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Robust Model of the Combination of Expectations and Conditional Value-at-Risk from Paddy Farming Risk Management Based on Climate Variability

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Agriculture is one sector that is vulnerable to risks, including production risk. Production risk is influenced by the uncertainty of climate variability, that results in reduced agricultural productivity. Risk management is an alternative method that can be used to reduce these risks. There are two risk management procedures that can be done, namely decision support systems and financial products. There are two risk measures used in these risk management practices, namely Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR). VaR is a risk measure that is widely used but has several weaknesses. The weakness of the VaR can be overcome by CVaR which is the expected loss in excess of VaR. However, in general information on the distribution of climate variables is not known with certainty. To overcome the climate uncertainty, work can be done through optimization modelling, with the Robust Conditional Value-at-Risk (RCVaR) approach. The aim of this study is to complete an agricultural risk management optimization model based on the uncertainty of climate variables with RCVaR under ellipsoid uncertainty. The solution of the robust expectations and CVaR models is stronger because it is the worst case of possible variability in climate variables.

Key words: *Conditional Value-at-Risk, Crop Insurance, Land allocation, Paddy Rice, Robust Optimization.*

Introduction

The agricultural sector is directly affected by climate variability, as well as agriculture in Indonesia, as it is a tropical country. Indonesia is very vulnerable to climate change (Measey, 2010). Agricultural production in Indonesia is strongly influenced by variations in climate variables. Drought conditions have an impact on agricultural output, farmer incomes and prices of staple foods (Naylor et al, 2007). Drought has a big impact on food loss, so much so that it can cause a world food crisis (D'Arrigo and Wilson, 2008). Because of the uncertainty of climate variables, farmers around the world are exposed to high risks during each agricultural season. Risk directly plays an important role in agricultural decision making (Naylor and Mastrandrea, 2009). Climate variability is a major risk for agricultural production (Alam et al, 2011). There are several types of risks in agricultural systems that are caused by climate variables, including crop loss and water shortages (Abid et al, 2016). The most appropriate way to deal with climate risk is through risk management. There are two types of risk management options that can be done, namely decision support systems and financial products such as insurance (Alam et al, 2011).

There are two risk measures used in risk management practices, namely Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) (Alexander et al. 2004). VaR is an important and widely used risk measure. However, VaR presents several weaknesses that can be overcome by using other risk measures, one of which is CVaR (Balbas et al. 2017). Rockafellar and Uryasev (2000), consider CVaR to be a more consistent measure of risk than VaR. For continuous distribution, CVaR is the expected loss exceeding VaR. For discrete distributions, CVaR is the average weighting of VaR and losses exceeding VaR. CVaR is the upper limit for VaR so that minimizing CVaR can reduce VaR to make arrangements to calculate VaR and optimize CVaR simultaneously(Larsen et al. 2001). This approach is carried out to maximize return expectations with CVaR constraints.

Risk measures provide results that are sensitive to errors in data, based on this several studies have been carried out by assuming that a portion of the return distribution is not known with certainty. This uncertainty can be resolved by Robust Optimization (Huang et al. 2010). El Ghaoui et al. (2003), use the worst-case VaR as the largest VaR that can be achieved. Zhu and Fukushima (2009), use the worst-case CVaR to handle these assumptions under the uncertainty of mixed distribution, box uncertainty, and ellipsoidal uncertainty. VaR and CVaR are widely applied in portfolios. Pac and Pinar (2013), consider the problem of optimal portfolio selection using CVaR and VaR when the average is known and the variance of returns is known, but the distribution of asset risk returns is unknown. Uncertainty (ambiguity) on the average return is modelled in the set of ellipsoidal uncertainty. Lotfi and Zenios (2018), developed a robust model of optimizing VaR and CVaR risk measures by minimizing return expectations under combined ambiguity in distribution, average returns, and covariance matrices for the set of the ellipsoid, polyhedral, and interval uncertainties.

This study uses a robust optimization approach in the selection of agricultural insurance products and rice planting activities by considering the CVaR risk measures and the uncertainty of the ellipsoid distribution. Previous research did not consider the uncertainty of climate distribution that affects the agricultural industries productivity, so the completion of this study will provide a robust solution to the distribution of climate variables in general. The solution of this optimization model is to obtain optimal insurance products and optimal land allocation for planting paddy and inland rice at certain planting periods.

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Measuring Risk of Climate Variables for Agriculture

Climate risk plays an important role in agriculture. Several factors can have an influence these risks, including basis risk, price uncertainty, and the effect of diversification (Berg and Schmitz, 2007). There are several instruments that can be used for agricultural risk management. The spread of risk through diversification of agricultural activities can be reduced by choosing a portfolio of activities that have results with little correlation. According to Artzner et al. (1999), the risk is the variability of the future value of a situation, due to changes in something due to uncertain events.

Value-at-Risk (VaR) is a measure of risk that has become popular in risk management. However VaR has several disadvantages, including that VaR is not sub-additive in the case of general distribution, so it is not a coherent risk measure (Artzner et al., 1999), VaR can have several local extremes for discrete distribution; and VaR is only a percentile of the distribution of losses, so there is not much to say about the nature of the extreme losses that go beyond it (Zhu and Fukushima, 2009; Ipole, Agba and Okpa, 2018). This weakness requires inconsistency with the principle of diversification that is well accepted (diversification reduces risk) and a bigger problem from the point of view of numerical tractability. To overcome this problem Rockafellar and Uryasev (2002), introduce a Conditional Value-at-Risk (CVaR) risk measure. With the same level of confidence, VaR is the lower limit for CVaR. In optimization applications, CVaR is superior to VaR (Rockafellar and Uryasev, 2000). CVaR is defined as the average tail distribution that exceeds VaR. CVaR is a risk measure that has some properties that are better than VaR (Rochafellar and Uryasev, 2000). Minimizing CVaR can be achieved by minimizing the assistive functions that are easier to trace without determining the appropriate VaR in advance, and at the same time, VaR can be calculated as a by-product. CVaR Minimization Formulation produces convex and linear programming. This causes CVaR to be applied in financial optimization and risk management. CVaR is a coherent risk (Rockafellar and Uryasev, 2002).

Definition of Conditional Value-at-Risk (CVaR) (Quaranta and Zaffaroni, 2008). Let $\mathrm{x} \in X \subset R^n$ be a decision vector, $\mathrm{y} \in Y \subset R^n$ is the future value of the number of variables, and $\forall \mathrm{x}$ is denoted by $\psi(\mathrm{x},\cdot)$ the loss distribution function $z = f(x,y)$

$$\psi(\mathrm{x},\alpha) = P\{y | f(x,y) \le \alpha\}$$

Given $\alpha > 0$, then $\alpha - CVaR$ of the loss associated with x is the average of the $\alpha -$tail distribution of the loss function, this means that the average distribution function $\psi_\alpha(\mathrm{x},\cdot)$ is defined as follows:

$$\psi_\alpha(\mathrm{x},\cdot a) = \begin{cases} 0 & \text{jika } a < a_\alpha(\mathrm{x}) \\ \dfrac{\psi(\mathrm{x}-a)-\alpha}{1-\alpha} & \text{jika } a < a_\alpha(\mathrm{x}) \end{cases} \quad (1)$$

and $a_\alpha(\mathrm{x})$ is $\alpha - VaR$ from loss related x.

Rockafellar and Uryasev (2002), prove that Conditional Value-at-Risk is a coherent measure of risk in general. Furthermore, the above formulation makes it possible to minimize CVaR using linear programming methods.

Rockafellar dan Uryasev (2002) define functions, so that

$$\mathrm{CVaR}_\beta(f(\mathbf{x},\xi)) = \max_{\gamma \in R} F_\beta(\mathbf{x},\gamma) \quad (2)$$

Where $F_\beta(\mathbf{x},\gamma) = \gamma + \frac{1}{1-\beta} E\left[\left(f(\mathbf{x},\xi)-\gamma\right)^+\right] = \gamma + \frac{1}{1-\beta}\sum_{i=1}^{n} p_i \left(f(\mathbf{x},\xi)-\gamma\right)^+$ with $[t]^+ = \max\{0,t\}$.

Definition of Worst-case CVaR (*WCVaR*) for respect to $x \in X$ defined as $\mathrm{WCVaR}_\beta(\mathbf{x}) = \sup_{p \in P} \mathrm{CVaR}_\beta(\mathbf{x})$.

Optimization Model of Agriculture Risk Management

The risk management model in this study refers to the loss and constraint function proposed by Liu et al (2007). The first constraint assumes that the amount of land allocated for the date of planting the type of agriculture is equal to the total area of land available. The second obstacle assumes that farmers can only buy one type of insurance policy for each crop. Agricultural risk management solutions are obtained from the optimization model, including the allocation of land for plants k on the date of planting d_k and having an optimal insurance policy for plants k every planting date d_k.

The optimization model is written as follows:

$$\begin{aligned} \min \quad & E\left(f\left(\mathbf{x},\mathbf{y}\right)\right) + d \cdot CVaR_{\beta}\left(f\left(\mathbf{x},\mathbf{y}\right)\right) \\ s.t. \quad & \sum_{d_k=1}^{D} x_{d_k} = q_k \\ & \sum_{i=1}^{I} \lambda_{ik} = 1, \qquad k = 1,\dots,K \\ & x_{d_k} \geq 0 \\ & \lambda_{ik} = \begin{cases} 1, & \text{If the farmer chooses a policy } i \text{ for crops } k \\ 0, & \text{otherwise} \end{cases} \end{aligned} \tag{3}$$

Use the function (2), so that the objective function of the optimization model (3) can be written as follows:

$$\begin{aligned} & \min_{\mathbf{x}} \; E(f(\mathbf{x},\mathbf{y})) + d \cdot CVaR_{\beta}(f(\mathbf{x},\mathbf{y})) \\ & = \min_{\mathbf{x},\alpha} \left[E(f(\mathbf{x},\mathbf{y})) + d \cdot \left(\alpha + \frac{1}{S(1-\beta)} \sum_{s=1}^{S} [f(\mathbf{x},y_s) - \alpha]^{+} \right) \right] \end{aligned} \tag{4}$$

The loss function expectation can be written as follows:

$$E(f(\mathbf{x},\mathbf{y})) = \sum_{s=1}^{S} f(\mathbf{x},y_s) \cdot p(y_s) \tag{5}$$

Furthermore, to complete the maximum function, we can introduce the auxiliary variable z_s and add additional constraints, so that the optimization model (3) can be written as follows:

$$\begin{aligned}
\min_{\mathbf{x},\alpha} \quad & \sum_{s=1}^{S} f(\mathbf{x}, y_s)\cdot p(y_s) + d\cdot\left(\alpha + \frac{1}{S(1-\beta)}\sum_{s=1}^{S} z_s\right) \\
s.t. \quad & \sum_{d_k=1}^{D} x_{d_k} = q_k \\
& \sum_{i=1}^{I} \lambda_{ik} = 1, \qquad k = 1,\dots,K \\
& f(\mathbf{x}, y_s) - \alpha \le z_s, \qquad s = 1,\dots,S \\
& x_{d_k} \ge 0 \\
& \lambda_{ik} = \begin{cases} 1, & \text{If the farmer chooses a policy } i \text{ for crops } k \\ 0, & \text{otherwise} \end{cases}
\end{aligned} \tag{6}$$

Robust Conditional Value-at-Risk

Risk measures provide results that are sensitive to errors in data. Based on this, it is assumed that a portion of the return distribution is not known with certainty. In the optimization model (6) expectations and Conditional Value-at-Risk (CVaR) involve opportunities, so that reformulation of the objective function is carried out.

The following is a reformulation for Conditional Value-at-Risk,

$$\min_{\mathbf{x},\alpha}\max_{p\in P_e}\left(\alpha + \frac{1}{1-\beta}\sum_{s=1}^{S} p_s z_s\right) = \min_{\mathbf{x},\alpha}\max_{p\in P_e}\left(\alpha + \frac{1}{1-\beta}\mathbf{p}^T\mathbf{z}\right) \tag{7}$$

Let the vector $\mathbf{p}$ is in the set ellipsoid (Qiu et al, 2014), then the objective function for CVaR (7) into

$$\min_{\mathbf{x},\alpha}\left\{\alpha + \frac{1}{1-\beta}\mathbf{p}_0^T\mathbf{z} + \frac{1}{1-\beta}\min_{\xi}\left(-\mathbf{z}^T\mathbf{A}\xi \,\middle|\, \mathbf{e}^T\mathbf{A}\xi = 0, \mathbf{p}_0 + A\xi \ge 0, \|\xi\| \le 1\right)\right\} \tag{8}$$

Following is the Lagrange function of the third term of equation (8)

$$L(\boldsymbol{\omega}, \rho, \mu; \xi) = -\mathbf{z}^T\mathbf{A}\xi + \boldsymbol{\omega}^T\left(-\mathbf{p}_0^T - \mathbf{A}\xi\right) + \rho\left(\|\xi\| - 1\right) + \mu\mathbf{e}^T\mathbf{A}\xi \tag{9}$$

The Dual Lagrange function of equation (7) is as follows:

$$g(\boldsymbol{\omega}, \rho, \mu) = \min_{\xi} L(\boldsymbol{\omega}, \rho, \mu) = \left(-\mathbf{p}_0{}^T\boldsymbol{\omega} - \rho\right) - h^*\left(\mathbf{A}^T\mathbf{z} + \mathbf{A}^T\boldsymbol{\omega} - \mathbf{A}^T\mathbf{e}\mu\right) \tag{10}$$

where $h^*\left(\mathbf{A}^T\mathbf{z} + \mathbf{A}^T\boldsymbol{\omega} - \mathbf{A}^T\mathbf{e}\mu\right)$ is a conjugate function, such that $\left\|\mathbf{A}^T\mathbf{z} + \mathbf{A}^T\boldsymbol{\omega} - \mathbf{A}^T\mathbf{e}\mu\right\| \le \rho$, $\boldsymbol{\omega} \ge 0$ and $\boldsymbol{\rho} \ge 0$, so that

$$
\begin{aligned}
&\max_{\boldsymbol{\omega},\rho,\mu} g(\boldsymbol{\omega},\rho,\mu) \\
&= \max_{\boldsymbol{\omega},\rho,\mu}\left\{-\mathbf{p}_0{}^T\boldsymbol{\omega} - \rho \middle| \left\|\mathbf{A}^T\mathbf{z} + \mathbf{A}^T\boldsymbol{\omega} - \mathbf{A}^T\mathbf{e}\mu\right\| \le \rho, \boldsymbol{\omega} \ge 0, \rho \ge 0\right\} \\
&= \min_{\boldsymbol{\omega},\rho,\mu}\left\{\mathbf{p}_0{}^T\boldsymbol{\omega} + \rho \middle| \left\|\mathbf{A}^T\mathbf{z} + \mathbf{A}^T\boldsymbol{\omega} - \mathbf{A}^T\mathbf{e}\mu\right\| \le \rho, \boldsymbol{\omega} \ge 0, \rho \ge 0\right\}
\end{aligned} \tag{11}
$$

Equation (8) is equivalent to the following equation:

$$
\begin{aligned}
&\min_{\mathbf{x},\alpha}\left\{\alpha + \frac{1}{1-\beta}\mathbf{p}_0^T\mathbf{z} + \frac{1}{1-\beta}\min_{\boldsymbol{\omega},\rho,\mu}\left\{\mathbf{p}_0{}^T\boldsymbol{\omega} + \rho \middle| \left\|\mathbf{A}^T\mathbf{z} + \mathbf{A}^T\boldsymbol{\omega} - \mathbf{A}^T\mathbf{e}\mu\right\| \le \rho, \boldsymbol{\omega} \ge 0, \rho \ge 0\right\}\right\} \\
&= \min_{\mathbf{x},\alpha,\boldsymbol{\omega},\rho,\mu}\left\{\alpha + \frac{1}{1-\beta}\mathbf{p}_0^T\mathbf{z} + \frac{1}{1-\beta}\left(\mathbf{p}_0{}^T\boldsymbol{\omega} + \rho\right) \middle| \left\|\mathbf{A}^T\mathbf{z} + \mathbf{A}^T\boldsymbol{\omega} - \mathbf{A}^T\mathbf{e}\mu\right\| \le \rho, \boldsymbol{\omega} \ge 0, \rho \ge 0\right\}
\end{aligned} \tag{12}
$$

Furthermore, the reformulation for the expected loss function is as follows:

$$
\max_{p\in P_e} E[f(\mathbf{x},\mathbf{y})] = \sum_{s=1}^{S} f(\mathbf{x}, y_s)\cdot p_s = \mathbf{f}^T\mathbf{p} \tag{13}
$$

where $\mathbf{f} = (f_1, f_2, \ldots, f_n)^T$

Equation (13) is equivalent to the following equation:

$$
\max_{p\in P_e} E[f(\mathbf{x},\mathbf{y})] = \max_{p\in P_e}\sum_{s=1}^{S} f(\mathbf{x}, y_s)\cdot p_s = \max_{p\in P_e}\mathbf{f}^T\mathbf{p} \tag{14}
$$

Considering the ellipsoid distribution, equation (14) is equivalent to the following equation:

$$
\begin{aligned}
&\min_{\mathbf{x},\alpha}\left\{\mathbf{f}^T\mathbf{p} - \max_{\boldsymbol{\omega},\rho,\mu}\left\{-\mathbf{p}_0{}^T\boldsymbol{\omega} + -\rho \middle| \left\|-\mathbf{A}^T\mathbf{f} + \mathbf{A}^T\boldsymbol{\omega} - \mathbf{A}^T\mathbf{e}\mu\right\| \le \rho, \boldsymbol{\omega} \ge 0, \rho \ge 0\right\}\right\} \\
&= \min_{\mathbf{x},\alpha,\boldsymbol{\omega},\rho,\mu}\left\{\mathbf{f}^T\mathbf{p} + \mathbf{p}_0{}^T\boldsymbol{\omega} + \rho \middle| \left\|-\mathbf{A}^T\mathbf{f} + \mathbf{A}^T\boldsymbol{\omega} - \mathbf{A}^T\mathbf{e}\mu\right\| \le \rho, \boldsymbol{\omega} \ge 0, \rho \ge 0\right\}
\end{aligned} \tag{15}
$$

The optimization model (6) can be rewritten as follows:

$$\min_{\mathbf{x},\alpha,\boldsymbol{\omega},\rho,\mu,\hat{\boldsymbol{\omega}},\hat{\rho},\hat{\mu}} \quad \left(\mathbf{f}^T\mathbf{p}+\mathbf{p}_0{}^T\boldsymbol{\omega}+\rho\right)+d\cdot\left(\alpha+\frac{1}{1-\beta}\mathbf{p}_0^T\mathbf{z}+\frac{1}{1-\beta}\left(\mathbf{p}_0{}^T\hat{\boldsymbol{\omega}}+\hat{\rho}\right)\right)$$

$$\begin{aligned}
s.t. \quad & \sum_{d_k=1}^{D} x_{d_k} = q_k \\
& \sum_{i=1}^{I} \lambda_{ik} = 1, \qquad k=1,\dots,K \\
& f\left(\mathbf{x}, y_s\right)-\alpha \le z_s, \qquad s=1,\dots,S \\
& \left\|-\mathbf{A}^T\mathbf{f}+\mathbf{A}^T\boldsymbol{\omega}-\mathbf{A}^T\mathbf{e}\mu\right\| \le \rho, \\
& \left\|\mathbf{A}^T\mathbf{z}+\mathbf{A}^T\hat{\boldsymbol{\omega}}-\mathbf{A}^T\mathbf{e}\hat{\mu}\right\| \le \hat{\rho}, \\
& \boldsymbol{\omega} \ge 0, \quad \rho \ge 0, \\
& \hat{\boldsymbol{\omega}} \ge 0, \quad \hat{\rho} \ge 0, \\
& z_s \ge 0, \quad x_{d_k} \ge 0, \\
& \lambda_{ik} = \begin{cases} 1, & \text{If the farmer chooses a policy } i \text{ for crops } k \\ 0, & \text{otherwise} \end{cases}
\end{aligned} \qquad (16)$$

Results and Discussion

In this study one type of agriculture was selected, namely paddy rice, so $k=1$. Based on the optimization model (1), we need some data including the cost of producing lowland rice (c), the land allocated for lowland rice (q), insurance policy premium i (r_i), the agricultural market price, or the price of milled unhusked rice in scenario s (p_s), the standard price of lowland rice (p^*), agricultural products for planting date d in the s (y_{ds}) scenario, and the agricultural products insured by policy i (y_i^*).

Assuming there are 11 annual scenarios (2008-2018). The planting date is divided into every month. There are two insurance policies, $I=\{1,2\}$, namely a loss insurance policy and climate index insurance policy. The loss insurance policy premium (r_1) is 180,000 IDR, while the climate index insurance premium will be carried out in a simulation.

According to the Ministry of Agriculture of the Republic of Indonesia, the annual production cost of planting rice (c) is 6,000,000 IDR. For example, farmers allocate $q=5$ ha to plant paddy rice every year. The standard price of lowland rice is determined using a Conditional Value-at-Risk measure of the 2008-2018 price, $p^*=5{,}500$. Assuming the insured agricultural yields for the two policies are the same, $y_1^*=y_2^*=67.67$. The price of milled unhulled rice, the results of semester 1 rice productivity and the results of semester 2 rice productivity are presented in Table 1.

Table 1: The average price of dry milled grain and rice productivity in semester 1 and semester 2.

Scenario	p_s	y_{1s}	y_{2s}	y_{3s}	y_{4s}	y_{5s}	y_{6s}	y_{7s}	y_{8s}	y_{9s}	y_{10s}	y_{11s}	y_{12s}
1	2792	67.82	72.04	68.79	67.31	65.29	66.54	64.24	59.70	53.26	54.68	64.45	68.14
2	2997	70.31	71.13	65.87	63.55	63.61	67.14	67.23	65.89	55.10	53.89	59.01	63.38
3	3532	67.96	64.70	64.77	61.05	61.12	62.91	66.16	62.01	64.26	62.69	64.33	68.21
4	4030	68.56	68.01	66.83	67.19	65.61	66.86	69.38	64.09	55.47	50.19	64.90	64.67
5	4457	67.85	68.06	67.15	64.66	64.62	71.04	64.75	60.67	59.08	54.69	57.62	65.27
6	4594	66.47	66.20	64.03	62.72	64.65	66.55	71.34	65.63	59.75	54.78	61.16	67.25
7	4748	72.78	71.99	66.57	64.98	62.66	65.79	68.67	68.96	59.60	52.35	57.96	63.90
8	5280	68.05	69.55	66.04	63.22	65.85	65.57	66.64	64.44	56.80	50.26	52.84	59.71
9	5458	62.58	64.50	63.03	58.87	60.25	66.70	66.15	63.33	63.51	63.94	62.00	63.54
10	5500	65.14	65.17	64.28	63.50	63.78	65.85	61.50	61.73	55.51	57.78	63.63	65.15
11	5501	64.10	68.52	66.08	62.13	61.47	63.05	67.39	61.69	54.16	48.11	58.21	62.65

Based on these data, the loss function for each scenario can be formulated as follows:

$$f(\mathbf{x}, y_s) = c \cdot q - \sum_{d=1}^{12} (x_d y_{ds}) \cdot p_s + \lambda_1 \left[r_1 \cdot q - \left(\sum_{d=1}^{12} x_d \left(y_1^* - y_{ds} \right) \right) \cdot p^* \right] + \lambda_2 \left[r_2 \cdot q - \left(\sum_{d=1}^{12} x_d \left(y_2^* - y_{ds} \right) x_1 \right) \cdot p^* \right] \quad (17)$$

Minimization Model of Combination Expectations and CVaR

Suppose opportunities each scenario is the same, then

$$p(y_s) = \frac{1}{11}, \qquad s = 1, \ldots, S$$

The optimization model (1) becomes

$$
\begin{aligned}
\min_{\mathbf{x},\alpha} \quad & \min_{\mathbf{x},\alpha} \quad \sum_{s=1}^{11} f\left(\mathbf{x}, y_s\right)\cdot p\left(y_s\right) + d\cdot\left(\alpha + \frac{1}{11\left(1-\beta\right)}\sum_{s=1}^{11} z_s\right) \\
s.t. \quad & \sum_{d=1}^{12} x_d = 5, \\
& \lambda_1 + \lambda_2 = 1, \\
& f\left(\mathbf{x}, y_s\right) - \alpha \le z_s, \qquad s = 1,\dots,11 \\
& x_d \quad \ge 0, \qquad d = 1,\dots,12 \\
& \lambda_1, \lambda_2 \in \{0,1\}
\end{aligned} \tag{18}
$$

In this numerical simulation experiment for three major conditions of insurance policy premiums agriculture and climate index insurance. The first condition is the loss of the insurance policy premium (r_1) is greater than the climate index insurance policy premium (r_2). For example $r_1 = 180000$, $r_2 = 166000$, and select several β values, between 0.80 and 0.99. In this condition, the results were obtained that the farmer must allocate 1 hectare of land for the 10th planting date, which is October, and choose a climate index insurance policy on the planting date. The VaR value of this condition is 5,364,317 IDR with a CVaR value of 5,851,983 IDR for all of these values. The second condition is the loss of insurance policy premium (r_1) is the same as the climate index insurance policy premium (r_2). For example $r_1 = r_2 = 180000$ and select several β values. This condition provides an optimal solution that farmers can choose between the two insurance policies by allocating 1 hectare of land for the October planting date. The VaR value is based on the value, including 5,377,151 IDR and the CVaR value is 5,865,983 IDR for all these values. The third condition is the loss insurance premiums (r_1) less than the climate index insurance policy premiums (r_2). Suppose $r_1 = 180000$, $r_1 = 194000$. By selecting several β values, a solution is obtained that the farmer must choose an insurance policy by allocating 1 hectare of land for the October planting date. The VaR value is 5,377,151 IDR and the CVaR value is 5,865,983 IDR for all these β values.

Based on the three conditions, it is concluded that farmers can choose to allocate 1 hectare of land for the date of October planting by choosing an insurance policy that has the smallest premium.

Robust Optimization Model of a Combination of Expectations and CVaR

The robust minimization model of the combination of expectations and CVaR in the model (16) can be rewritten as follows:

$$\min_{\mathbf{x},\alpha,\boldsymbol{\omega},\rho,\mu,\hat{\boldsymbol{\omega}},\hat{\rho},\hat{\mu}} \left(\frac{1}{11}\sum_{s=1}^{11} f\left(\mathbf{x}, y_s\right)+\frac{1}{11}\sum_{s=1}^{11}\omega_s+\rho\right) + d\cdot\left(\alpha+\frac{1}{11(1-\beta)}\sum_{s=1}^{11} z_s+\frac{1}{11(1-\beta)}\sum_{s=1}^{11}\hat{\omega}_s+\frac{1}{1-\beta}\hat{\rho}\right)$$

$$\begin{aligned}
s.t.\quad & \sum_{d=1}^{12} x_d = 5, \\
& \lambda_1+\lambda_2=1, \\
& f\left(\mathbf{x}, y_s\right)-\alpha \le z_s, \qquad s=1,\ldots,11 \\
& \left\|-\mathbf{A}^T\mathbf{f}+\mathbf{A}^T\boldsymbol{\omega}-\mathbf{A}^T\mathbf{e}\mu\right\| \le \rho, \\
& \left\|\mathbf{A}^T\mathbf{z}+\mathbf{A}^T\hat{\boldsymbol{\omega}}-\mathbf{A}^T\mathbf{e}\hat{\mu}\right\| \le \hat{\rho}, \\
& \boldsymbol{\omega} \ge 0, \quad \rho \ge 0, \\
& \hat{\boldsymbol{\omega}} \ge 0, \quad \hat{\rho} \ge 0, \\
& z_s \ge 0, \quad s=1,\ldots,11 \\
& x_d \ge 0, \quad d=1,\ldots,12 \\
& \lambda_1,\lambda_2=\{0,1\}
\end{aligned} \tag{19}$$

Consider the fourth and fifth constraints of the optimization model (21), reformulation of the fourth and fifth constraints, Suppose matrix $\mathbf{A}_{11\times 11}=\frac{1}{2}\mathbf{I}$, so that

$$\left\|-\mathbf{A}^T\mathbf{f}+\mathbf{A}^T\boldsymbol{\omega}-\mathbf{A}^T\mathbf{e}\mu\right\|=\sqrt{\sum_{s=1}^{11}\left(-\frac{1}{2}f_s+\frac{1}{2}\omega_s-\frac{1}{2}\mu\right)^2}\le\rho \text{ and}$$

$$\left\|-\mathbf{A}^T\mathbf{f}+\mathbf{A}^T\boldsymbol{\omega}-\mathbf{A}^T\mathbf{e}\mu\right\|=\sqrt{\sum_{s=1}^{11}\left(\frac{1}{2}\hat{z}_s+\frac{1}{2}\hat{\omega}_s-\frac{1}{2}\hat{\mu}\right)^2}\le\hat{\rho}$$

The optimization model (19) is equivalent to the following optimization model:

$$
\begin{aligned}
\min_{\mathbf{x},\alpha,\boldsymbol{\omega},\rho,\mu,\hat{\boldsymbol{\omega}},\hat{\rho},\hat{\mu}} \quad & \left(\frac{1}{11}\sum_{s=1}^{11} f\left(\mathbf{x},y_s\right)+\frac{1}{11}\sum_{s=1}^{11}\omega_s+\rho\right) \\
& +d\cdot\left(\alpha+\frac{1}{11(1-\beta)}\sum_{s=1}^{11} z_s+\frac{1}{11(1-\beta)}\sum_{s=1}^{11}\hat{\omega}_s+\frac{1}{1-\beta}\hat{\rho}\right) \\
s.t. \quad & \sum_{d=1}^{12} x_d = 5, \\
& \lambda_1+\lambda_2=1, \\
& f\left(\mathbf{x},y_s\right)-\alpha \le z_s, \qquad s=1,\ldots,11 \\
& \sqrt{\sum_{s=1}^{11}\left(-\frac{1}{2}f_s+\frac{1}{2}\omega_s-\frac{1}{2}\mu\right)^2} \le \rho, \\
& \sqrt{\sum_{s=1}^{11}\left(\frac{1}{2}\hat{z}_s+\frac{1}{2}\hat{\omega}_s-\frac{1}{2}\hat{\mu}\right)^2} \le \hat{\rho}, \\
& \boldsymbol{\omega} \ge 0, \quad \rho \ge 0, \\
& \hat{\boldsymbol{\omega}} \ge 0, \quad \hat{\rho} \ge 0, \\
& z_s \ge 0, \quad s=1,\ldots,11 \\
& x_d \ge 0, \quad d=1,\ldots,12 \\
& \lambda_1,\lambda_2=\{0,1\}
\end{aligned} \tag{22}
$$

In this numerical simulation experiment for the three major conditions of the insurance policy premiums, agriculture, and climate index insurance. The first condition is the loss insurance policy premium (r_1) is greater than the climate index insurance policy premium (r_2). For example $r_1 = 180000$, $r_2 = 166000$. The results were obtained that the farmer must allocate 1 hectare of land for the 10th planting date, which is October and choose a climate index insurance policy on the planting date. The VaR value of this condition is 5,941.888 IDR with the combined value between expectations and the CVaR loss function is 5,851,983 IDR. The second condition is the loss insurance policy premium (r_1) is the same as the climate index insurance policy premium (r_2), for example $r_1 = r_2 = 180000$. This condition provides an optimal solution that farmers can choose between the two insurance policies by allocating 1 hectare of land for the October planting date. The VaR value is 5,959,683 IDR if you choose a general insurance policy and 5,955,888 IDR, while the combined value of expectations and Conditional Value-at-Risk (CVaR) is 5,865,983 IDR. The third condition is the loss of insurance premiums (r_1) is less than the climate index insurance policy premiums (r_2). Suppose $r_1 = 180000$ and $r_1 = 194000$, then obtained a decision that the farmer must choose an insurance policy losses by allocating one hectare of land for the planting date of October. The VaR value is 6,108,932 IDR and the CVaR value is 5,865,983 IDR for all these β values.

Conclusion

This study looked at the risk management in determining the allocation of land at each planting date and choosing the optimal insurance policy based on the variability of climate variables. The optimization model aims to minimize the expected loss function by considering the Conditional Value-at-Risk, then we reformulated the model into a robust optimization model under the uncertainty of the ellipsoid distribution. Based on the numerical experiments, the solution of the robust model is stronger because it is the worst case of possible climate variable variability, so farmers can be better prepared to face all of the identified risks. The risk management optimization model that we propose can be expanded by considering fluctuations in market prices. In addition, future research can consider the distribution uncertainty so that a more robust solution is obtained.

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