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Mean-VAR Portfolio Optimisations: An Application of Multiple Index Models with Non-constant Volatility and Long Memory Effects

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One strategy by investors facing risky investments is portfolio optimisation. Therefore, this study aims to analyse the formulation of Mean-VaR portfolio optimisation under multiple index models with non-constant volatility (ARFIMA) and long memory effects (GARCH). Further, the mean and variance of stocks is estimated using multiple index models. The value of mean and variance is used to calculate the Value-at-Risk (VaR), as a measure of risk. Portfolio optimisation is determined using Mean-VaR model, the Lagrangian multiplier approach, and Kuhn-Tucker's theorem. The optimisation analysis result shows that the minimum risk for weight vector composition as x' is (0.1191, 0.2393, 0.0855, 0.2626, 0.2936), with a portfolio mean of 0.0244 and Value-at-Risk of 0.0457. In conclusion, the investor must consider the composition of the weighted index to identify the minimum risk for the investment capital allocation.

Key words: *Asset return, multiple index models (ARFIMA and GARCH), Lagrangian Multiplier, Kuhn-Tucker, portfolio optimisation.*

Introduction

One interesting form of investment is stock investment through the capital market. Usually, investors prefer to buy stocks from companies that go public, because the stocks as investment commodities promise a high return (Bansal et al., 2014). However, the commodities that promise high return, also have a high risk. It is caused by the nature of the commodity which is very sensitive to changes. Numerous companies list on the stock market, leading to an increase in the availability of various combinations of preferred stocks for investors in the stock market (Baweja and Saxena, 2015). That is based on the fact that investors generally do not invest all their funds in one type of stock, but diversify stocks to reduce their risk (Gambrah and Pirvu, 2014). Portfolio analysis is one of the best tools for minimising risk, and so maximising profit. The purpose of forming a portfolio is to maximise profits through the same level of uncertainty among existing stocks (Hult et al., 2012). The results of this analysis will determine the accuracy of investment decisions taken by investors, for obtaining return on investments (Ahmadi and Sitdhirasdr, 2016). Return is the primary goal of their investment activities. However, through observation, stock returns fluctuate in the direction of the market price index and other economic indices. In particular, it can be observed that most stock returns tend to increase, if the stock price index and economic index rise. Conversely, if the stock price index and other economic indices fall, most stocks decline (Pinasthika and Surya, 2014). Stock indices, and other economic indices, often have characteristics that follow the time series model, and are even influenced by long memory effects.

Sensitivity to the influence of stock price indices and other economic indices causes uncertainty as to future stock returns. The existence of future uncertainty can cause risks, in investing, especially in financial assets that are always marketed in the capital market (Plunus et al., 2014). Referring to research conducted by Balibey & Turkyilmaz (2014) and Kasman (2009), market risk can be measured using Value-at-Risk (Balibey and Turkyilmaz, 2014; Kasman, 2009). To date Value-at-Risk is very popular, and is often used for investment risk measurement. Value-at-Risk measures at least two parameters, the mean and volatility of stock returns. Since stock return is influenced by the stock price index and other economic indexes, then the mean and volatility of stock return value also depend on the mean and volatility return index of stock price and other economic indices. Such characteristics can be analysed using multiple index models. Mandal (2013), Nalini (2014), and Sathyapriya (2016) have researched and analysed investment portfolio optimisation, using the Sharpe index model approach (Mandal, 2013); (Nalini, 2014); (Sathyapriya, 2016). Meanwhile, the returns characteristic of the stock price index and other economic indices follow the time series model, and are influenced by long memory effects. Therefore the average and volatility of stock index returns and other economic indices can be estimated, using the autoregressive fractional integrated moving average (ARFIMA) model, and a model of generalised autoregressive conditional heteroscedasticity (GARCH). Goudarzi (2010) has conducted capital market analysis in India involving a long memory model. Subsequently, Gokbulut & Pekkaya (2014) and Makiel (2012) have conducted risk estimation analysis (volatility) involving the ARIMA-GARCH model (Gokbulut and Pekkaya, 2014); (Makiel, 2012). Furthermore, for investment portfolio optimisation, Golafshani and Emamipoor (2015), Qin (2015), and Shakouri and Lee (2016), have optimised their investment portfolio, using mean-variance models (Golafshani and Emamipoor, 2015); (Qin, 2015); (Shakouri and Lee, 2016; Jake, 2017).

Risk is measured using Value-at-Risk (VaR). Given the above discussion, we in this paper seek to analyse portfolio optimisation using a Mean-VaR model. It is assumed that the return of stock assets follows a multiple index model. It is also assumed that the stock price index returns and other economic indices have non-constant volatility. Further, there is a long memory effect, as the object of research is the closing price of some stocks traded on the capital market in Indonesia. The goal is to get the proportion (weight) of the allocation of funds that is to be invested in portfolio formation. The maximum return of portfolio expectation and minimum risk level can then be obtained.

Methodology

In this section, we discuss the methodology used in the analysis of stock return data. The discussion includes the model of stock asset return, identification of long memory patterns, mean, volatility, multiple index, and Value-at-Risk models, and investment portfolio optimisation.

Asset Return Determination Model

This section discusses the return of stock assets. Suppose the asset price (stock or index factor) on the day to t is the amount of X_t. Using a daily horizon time-frame, for the analysis of financial data, asset return of Y_t is often given in the form of continuous compound return or log return, with the following formula:

$$Y_t = \ln\left(\frac{X_t}{X_{t-1}}\right) \quad \textbf{(1)}$$

with $t = 1, 2, \ldots, T$ where T the number of data observations, and it is assumed $X_0 = 1$ (Sukono et al., 2017a); (Sukono et al., 2017b). Asset return data is used for modelling in the following sections.

Identification of Long Memory Effect

In this section, we discuss the identification of the effects of the long memory of return data index factor, on the index factor return data F_{it}. Identification of long memory effect was performed using the Rescale Range (R/S) method (Tsay, 2005); (He et al., 2016). To estimate the fractional different parameters, *d* is performed using the maximum likelihood estimator method. The process of fractional differentiation is defined as:

$$(1-B)^d F_t = u_t ; \ -0.5 < d < 0.5, \qquad \textbf{(2)}$$

where $t = 1, 2, \ldots, T$ with T the number of data observations, $\{u_t\}$ sequence of white noise residuals and B declared the back shape operator. The binomial theorem of the fractional power used is [13]:

$$(1-B)^d = \sum_{k=0}^{\infty} (-1)^k \binom{d}{k} B^k ; \binom{d}{k} = \frac{d(d-1)\ldots(d-k+1)}{k!}.$$

Meanwhile, the confidence interval of $(1-\alpha)\%$ for different fractional parameters d determined by the inequality:

$$d - z_{\frac{1}{2}\alpha} .SE(d) < d < d + z_{\frac{1}{2}(1-\alpha)} .SE(d), \qquad \textbf{(3)}$$

where $SE(d)$ is a square error of different fractional d

Mean Model in Time Series

This section discusses the mean model in time series data. Let F_t equal the return of the index factor at a time t ($t = 1, 2, \ldots, T$ with T the number of observations data). When $(1-B)^d F_t$ follows the model of Autoregressive Moving Average, ARMA(*p*, *q*), then F_t is said to follow the process (model) of Autoregressive Fractional Integrated Moving Average ARFIMA(*p*, *d*, *q*), where Autoregressive Integrated Moving Average ARIMA(*p*, *d*, *q*) is a general model for fractional *d* non-negative integers. Model of ARMA(*p*, *q*) can generally be expressed in the following equation (Tsay, 2005); (He et al., 2016):

$$F_t = \phi_0 + \sum_{i=1}^{p} \phi_i F_{t-i} + u_t - \sum_{j=1}^{q} \theta_j u_{t-j}. \qquad \textbf{(4)}$$

where $\{u_t\}$ the residual sequence assumed to have normal white noise distribution with mean 0 and variance σ_u^2. Non-negative integer *p* and *q* are an ARMA order. The AR and MA models are model specific cases ARMA(*p, q*).

Modelling Stages for mean models in time series. In broad outline, according to Tsay (2005), the mean modelling stage is as follows: (i) identification of the model, determining the value of the order *p* and *q* by using the plot autocorrelation function (ACF) and partial autocorrelation function (PACF); (ii) parameter estimation can be done by the least squares method or maximum likelihood; (iii) diagnostic test, with white noise test and serial correlation to residual u_t; and (iv) prediction, if the model is suitable then it can be used for prediction done recursively (Tsay, 2005).

Volatility Model in Time Series

This section deals with volatility models in time series data. The autoregressive conditional heteroscedasticity (ARCH) model, introduced by Engle in 1982. In the ARCH model, the following conditional variance is an autoregressive process. The ARCH(*m*) model assumes the returns $F_1, F_2, \ldots$ described by the process (Gokbulut and Pekkaya, 2014); (Makiel, 2012):

$$F_t = \mu_t + u_t\,,\quad u_t = \sigma_t v_t \text{ and}$$

$$\sigma_t^2 = c_0 + \sum_{i=1}^{m} c_i u_{t-i}^2 + v_t \qquad \textbf{(5)}$$

where μ_t expectation of F_t, σ_t^2 the variance of F_t, v_t random residuals with mean 0 and variance 1. The general assumption is $\{v_t\} \sim iid\ N(0, 1)$, c_0 and c_i constants, $c_0 > 0$ and $c_i \geq 0$, $i = 1, \ldots, m$.

ARCH Effect Testing. The widely used test for ARCH effect detection is the Lagrange-Multiplier (ARCH-LM) test. Under the null hypothesis, the error term is assumed as a normal white noise distributed process, $v_t \mid F_{t-1} \sim NWN(0, \sigma^2)$. Alternative hypothesis that the error rate is influenced by the ARCH(*m*) model, so that:

$$u_t = \sigma_t v_t\,,\quad \sigma_t^2 = c_0 + \sum_{i=1}^{m} c_i u_{t-i}^2 + v_t\,.$$

where $\{v_t\}$ sequence of normal and random variable with mean 0 and variance σ_v^2. The test for the ARCH(m) effect is based on the null hypothesis $H_0 : c_0 = c_1 = \cdots = c_m = 0$ against the alternative $H_1 : \exists c_0 \neq c_1 \neq \cdots \neq c_m \neq 0$ (Gokbulut and Pekkaya, 2014); (Makiel, 2012).

The ARCH-LM test statistic of this hypothesis shows asymptotically equivalent to the test statistic of $T \times R^2$, where T sample size, and R^2 calculated from the regression (Tsay, 2005):

$$\hat{v}_t^2 = a_0 + a_1 \hat{v}_{t-1}^2 + ... + a_m \hat{v}_{t-m}^2 + \varepsilon_t .$$

Under the null hypothesis, there is no ARCH effect, ARCH-LM and statistic test of $T \times R^2$ asymptotical distribution $\chi^2(m)$. As an alternative to the ARCH-LM test form, it could use asymptotically equivalent the portmanteau test, as the Ljung and Box test statistic in 1978, for v_t^2 (Tsay, 2005).

GARCH Model. Introduced by Bollerslev in 1986, is a general or generalized form of the ARCH model. In general, the GARCH(m, n) model can be written as follows (Gokbulut and Pekkaya, 2014); (Makiel, 2012):

$$u_t = \sigma_t v_t, \sigma_t^2 = c_0 + \sum_{i=1}^{m} c_i u_{t-i}^2 + \sum_{j=1}^{n} b_j \sigma_{t-j}^2 + v_t. \quad \textbf{(6)}$$

Based on equations (5) and (6), the conditional expectation and variance of v_t is:

$$E(v_t | \mathrm{F}_{t-1}) = 0 \quad \textbf{(7)}$$

$$Var(v_t | \mathrm{F}_{t-1}) = E(v_t^2 | \mathrm{F}_{t-1}) = \sigma_t^2 \quad \textbf{(8)}$$

The ARCH model, considered simple because it uses fewer parameters (Tsay, 2005)

Multiple Index Model

This section discusses multiple index models, assuming that there is an index factor affecting stock assets. If Y_t the return of the stock assets at the time t ($t = 1, 2, \ldots, T$ with T the number of data observations), then multiple index model is expressed as follows (Mandal, 2013); (Nalini, 2014):

$$Y_t = \beta_0 + \sum_{l=1}^{L} \beta_l F_{lt} + e_t \quad \textbf{(9)}$$

where β_0 unique return of stock assets, β_l ($l = 1,\ldots,L$ with L the number of index factors) is the degree of sensitivity of return of stock assets to changes in index factors of F_t at time *t*, and e_t residuals from the unique return of stock assets at the time *t* ($t = 1,\ldots,T$ with T the number of data observations) (Sathyapriya, 2016).

Stock Assets Return Expectation. The expected value of a return of a stock asset under the multiple index model of equation (9) is:

$$\mu_Y = E[Y_t] = \beta_0 + \sum_{l=1}^{L} \beta_l \mu_{F_l} \quad \textbf{(10)}$$

where $E[e_t] = 0$, and μ_{F_l} is mean of the index factor F_{lt} ($l = 1,\ldots,L$ with L the number of index factors) as well ($t = 1,\ldots,T$ with T the number of data observations).
The variance of Stock Assets Return. The equation of return variance of individual stock assets under multiple index model can be determined by the following procedure (Sathyapriya, 2016):

$$\begin{aligned}\sigma_Y^2 &= E[(Y_t - \mu_Y)^2] \\ &= \sum_{l=1}^{L} \beta_l^2 \sigma_{F_l}^2 + \sigma_e^2 + \sum_{k=1}^{L} \sum_{l=1}^{L} \beta_k \beta_l \sigma_{kl}; k \neq l, \end{aligned} \quad \textbf{(11)}$$

where $\mu_{F_l}^2$ variance of index factors F_{lt} ($l = 1,\ldots,L$ with L the number of index factors) as well ($t = 1, 2,\ldots,T$ dengan T the number of data observations), σ_e^2 residual variance e_t, and σ_{kl} covariance between index factors F_{kt} and F_{lt}.
Covariance Between Stock Assets Return. Covariance between the return of stock assets Y_{it} and Y_{jt} can be expressed by the following equation (Sathyapriya, 2016):

$$\begin{aligned}\sigma_{ij} &= E[(Y_i - \mu_{Y_i})(Y_j - \mu_{Y_j})] \\ &= \sum_{k=1}^{L} \sum_{l=1}^{L} \beta_k \beta_j \sigma_{kl}. \end{aligned} \quad \textbf{(12)}$$

Value-at-Risk Model of Investment Portfolio

This section discusses the Value-at-Risk on investment portfolio returns. View, for example, N stock assets, and return of each asset is given by a random variable stock of $Y_{1t},\ldots,Y_{Nt}$. If an entrepreneur forms a portfolio with weighted vectors as: $\mathbf{x}'=(x_1,\ldots,x_N)$ with $\mathbf{x}'\mathbf{e}=1$, where $\mathbf{e}'=1,\ldots,1$, then portfolio return is given by random variable (Ogryczak and Sliwinski, 2010):

$$Y_p=\sum_{i=1}^{N}x_iY_{it}=\mathbf{x}'\mathbf{Y} \quad \textbf{(13)}$$

where $\mathbf{Y}'=(Y_{1t},\ldots,Y_{Nt})$. The expected value of equation (13) is given as:

$$\mu_p=E[Y_p]=\sum_{i=1}^{N}x_i\mu_i=\mathbf{x}'\boldsymbol{\mu} \quad \textbf{(14)}$$

where $\boldsymbol{\mu}'=(\mu_1,\ldots,\mu_N)$ with $\mu_i=E[Y_{it}]$ and $i=1,\ldots,N$, while the variance of the portfolio is given as:

$$\sigma_p^2=Var[Y_p]=\sum_{i}^{N}\sum_{j=1}^{N}x_ix_jCov(Y_{it},Y_{jt})=\mathbf{x}'\mathbf{C}\mathbf{x} \quad \textbf{(15)}$$

where $\mathbf{C}=(\sigma_{ij})$ variance-covariance matrix, with $\sigma_{ij}=Cov(Z_{it},Z_{jt})$ (Sukono et al., 2017a); (Mustafa et al., 2015).
It is assumed that the asset return has a specific distribution, and the risk of the portfolio is measured using the Value-at-Risk (VaR). According to Kasman (2009) and Balibey & Turkeyilmaz (2014), risk measurement model of the Value-at-Risk for portfolios formulated as $VaR_p=-V_0\{\mu_p+z_\alpha\sigma_p\}$ (Balibey and Turkyilmaz, 2014); (Kasman, 2009).
Refer to equation (14) and (15), the Value-at-Risk for the portfolio can be expressed as:

$$VaR_p=-V_0\{\mathbf{x}'\boldsymbol{\mu}+z_\alpha(\mathbf{x}'\mathbf{C}\mathbf{x})^{1/2}\} \quad \textbf{(16)}$$

where the sign (-) stated losses, V_0 the initial capital invested, and z_α percentile of the standard normal distribution when the given level of significance $(1-\alpha)\%$

Investment Portfolio Optimisation

This section discusses the optimisation of investment portfolios, where Value-at-Risk measures portfolio risk.

Referring to Sukono et al. (2017a), a portfolio *p** called (Mean-VaR) efficiently if there is no portfolio *p* with $\mu_p \geq \mu_{p*}$ and $VaR_p < VaR_{p*}$ (Sukono et al., 2017a).

Therefore, if the investment portfolio risk is measured using Value-at-Risk, then the investment portfolio optimisation problem to be solved, it is in the form of:

$$\text{Maximum } \{2\tau\mu_p - VaR_p\}$$

$$\text{Subject to} \sum_{i=1}^{N} w_i = 1$$

with $\tau \geq 0$ Risk tolerance factor owned by investors. Suppose, if the initial capital invested is equal to $V_0 = 1$ units of money, referring to equations (14) and (15), then objective functions can be expressed as:

$$\begin{aligned}&\text{Maximum } \{(2\tau+1)\mathbf{x}'\boldsymbol{\mu} + z_\alpha(\mathbf{x}'\mathbf{C}\mathbf{x})^{1/2}\}\\ &\text{Subject to} \, \mathbf{x}'\mathbf{e} = 1\end{aligned} \qquad \textbf{(17)}$$

Solutions to the problem of investment portfolio optimisation of equation (17), can be found by forming Lagrangean multiplier function as follows:

$$\begin{aligned}L(\mathbf{x},\lambda) = {} & (2\tau+1)\mathbf{x}'\boldsymbol{\mu} + z_\alpha(\mathbf{x}'\mathbf{C}\mathbf{x})^{1/2}\\ & + \lambda(\mathbf{x}'\mathbf{e} - 1).\end{aligned} \qquad \textbf{(18)}$$

Based on Kuhn-Tucker's theorem, the optimality requirements are:

$$\frac{\partial L}{\partial \mathbf{x}} = (2\tau+1)\boldsymbol{\mu} + \tfrac{1}{2}.2.z_\alpha \frac{\mathbf{C}\mathbf{x}}{(\mathbf{x}'\mathbf{C}\mathbf{x})^{1/2}} + \lambda\mathbf{e} = 0 \qquad \textbf{(20)}$$

and

$$\frac{\partial L}{\partial \lambda} = \mathbf{x}'\mathbf{e} - = 0. \qquad \textbf{(21)}$$

Completing the system of equations (20) and (21), we obtain the vector of investment portfolio weighting as follows:

$$\mathbf{x} = \frac{(2\tau+1)\mathbf{C}^{-1}\boldsymbol{\mu} + \lambda\mathbf{C}^{-1}\mathbf{e}}{(2\tau+1)\mathbf{e}'\mathbf{C}^{-1}\boldsymbol{\mu} + \lambda\mathbf{e}'\mathbf{C}^{-1}\mathbf{e}} \qquad \textbf{(22)}$$

where $\mathbf{C}^{-1}$ is the inverse of a matrix **C**. Also based on the solution of system equations (20) and (21), multiplier values are obtained λ, as follows:

$\lambda_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; with the condition $\lambda \geq 0$,

where $a = \mathbf{e}'\mathbf{C}^{-1}\mathbf{e}$, $b = (2\tau+1)\{\mathbf{e}'\mathbf{C}^{-1}\boldsymbol{\mu} + \boldsymbol{\mu}'\mathbf{C}^{-1}\mathbf{e}\}$ and $c = (2\tau+1)\boldsymbol{\mu}'\mathbf{C}^{-1}\boldsymbol{\mu} - z_{\alpha}^{2}$. Furthermore, if the vector **w** equation (22) is substituted into equations (14) and (16), then there will be a return on expectation and a value-at-risk efficient investment portfolio. Such weight vector sets can be used to establish the efficient frontier in the investment portfolio.

Result and Discussion

This section discusses the analysed data, identifies long memory effects, estimates of mean models and volatility of time series data, estimates of multiple index models, estimates of mean and variance returns of stock assets, and investment portfolio optimisation.

Data Analysed

The data analysed here is obtained through the website http://www.finance.go.id//, for the period from January 2, 2013, up to March 31, 2017. Data includes AALI, LSIP, ASII, BMRI, and UNTR, hereinafter symbols are given A_1 up to A_5. While the index factor data used include Composite Stock Price Index (IHSG), rupiah exchange rate against USD, EURO and YEN, then respectively given the symbol F_1 up to F_4. Both the stock asset data, as well as the index factor data, are each determined by return using equation (1), and then used for the following analysis.

Identify Long Memory Effect on Return of Index Factors

It started with the stationary test of index factor return data. Factor index return data has been determined by using equation (1), then performing statistical analyses using a stationary Augmented Dickey-Fuller (ADF). The test results show that the return of the four index factors analyzed is stationary at the level of significance $\alpha = 5\%$.

To identify the effect of long memory, estimated different fractional parameters *d* in equation (1). Fractional different values *d*, calculated using the Geweke and Porter-Hudak method, with the help of software R. The result, for data return factor index F_1 obtained by different fractional value $\hat{d}_1 = 0.3613183$ and the standard errors $SE(\hat{d}_1) = 0.01462239$. To assure the existence of long memory pattern, hypothesis test $H_0 : \hat{d}_1 = 0$, against $H_1 : \hat{d}_1 \neq 0$. Based on the calculation, statistics are obtained $z = -5.86$, While at the level of significance $\alpha = 95\%$, from the standard normal distribution table obtained value $z_{\frac{1}{2}(5\%)} = -1.96$. Because the value *z* is smaller than the value of $z_{\frac{1}{2}(5\%)}$, it is concluded that the test result is significant; it means data return factor index F_1 there is a long memory effect. Confidential interval 95% for different fractional parameters $\hat{d}_1$ is determined by inequality (3), and the result is $0.332658 < \hat{d}_1 < 0.389978$. Because $\hat{d}_1$ lies within the interval $-0.5 < d_1 < 0.5$, it is concluded that $\hat{d}_1$ is significant. As for the index factors F_2, F_3 and F_4 not significantly there is a long memory effect. The next step, using data that has been different fractional $\hat{d}_1 = 0.3613183$ to estimate models of the mean and volatility.

Estimation of Mean and Volatility Models in Time Series

Return data from index factors F_1 up to F_4 estimated the mean model. Step (i), model identification, is done with the sample autocorrelation function (ACF) and partial autocorrelation function (PACF). From the correlogram, a possible tentative model is selected for the index factor return data F_1 up to F_4, are ARMA(1, 1), ARMA(1, 1), ARMA(1, 1) and ARMA(2, 2) models respectively. Step (ii), based on the tentative model, estimates the model, and it can be concluded that the ARMA(1, 1), ARMA(1, 1), AR(1) and ARMA(2, 1), ARMA significant. Steps (iii), diagnostic tests of estimation models, and test results show that the residuals of each model are white noise. Furthermore, normality tests were performed on the residuals of each model. The test results show that the residuals of each model are normally distributed with mean 0 and certain variance. So there is no need to look at other alternative flat models.

Modelling of volatility. The residual data of each mean model is used for volatility modelling. Step (i), identification of the existence of ARCH effect is made by an ARCH-LM test. The result shows that four index data F_1 up to F_4 there is an ARCH effect. Step (ii), identification and estimation of volatility model, carried out through ACF and PACF residual data square of each mean model of index factors, then made estimation. The result of identification and estimation simultaneously with mean and volatility model to four index factor return data are obtained as follows, i.e. F_1: model of ARFIMA(1, $\hat{d}_1$, 1)-GARCH(1, 1), where $\hat{d}_1 = 0.3613183$, F_2: model of ARMA(1, 1)-GARCH(1, 1), F_3: model of ARMA(1, 1)-ARCH(1) and F_4: model of ARMA(2, 2)-GARCH(1, 1).

It is based on the ARCH-LM test, the residuals of the four volatility models are white noise. Furthermore, the equations of the mean and volatility models are used for estimation of values $\hat{\mu}_{F_l}$ and $\hat{\mu}^2_{F_l}$ ($l = 1, \ldots, 4$) 1-step forward recursively.

Estimates of the Multiple Index Model

In this section, we estimate the multiple index model, by determining the regression model of each return data from the five stock assets, to the return data of the four index factors. The estimates were made referring to equation (9), and with the help of Evies 8. The results are given in Table 1.

Table 1: Estimator of the Multiple Index Model

Multiple Index Model
$Y_1 = 0.003 + 0.032F_1 + 0.793F_2 + 0.099F_3 + 0.202F_4 + e_1$
$Y_2 = 0.003 + 0.125F_1 + 0.082F_2 + 0.161F_3 + 0.217F_4 + e_2$
$Y_3 = 0.001 + 0.045F_1 + 0.450F_2 + 0.003F_3 + 0.069F_4 + e_3$
$Y_4 = 0.002 + 0.182F_1 + 0.911F_2 + 0.147F_3 + 0.130F_4 + e_4$
$Y_5 = 0.002 + 0.132F_1 + 0.716F_2 + 0.002F_3 + 0.140F_4 + e_5$

Based on ANOVA test results it can be shown that the five regression models in Table 1 above are quite significant. It can also be shown that residual e_i $(i = 1, \ldots, 5)$ of each regression model is normally distributed, with mean 0 and variance as given in Table 2, column σ^2_e. This regression model is then used for the following analysis.

Estimation of Values of Mean and Variance of Return on Stock Assets

Estimation of values of mean and variance return of index factors, was conducted as based on mean model and variance of four index factors F_1 up to F_4. It has been discussed in the modelling of the mean and volatility models of returns from the above index factors. The results are given in Table 2.

Table 2: Mean, Variance and Residual Variance from Returns of Index Factors

F_l	$\hat{\mu}_{F_l}$	$\hat{\sigma}^2_{F_l}$	σ^2_e
F_1	0.151140	0.001798	0.000600
F_2	0.002334	0.000009	0.000630
F_3	0.000343	0.000055	0.000558
F_4	0.012470	0.000261	0.000560

Furthermore, based on the values in Table 2, the mean values $\hat{\mu}_{F_l}$ were used to estimate the mean value $\hat{\mu}_i$ $(i = 1, \ldots, 5)$ using equation (10). The value of variance $\hat{\sigma}^2_{F_l}$ and σ^2_e were used to estimate the value of variance $\hat{\sigma}_i$ by equation (11). The calculation results are given in Table 3.

Table 3: Mean and Variance of Stock Returns

Stocks	$\hat{\mu}_i$	$\hat{\sigma}^2_i$
Y_1	0.012300	0.000618707
Y_2	0.024535	0.000671891
Y_3	0.009733	0.000564709
Y_4	0.033305	0.000869633
Y_5	0.025638	0.000601067

The values in Table 3 are then used for the following Mean-VaR portfolio optimisation process.

Investment Portfolio Optimisation

For optimisation purposes here is a defined vector $\mathbf{e}' = (1, 1, 1, 1, 1)$. Based on the values in Table 3, column $\hat{\mu}_i$, can be formed mean vector $\boldsymbol{\mu}'$ = (0.012300, 0.024535, 0.009733, 0.033305, 0.025638). The values in Table 3, column $\hat{\sigma}^2_i$ and the covariance between the return of stock assets, rounded up to four decimals, are used to form the variance-covariance matrix, and the inverse is as follows:

$$\mathbf{C} = \begin{pmatrix} 0.0006 & 0.0001 & 0.0003 & 0.0001 & 0.0002 \\ 0.0001 & 0.0007 & -0.0001 & 0.0001 & 0.0003 \\ 0.0003 & -0.0001 & 0.0006 & 0.0002 & 0.0001 \\ 0.0001 & 0.0001 & 0.0002 & 0.0009 & 0.0003 \\ 0.0002 & 0.0003 & 0.0001 & 0.0003 & 0.0006 \end{pmatrix}$$

$$\mathbf{C}^{-1} = 10^3 \times \begin{pmatrix} 2.5258 & -0.3333 & -1.3016 & 0.2381 & -0.5774 \\ -0.3333 & 2.0000 & 0.6667 & -0.0001 & -1.0000 \\ -1.3016 & 0.6667 & 2.6032 & -0.4762 & -0.0952 \\ 0.2381 & -0.0001 & -0.4762 & 1.4286 & -0.7143 \\ -0.5774 & -1.0000 & -0.0952 & -0.7143 & 2.7321 \end{pmatrix}$$

Furthermore, if the level of significance is established $\alpha = 5\%$, then from the standard normal distribution, percentile values are obtained $z_{5\%} = -1.645$. It determines the weight vector **x**, calculated using equation (22). For risk tolerance $\tau = 0$ the weight vector obtained is **x**' = (0.1191, 0.2393, 0.0855, 0.2626, 0.2936) and when substituted into equation (14) obtained the portfolio mean return of 0.0244. When substituted into equation (16), it obtained Value-at-Risk of 0.0457. For risk tolerance $\tau = 0.1$ we get the optimum weight vector **x**' = (0.0911, 0.2529, 0.0483, 0.02942, 0.3135), and the mean portfolio return of 0.0256, and Value-at-Risk of 0.0482. For risk tolerance $\tau = 0.2$ we get the optimum weight vector **x**' = (0.0565, 0.2698, 0.0024, 0.3332, 0.3380) and the mean portfolio return of 0.0271 and Value-at-Risk of 0.0516. While for risk tolerance $\tau = 0.3$ we get the optimum weight vector **x'** = (0.0102, 0.2924, -0.0592, 0.3856, 0.3710). If a short sale is forbidden, then this last weight is not feasible, because there is a negative weight. Increased risk tolerance $\tau = 0$ become $\tau = 0.1$ and $\tau = 0.2$ has brought consequences of changes in vector weight composition and an increase in mean portfolio return and Value-at-Risk.

Conclusion

In this paper, we have analyzed the formulation of Mean-VaR portfolio optimisation under multiple index models, with non-constant volatility and long memory effects. Based on the results of the above analysis, it can be concluded that for the data return factor index F_1 significantly there is a long memory effect, and it follows the model of ARFIMA(1, $\hat{d}_1$, 1)-GARCH(1, 1) and $\hat{d}_1$= 0.3613183. Index factor returns F_2 up to F_4 not significantly existing long memory effect, and sequentially follow the models of ARMA(1, 1)-GARCH(1, 1), ARMA(1, 1)-ARCH(1) and ARMA(2, 2)-GARCH(1, 1). Each share asset of A_1 up to A_5 correlates reasonably well against the four index factors F_1 up to F_4. Therefore, the values of the mean and variance estimates of each stock asset A_1 up to A_5. It depends on the estimation values of the mean and the variance of the four index factors F_1 up to F_4. At-risk tolerance $\tau = 0$ obtained the mean return portfolio of 0.0244 and Value-at-Risk of 0.0457. For at-risk tolerance $\tau = 0.1$, the mean return of the portfolio is 0.0256 and Value-at-Risk is 0.0482. At-risk tolerance $\tau = 0.2$ obtained the mean return portfolio of 0.0271 and Value-at-Risk of 0.0516. Meanwhile, at-risk tolerance $\tau = 0.3$ is not feasible, because there is a negative weight.

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