins learning by conveying the learning objectives and giving apperception by linking the knowledge that has been learned previously with the new material to be learned. In addition, the teacher also explores the level of student mastery of the required material by asking a number of questions.

Representation is used by the teacher to communicate the concept of function into certain forms to make it easier for students to understand. The teacher employs verbal representation by presenting material with examples and questions to build students' understanding of the function materials. The teacher provides examples through questions and assignments according to the teaching material and learning objectives. To help comprehend the learning material, the teacher gives examples and practices in groups while measuring the learning achievement independently. This is in line with Kilic (2011), who stated that teachers with excellent knowledge of learning strategies are able to meet the needs of various types of students by choosing the right examples, assignments and representations to build on existing knowledge and facilitate student understanding. Therefore, the teachers should choose a learning strategy based on the characteristics of students and the teaching material. Besides,

the use of learning strategies can increase motivation, interaction and activities of students in learning.

In terms of managing learning, the findings obtained include: (1) to explore the students' initial knowledge, the teacher or the class starts learning with apperception; (2) the teacher conveys the learning objectives so that students know the target of competence that must be achieved; (3) representing functions into some examples is one alternative to bring concepts function to students; (4) to test understanding, teachers provide group assignments, while individual assignments are used to measure the achievement of the learning objectives. Based on the four findings above, it can be concluded that the PCK aspect of the teacher, in terms of the KP, allows the teacher to compile and implement exploratory learning scenarios that begin with the application of apperception, representation of material through examples, and finish with fairly unique evaluation techniques. It is unique, because the test technique is used for two different purposes. Group assignments are used to test the level of classical understanding, while individual tasks are used to measure the achievement of individual learning goals.

**Knowledge of Students (KS)**

Knowledge of students is the teacher's knowledge of students' mistakes and misconceptions. KS includes teacher's knowledge about types, causes and ways to overcome errors and misconceptions of students on the materials to determine the value of functions and draw the graphs. The teacher's knowledge of the types of errors and student misconceptions in the function materials is indicated by the teacher's ability to identify it when the students are determining the value of the function. The teacher identifies student procedural errors in determining the addition and subtraction of integers in the process of identifying function values and drawing graphs of linear functions.

Knowledge of students is related to the causes of errors and misconceptions of students. Teachers who identify these elements lack the basic knowledge of their students regarding the rules and objectives of a mathematical concept, resulting in procedural and conceptual errors. The procedural errors occur as they are not thorough in performing algebraic operations, while conceptual errors occur because they have not understood the basic concepts related to determining function values and drawing graphs of functions.

Knowledge of students requires an understanding of teaching material and pedagogy. The teacher discovers the causes of students' errors and misconceptions in the function materials in terms of conceptual and procedural knowledge. Namely, the lack of basic knowledge of students, such as addition and subtraction of integers and they do not yet understand the required material in determining function values and drawing function graphs. Developments

that occur in the form of increasing knowledge, errors in concepts and procedures as well as cognitive conflict happens when the concept in the cognitive structure is not well organised. These symptoms cannot be developed without guidance from the teacher. According to Shulman (in Ma'rufi, 2017), the teacher must know the causes of errors and student misconceptions as well as ways to overcome them.

As a professional, the teacher must continuously improve his or her service for the improvement of students. The changing times demand teachers to continue learning how to teach the students. Besides that, students must also master a variety of life skills that UNESCO formulates in the form of the four pillars of education. Namely, learning to be, learning to know, learning to do and learning to live together. Teachers must develop professionalism continually, and it is important to have a variety of knowledge and skills to plan and implement learning strategies that can encourage student understanding and learning. However, teachers not only need to have knowledge of good teaching material, pedagogy, curriculum and students, but they must also be able to implement their knowledge effectively in the teaching-learning process. Shulman (in Ma'rufi, 2017) found one of the teacher's knowledge as a basis for effective teaching. Namely, PCK in which PCK is a way of presenting and formulating scientific material that makes it easier for students to understand.

The learning process is inseparable from events between individuals and their experience, so before starting a new lesson as a stepping stone, the teacher should try to connect it with the learning material that has been mastered by students from the previous lesson or from their previous experience. This then allows the learning process to work effectively. Renold (2002) argued that effective teachers connect what students know with the new information and rely on knowledge of teaching material (KSM) to create effective learning, while providing good explanations for students.

Several findings related to the KS aspects in PCK include: (1) the teacher identifying and clarifying the types of errors that occur in students, including misconceptions and procedural errors; (2) the teacher revealing the causes of the two types of errors; and (3) the occurrence of multi-errors made by students with the same cause. Namely, a lack of mastery on the materials required in terms of the ability to manipulate algebraic forms. Based on the findings, it can be concluded that the process of learning interactions between teachers and students occurs intensely. Then, the results of these interactions allow teachers to recognise the level of their students' abilities in detail. The detailed information on students' knowledge tends to trigger the teacher to do metacognition to improve previous learning and to build a better learning culture in the future.

## Conclusion

Pedagogical content knowledge is a type of knowledge which integrates mathematical knowledge, pedagogy and knowledge about students. Teachers understanding of function materials is integrated with teacher knowledge about students, which has an impact on teacher knowledge about pedagogy with indications that: (1) teachers integrate conceptual and procedural knowledge through expository methods, question and answer, and apperception in presenting function materials (providing apperception by exploring pre-requisite material helps students to determine the value of functions and drawing graphs of functions); (2) using verbal representations accompanied by examples through questions by explaining concepts to build understanding in students; and (3) giving assignments in the form of multi-practices by paying attention to the difficulty level of each question to deepen students' understanding of the material in determining function values and drawing graphs of linear functions.

Teachers' PCK in learning function materials is explained in three domains. First, KSM teachers have a role in developing students' conceptual and procedural abilities. Teacher's conceptual knowledge allows each student to trace the relationship between the concept of function, the equation and the presentation of functions in the form of variables. While, the procedural knowledge of the teacher allows each student to be involved in determining the value of a function using algebraic operations. Second, KP allows the teacher to compile and implement exploratory learning scenarios that begin with the application of apperception, represent the material through various examples, and ends with a fairly unique evaluation technique. It is assumed to be unique because the test technique is used with two different purposes. There are group assignments used to test the level of classical understanding and individual tasks which are used to measure the achievement of individual learning goals. Finally, KS allows teachers to recognise the level of their students' abilities in detail. The detailed information on students' understanding tends to trigger the teacher to carry out metacognition in improving previous learning to build a better learning culture in the future.

In this case, the teacher, as the subject of research, certainly has different capacities. The difference in capacity certainly results in different types of PCK. As an extension of the study of PCK, researchers suggest further research to consider the selection of other materials, teachers' education background and training programs previously attended by the teachers. This can provide more diverse information that can help teachers in planning, managing, and evaluating better mathematics learning.

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**REFERENCES**

An, S., Kulm, G., Wu, Z., Ma, F. & Wang, L. (2002). *A comparative study of mathematics teachers' beliefs and their impact on the teaching practice between the U.S. and China*. Invited paper presented at the International Conference on Mathematics Instruction, Hong Kong.

An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teacher in China and The USA. *Journal of Mathematics Teachers Education* 7: 145-172

Arends, R. I., Wenitzky, N. E., & Tannenboum, M. D. (2001). *Exploring teaching: An introduction to education.* New York: McGraw-Hill Companies, Inc.

Arends, Richard I. (1997). *Classroom instruction and management*. New York: MC Grow-Hill Companies, Inc.

Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 83–104). Westport, UK: Ablex.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education,* 59(5), 389-407.

Fernandez, C. & Yoshida, M. (2004). *Lesson Study: A Japanese Approach to Improving Mathematics Teaching and Learning*. London: Lawrence Erlbaum Associated, Inc.

Fernandez, C., Yoshida, M., Chokshi, S., & Cannon, J. (2001). *An Overview of Lesson Study*, ppt. online lsrg@columbia.edu, www.tc.edu/lessonstudy.

Hawkins, W.J. (2012). *An investigation of primary teachers' pedagogical content knowledge whan teaching measurement three and four.* 12th International on Mathematics Education

Isiskal, M. & Cakirogu, E. (2011). The nature of prospective mathematics teachers' pedagogical content knowledge: the case of multiplication of fractions. *Journal Math Teacher Education*. 14(3). Hal. 213 – 230.

Jakobsen, A., Thames, M.H., & Ribeiro, M. (2013). Delineating issues related to horizon content knowledge for mathematics teaching. *Paper presented at the Eight Congress of European Research in Mathematics Education* (CERME-8). Antalya, Turkey.

Kilic, H., (2011). Pre-service secondary mathematics teachers' knowledge of students. *Turkish Online Journal of Qualitative Inquiry*, April 2011, 2(2).

Lewis, C. (2002). *Lesson study: A handbook of teacher-led instructional change*. Philadelphia: Research for Better Schools.

Lewis, C., Perry, R., Hurd, J., & O'Connel, M. P. (2006). *Teacher collaboration: Lesson study comes of age in North America*.

Ma'rufi, (2016). Pedagogical content knowledge (PCK): Special knowledge type of teaching effective learning (case study of high school). *International Journal of Library Science Volume 14, Issue 2*. India: Ceser Publication.

_____, (2016). Teacher's pedagogical content knowledge on students-matter on quadratic function materials. *Proceedings 3rd ICRIEMS "The Global Challenges on the Development and the Education of Mathematics and Science*. Yogyakarta: Faculty of Mathematics and Science.

Ma'rufi, Budayasa, I. K., & Juniati, D. (2017, February). Pedagogical content knowledge: Knowledge of pedagogy novice teachers in mathematics learning on limit algebraic function. In *AIP Conference Proceedings* (Vol. 1813, No. 1, p. 050003). AIP Publishing.

Mishra. P dan Koehler. M.J. (2009). *What is technological pedagogical content knowledge*? *Contemporary issues in technology and teacher education*. 9(I). 60-70.

Ma'rufi, Budayasa, I. K., & Juniati, D. (2017, August). The analysis of mathematics teachers' learning on algebra function limit material based on teaching experience difference. In *AIP Conference Proceedings* (Vol. 1867, No. 1, p. 020056). AIP Publishing.

NCTM. (2000). *Principle and standards for school mathematics.* Reston: The National Council of Teacher Mathematics, Inc.

Renold, E. (2002). Using vignettes in qualitative research. *Building Research Capacity*, 3. 3-5.

Tirosh, D. (2000). Enhancing prospective teachers' knowledge of children's conceptions: The case of division of fractions. *Journal for Research in Mathematics Education*, 31, 5–25.

Yeo, K.K.J., (2008). Teaching area and perimeter: Mathematics-pedagogical-content knowledge-in-action. In M. Goos, R. Brown, & K. Makar (Eds.), *Proceeding of the 31st Annual Conference of the Mathematics Education Research Group of Australasia*. Merga, 621-627.