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Characteristics of Mathematical Representation Translation from Verbal to Graph

Dwi Rahmawati[a], Rahmad Bustanul Anwar[b], Jazim Ahmad[c],
[a,b,c]Universitas Muhammadiyah Metro, Jl. Ki Hajar Dewantara No 116, Iringmulyo, Metro Timur, Metro, Lampung, Indonesia, Email: [a]dwirahmawati1083@gmail.com, [b]rarachmadia@gmail.com, [c]jazimmetro@gnail.com

This study aims to describe the translation characteristics of verbal, mathematical representations into graphs. The translation between mathematical representations is one of the indicators in understanding mathematical concepts. Understanding the things related to the process of student representation translation is very important in learning mathematics. This study aimed to describe the characteristics of mathematical representation translation from verbal to the graph. The approach used in this study was qualitative. The subjects of this study were eighth-graders at some Junior High School. Tests and interviews collected the data. Students did the test while expressing verbally what they were thinking (think aloud). Then the researchers did an interview related to the results of the students' answers. Interviews in this study were semi-structured interviews. The results of the study showed that the process of representation translation from verbal to graph occurred through four stages: unpacking the source, preliminary coordination, constructing the targets, and determining equivalence. There were two characteristics of representation translation from verbal to the graph, namely numerically and algebraically. This characteristic of representation translation from verbal to graph is expected to be used by the teacher as a consideration to make the appropriate instructional design, especially on function. So that teachers can optimise students 'thinking and minimise students' difficulties in the process of representation translation from verbal to the graph. Future research can examine the representation translation of the graph into verbal, verbal to symbolic and otherwise, verbal to the table and otherwise, or symbolic to the graph.

Keywords: *Translation, Verbal, Graph, Numerically, Algebraically*

Introduction

Representation has an important role in mathematics learning. NCTM (2000) decides representation as one of the standard processes in mathematics learning. Mathematical learning aims to connect real life with mathematical concepts (Oflaz et al., 2016). Therefore, the ability of representation needs to be embedded early, because it can help students to deepen understanding of concepts, communicate mathematical ideas, recognise the interrelationship between mathematical concepts, and help in problem-solving. Goldin (2002) defines that representation is a configuration that can represent something in another form. While Kaput (1998), mentioned that representation as a tool used to organise and make the situation more meaningful. Representation is a model or a substitute form of a problem situation that is used to find a solution (Jones & Knuth, 1991). The ability of representation needs to be instilled early on because it can help students to deepen understanding of concepts, communicate mathematical ideas, recognise the relationships between mathematical concepts, and help in problem-solving. Representations can be divided into three, namely visual representations (pictures, graphs, tables), verbal (words/written text), symbolic (mathematical notation, algebra, numeric) (Goldin & Shteingold, 2001). Representations can be divided into four namely visual (images, graph), verbal (words/texts), symbolic (mathematical notation, algebra, mathematical equations), numeric (tables) representations (Kaput, 1987; Pape & Tchoshanov, 2001). In learning, the use of representations is not only tied to a single representation but can use multiple representations. For example, given representation in the form of graphs, then the teacher can ask students to make other representations such as a table, verbal, and symbolic. Understanding the interrelationships between representations will help in managing information (Dreyfus, 1991).

Translation between forms of representation is a process that occurs in representation. According to Janvier (1987), translation is a psychological process that involves the change from one form of representation to another, such as translation from a mathematical equation to a graph. While Ainswort (1999) states that translation must be able to see the relationship between two forms of representation, this shows that translation involves at least two different forms of representation. Besides, translation consists of the activity of constructing new representations that are not given, such as constructing graphs from tables or mathematical equations (Bosse et al., 2014). Translation between representations sometimes requires more than one translation process, or in other words, translation occurs indirectly (Janvier, 1987) or through intermediaries (Bosse et al., 2011). And there is also a translation between representations that require only one translation process. In other words, translation occurs directly (Janvier, 1987) or without going through intermediaries (Bosse et al., 2011). The use of an intermediate representation is one of the difficult factors of the translation process (Bosse et al., 2011). Janvier (1987) states that activities in translation include measuring, sketching, curve fitting, modelling, interpretation, fitting, computing, reading off, parameter recognition, and plotting.

NCTM (2000) determines one of the standards of representation for a mathematics learning program is to enable students to translate between forms of mathematical representation in problem-solving. In Indonesia, representation has also been included in the objectives of mathematics learning in schools. According to Janvier (1987), translation is a psychological process that involves changing from one form of representation to another, such as from mathematical equations to graphs. The ability to doing mathematical representation translation is a fundamental ability that must be possessed by students to construct a concept and think mathematically. In mathematics learning, students are expected to be able to do the translation of mathematical representation. The ability of translation from one form of representation to another is needed in understanding the mathematical concept (Lesh, Post & Behr, 1987; Hiebert & Carpenter, 1992). If students can do the translation between forms of representation, then students can access mathematical ideas well. This corresponds with Ryken (2009) that the ability to translate between representations is the key to construct a broad and deep understanding of mathematics.

The subject of function is core in both mathematics and other disciplines (Birgin, 2012). Students begin to recognise the function concept formally in the eighth grade. Functions can be presented in several representations, namely verbal, table, symbolic, and graphic. Students must be able to understand the information presented in various representations and be able to translate between representations (Birgin, 2012). The ability of translation between multiple representations as a tool to show students' mental construction of the function concept (Cawley, 2016). Carlson (1998) explains the factors in understanding the concept of functions, among others: the ability to characterise the relationship between real events and function notation; the ability to use multiple representations of functions such as formulas, tables, graphs, and verbs; the ability to translate from among representations in the function representation; the ability to represent and interpret changes in one variable affects other variables set the standard representation for mathematics learning programs, one of which is to enable students to translate between forms of mathematical representation in solving problems.

Several studies such as Çikla and Çakiroğlu (2006), Anastasiadou (2008), Çelik and Sağlam-Arslan (2012), Biber (2014), Bal (2014, 2015), and Dündar (2015) have focused on the problem of translation in mathematical representation, and emphasises the importance of changes between representations and connections. However, the reality on the ground shows that the translation ability of students' mathematical representations is still very low. Nevertheless, the fact in the field indicates that the translation ability of students' is still very low. Students always have difficulty in translation from verbal to graph. This was shown from the observations made by researchers on eighth-grade students in some junior high schools in some cities.This is shown from the results of observations made by researchers on SMP/MTs class VIII in several schools in Malang and the metro. This has also shown results in several studies relating to the problem of representation translation. The results of Gurbuz and Sahin

(2015) found that translations of verbal, tables, equations to graphs were the most difficult. While Bal (2015), in his research, found that students usually succeed in translation between representations but did not succeed in translation, especially verbal to other representations. Most students succeed in problem-solving on symbolic representation and at least on verbal representation. This is also shown in Dündar's (2015) study that found that most students had success on symbolic tests and at least on verbal tests. While the results of research Çelik and Sağlam-Arslan (2012), found that students have not been able to do the translation between types of representation, especially translations into the graph representation less successful. Students are more successful in determining the correct graph, among others than constructing the graph. Bosse et al. (2011), also states that representation translation involving verbal representation as either a source or target representation is a difficult translation.

Bosse et al. (2014), in his research, found four activities done by students in performing translation from the graph to symbolic, which consists of unpacking the source, preliminary coordination, constructing the targets, and determining equivalence. Unpacking the source stage is the stage of unpacking information in the source representation, outlining what is known and what is asked, identifying the related information. In the Preliminary coordination stage, students connect the information that has been unpacked already with the concept which has been understood, and students prepare information that may be used to construct the target representation. In the activity of constructing the target, students transfer information on the source representation to the target representation and complete the description on the target representation. In determining equivalence, students check the similarity in source representation and target representation and consider the same idea in source representation and target representation.

Based on the result of observations and previous research described above show that the ability of translation representation is still very limited, especially verbal and graph representation. To be able to know in depth the ability and difficulty of representation translation from verbal to graph then previously need to know in detail characteristic of mathematical representation translation from verbal to graph. To our knowledge, no research has examined more detailed characteristics of mathematical representation translation from verbal to graph. So it is necessary to research to find out how the characteristics of representation translation from verbal to graph.

This study described the characteristics of the process of representation translation from verbal to graph based on the four stages translation activities Bosse, et al. (2014), i.e. unpacking the source, preliminary coordination, constructing the target, and determining equivalence. The results of this study are expected to complement the results of previous studies related to representation translation. They can be used as a consideration of making appropriate learning related to representation translation, especially the representation translation from verbal to graph so that it can minimise student difficulties in representation translation.

Methods

Research Design

This study was qualitative research. This study aims to reveal the characteristics of representation translation from verbal to graph.

Research Sample

This study was conducted on the even semester of the academic year 2016/2017. This study involved 37 students in the eighth grade and the selection of subjects using a purposive sampling technique.

The subjects of this study were chosen by considering certain criteria by the objectives, and subjects can communicate that the expression of representation translation from verbal to graph can be done well. The selection of the subject of this study considers the results of work and communication skills. Information related to the subject's communication skills were obtained from the mathematics teacher who taught in class VIII. The choice of eight grade students of junior high school because the students have taken material function where students are required to be able to do the translation between the representation of functions, which one of them is the translation from verbal to the graph.

To determine the subjects of this study, the researchers gave the test to the students to solve the problem related to mathematical representation translation from verbal to the graph. Students did the test while expressing verbally what they were thinking (think aloud). Then, the researchers conducted interviews based on the results of student answers to reveal the necessary things related to the translation process.

The results of student work are analysed to be classified into each characteristic of representation translation from verbal to the graph; then, the researchers chose two students for each classification. The selection of the subject of this study considers the results of work and communication skills. From the selection, the result obtained four subjects were divided into two groups. Group of subjects 1 consisted of S1 and S2. While the group of subjects 2 consisted of S3 and S4.

Research Instruments and Procedures

The instrument of this study is the main instrument and auxiliary instrument. The main instrument is the researcher himself, while the auxiliary instrument is the test and the interview. Researchers act as instruments that collect data, analyse data, interpret data, and report research results. The instrument test of this study is questions related to the representation translation of verbal to the graph. Interviews in this study were semi-structured interviews. An interview is used to clarify the data obtained about the assigned test. Interview questions are given to find out the reason the subject when using the task completion step uses the test completion steps.

Questions posed in the interview developed the researcher according to the written answer of the subject. The question could be, "How do you solve this problem by using the information you get from the problem? Is there any other way than you do? Why did you take this step?". Researchers act as instruments that collect data, analyse data, interpret data, and report research results. Before the test and interview guidelines were used, it is validated by an expert. Validation is directed to the conformity of the problem with objective conformity, material construction, and language conformity used. Based on the results of expert validation, the instruments that researchers have developed are worthy of use.

The process of collecting data in this study by giving the test with think-aloud related to the representation translation from verbal to the graph, then interview related to the completion of the test. In completing the test, students express loudly what is being thought (think aloud). Researchers record all verbal expressions and record student behaviour. Students who are translating verbal representations to graphics correctly are selected as research subjects. Then, if they still needed more in-depth information about what is being considered, then the researcher conducted a test-based interview. Interview activities are recorded with a tape recorder. Furthermore, researchers examined the written and verbal data obtained. If there is inconsistent data, then conducted interviews to clarify.

Data Analysis

The data of this study were data of written answers, think aloud, and interview. The data that had been obtained described based on the real condition to obtain the characteristics of representation translation from verbal to the graph. The process of data analysis in this study includes the steps of transcoding the collected data, reducing the data, compiling each part of the data in units that are then encoded, checking the validity of the data or triangulation, studying the data, analysing the representation translation and making conclusions.

Triangulation in this research is done by comparing or checking to think aloud data, written results, and interview results. The researcher classifies and identifies the data by encoding in a particular order of the subject's written result as well as the interview and think aloud transcript. This encoding is used to construct a description of the characteristics of representation translation from verbal to the graph. The term and coding in the research can be presented in Table 1 below:

Table 1. Description of The Encoding

Term	Code
Reading	
Identifying	
Determining mathematical equations	
Specifying the domain	
Doing substitution	
Determining X axis	
Determining Y-axis	
Sketching	
Specifying the value	Plh
Determining the relationship pattern	Pl
Creating a scheme	Skm
Interpret	Tfr
Checking graph suitability	Kes
Checking and recalculating	Prsk, hit ul

Results and Discussion

The subjects of this study were 37 students who were given problems related to the translation of mathematical representations from verbal to the graphic.

Subjects were given problems related to mathematical representation translation from verbal to the graph. The process of mathematical representation translation from verbal to graph, in general, was carried out through four stages: unpacking the source, preliminary coordination, constructing the targets, and determining equivalence. From 37 students' answers, in the preliminary coordination stage, there are two characteristics of the students, namely numerically by determining the pattern of the relationship between the distance of the cars to the time carried out by subject group 1 and algebraically by determining the function formula performed by the subject group 2. In preliminary coordination stage, there were two characteristics of the students, namely numerically by determining the relationship pattern between the distance a toy car toward the time carried out by the group of subjects 1 and algebraically/symbolically by determining the function formula performed by the group of subjects 2. This discussion four correct answers were selected, and they represented two characteristics to be presented.

Table 2. Description of The Encoding on The Process of Representation Translation from Verbal to Graph of Subject in Group 1

Term	Code
Reading the problem	
Identifying	
Creating a scheme	
Specifying the value of x (time)	
Determining the relationship pattern between time and distance	Pl
Determining the X-axis	Sb X
Determining Y-axis	
Sketching	
Drawing a graph	
Checking graph suitability	
Checking and recalculating	

Table 3. Description of The Coding on The Process of Representation Translation from Verbal to Graph by Subject in Group 2

Term	Code
Reading the problem	Bc
Identifying	
Determining mathematical equations	
Specifying the domain	
Doing substitution into the equation	
Determining X-axis	
Determining Y-axis	
Sketching	Sket
Drawing a graph	
Interpret for Checking graph suitability	

Unpacking The Source Stage

While doing the mathematical representation translation from verbal to the graph, first of all, students unloaded the information in the given verbal representation (unpacking the source). In doing unpacking the source, the students began by reading the verbal problem that had been given repeatedly and marked the words that were considered important. The student then selected the keywords that would be used to describe the appropriate graph. In the group of subjects 1, the students selected the keywords by underlining the keywords on the given worksheets. While in group 2, students chose keywords by writing the words on the answer sheets. The keywords were the toy cars, 2 cm from the edge of the room, and the speed was 5cm/s. It is shown from the result of students' written answers in Figure 1 and 2:

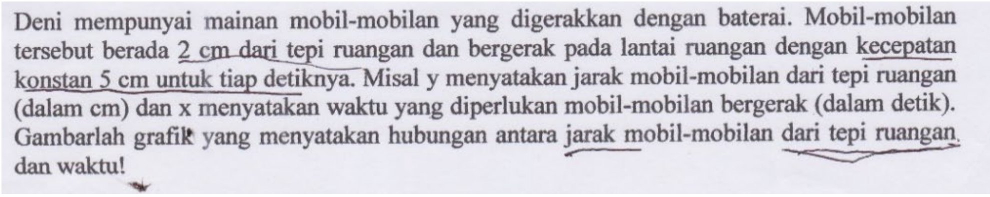
Deni mempunyai mainan mobil-mobilan yang digerakkan dengan baterai. Mobil-mobilan tersebut berada 2 cm dari tepi ruangan dan bergerak pada lantai ruangan dengan kecepatan konstan 5 cm untuk tiap detiknya. Misal y menyatakan jarak mobil-mobilan dari tepi ruangan (dalam cm) dan x menyatakan waktu yang diperlukan mobil-mobilan bergerak (dalam detik). Gambarlah grafik yang menyatakan hubungan antara jarak mobil-mobilan dari tepi ruangan dan waktu!

Figure 1. Written Answers of the Subjects of Group 1 When Choosing Keywords

Deni has a battery-driven toy car. The toy car is 2 cm from the edge of the room and moves on the floor of the room with a constant speed of 5 centimetres for each second. Suppose y represents the distance of cars from the edge of the room (in centimetre), and x represents the time it takes the moving car (in seconds). Draw a graph that states the relationship between the distance of cars from the edge of the room and time!

Description: English Translation of Figure 1

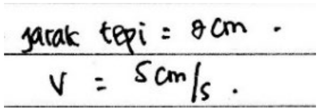

Figure 2. Written Answers of the Subjects of Group 2 When Selecting Keywords

Edge distance = 2 centimeter

Description: English Translation of Figure 2

Based on the written answer, the two subject groups chose the keywords differently. And the keywords selected by both groups, both the 1st and 2nd subject groups, were identical; namely the distance of the car toy from the edge of the room and the speed of the car toy every second. The subjects' ability to understand the important information in the given problem indicated that the subjects had been able to do unpacking the source well. Besides, the subjects were also able to understand the questions posed. It is shown in the following interview results:

Q: Do you understand the question of the problem? Explain!
S1: Yes, Ma'am, draw a graph of the relationship between the distance of the two toy cars from the edge of the room and time.
Q: Is the information available enough to answer the question given?
S1: Yes Ma'am (while nodding his head)

Preliminary Coordination Stage

Further to the second stage, namely preliminary coordination, subjects did differently. There are two different characteristics at this stage, namely determining the pattern and determining the function formula. The subject of group 1, i.e. S1 and S2 do preliminary coordination stage by determining the pattern of relationship between car distance to time. Both subjects determine the distance of cars from the edge of the room for some time. The subject determines the distance of the cars from the edge of the room when the time is 0 seconds is 2 cm, then when the time of 1 second is 7cm and the time is 2 seconds is 12 cm and at 3 seconds is 17 cm. Based on the results obtained, both subjects concluded the relationship of time changes to the distance of cars from the edge of the room. The following in Figure 3 is the written answer to subject group 1.

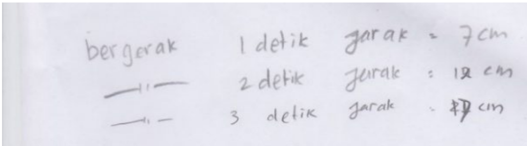

Figure 3. Written Answer of S3 When Determining the Distance for Some Time

move 1-second distance 7 centimetres
move 2-second distance 12 centimetres
move 3-second distance 17 centimetres

Description: English Translation of Figure 3

The subject of group 2, i.e. S3 and S4 perform preliminary coordination by making the function formula of the given verbal situation. Subjects connected with the previously understood concept and conducted a series of algebraic operations to obtain the mathematical equation. Both subjects chose the form of the function formula is f (x) = ax + b. Furthermore, the subject determines the values of a and b are 5 and 2, which are known in the problem. It showed that the subject already understood the values of a and b for f (x) = ax + b. It can be seen in the subjects' written answers to the following in Figure 4.

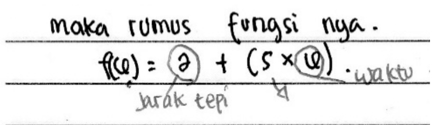

Figure 4. The Written Answer Of S3 When Determining Function Formula

The Formula of funtion:
f(x) = 2 + (5 x X)
edge distance time

Description: English Translation of Figure 4

The second characteristic is the process of translation carried out by the subject when conducting preliminary coordination by determining the formula of the relationship function between the distance of the cars from the edge of the room and time.

The following interview also supported it:

> *Q: What do you do to draw the graph?*
> *S1: by making this formula, f (x) = ax + b it can be obtained f (x) = 5x + 2. Then inserted x = 1, f (1) = 7, x = 2 f (x) = 12, x = 3 f (x) = 17 and so on*

Constructing the Target Stage

When constructing the target, the students did by determining x, which stated time as X-axis and determining y, which stated toy car distance to the edge of the room as axis Y. Next, subjects determined the coordinates of points that had been obtained in the previous stage, namely (0, 2), (1, 7), (2, 12), and (3, 17). Then the coordinates of the points were placed in cartesian coordinates and then drew a line connecting the coordinates of the points. Both subjects stated that the graph would rise. This is based on the conclusions obtained in previous stages that the distance of cars from the edge of the room. The following is the written answer of S1 and S3 when constructing the target in Figures 5 and 6.

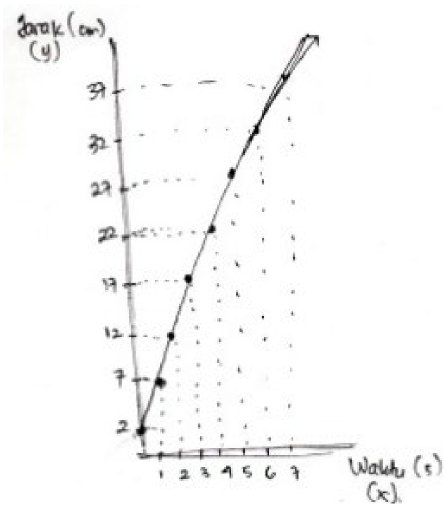

Figure 5. Written Response of S1 (Grup 1) When Doing Constructing the Target

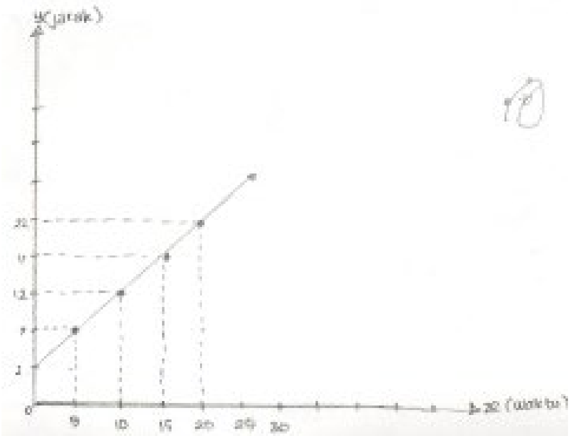

Figure 6. Written Answer of S3 (Grup 2) When Doing Constructing the Target

The following is the interview result of the subject of group 1 related constructing graph:

> *Q: What do you do to draw a graph which is suitable for the problem after you got the pattern?*
> *S1: Making X and Y axis first, then putting the pairs of points obtained namely (0, 2), (1, 7), (2, 12) and (3, 17), then they here are connected like this (while pointing the graphic image)*

While excerpts of the interview result of the subject of group 2 related constructing graph as follows:

> *Q: What do you do to draw a graph which is suitable for the problem after you got the formula?*
> *S1: Putting the value of its x, namely 1, 2, 3 on the formula like this.*

Then the graphic is drawn by drawing sb-X and sb-Y and draw it like this (while pointing at the picture).

Determining Equivalence Stage

Students performed this stage by checking all the steps that had been done before to trace whether there was still a mistake or not in the process of operation. Then the students also checked the suitability of the graph with the information in the verbal representation (source). However, some students did not perform determining equivalence. The interview results in the subject of group 1 are as follows:

> *Q: Are you sure about the answer you got?*
> *S1: yes.*
> *Q: How do you check that the graph you're getting is right? Explain*
> *S1: At first, the car is 2 cm from the edge of the room, then it moves 5 cm every second. It means after 1 second the car is 7cm, then 12 and so on (while showing the scheme that has been made). So the graph is like this (while showing the graph obtained).*

From the above exposure, the structure of the process of mathematical representation translation from verbal to the graph by subjects of groups 1 and 2 can be described in Figure 7 and 8.

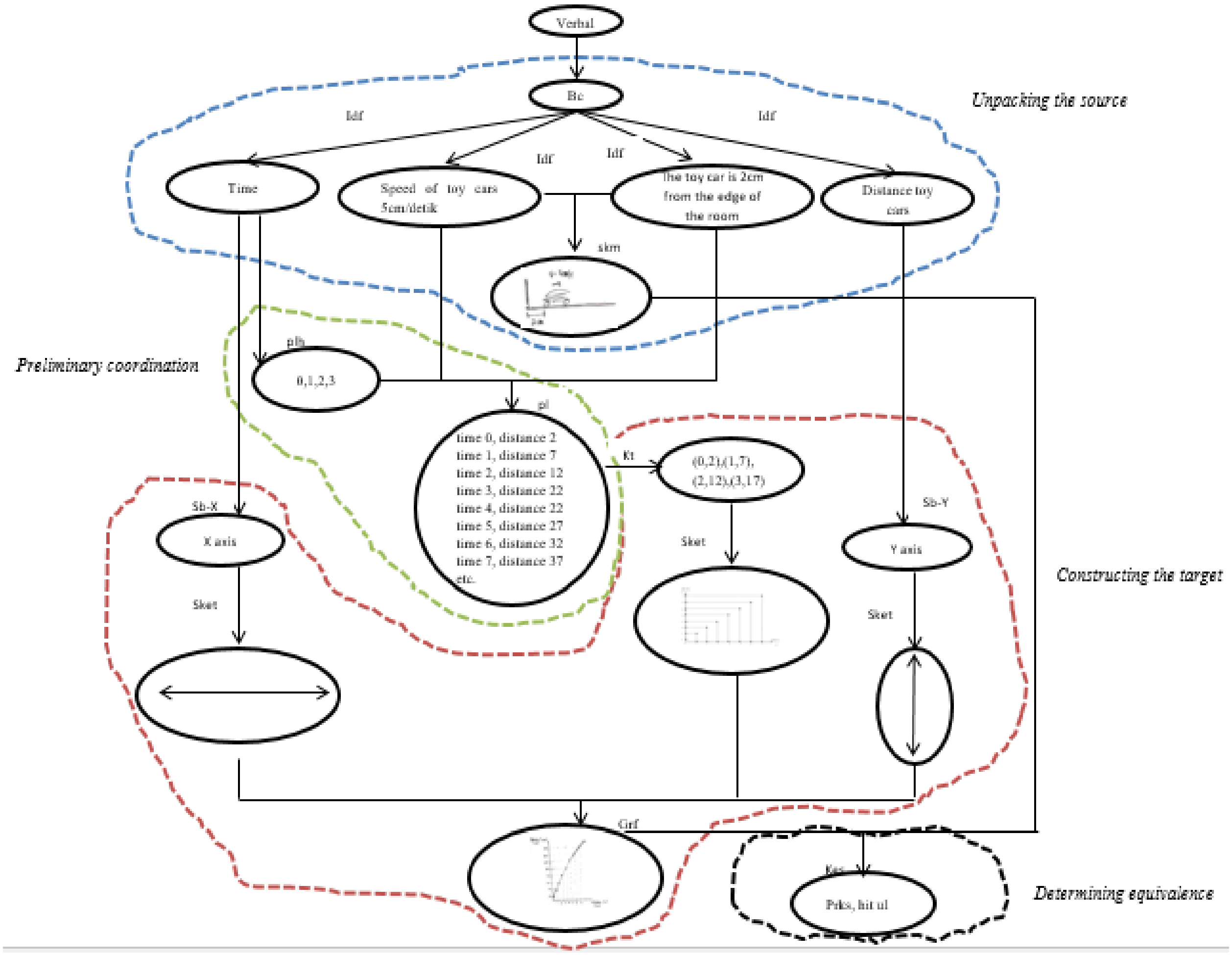

Figure 7. Structure of Process of Mathematical Representation Translation from Verbal to Graphic of Subject in Group 1

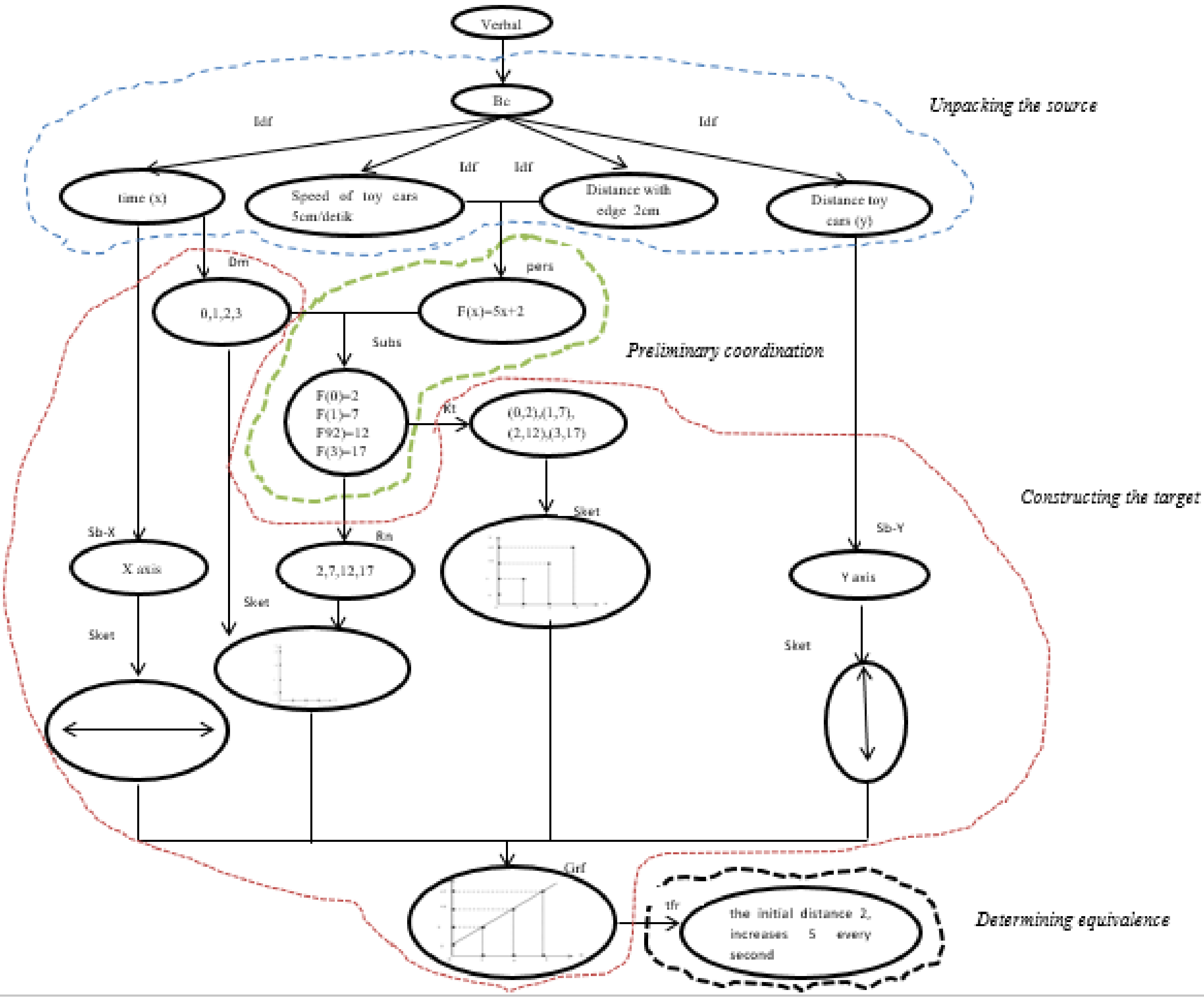

Figure 8. Structure of Process of Mathematical Representation Translation from Verbal to Graphic of Subject in Group 2

The result showed that the subjects did the process of representation translation from verbal to graph well through four stages of translation Bosse, et al. (2014), namely unpacking the source, preliminary coordination, constructing the target, and determining equivalence. All students did the stages of unpacking the source, preliminary coordination, and constructing the target. Based on the translation stage, some subjects performed different activities on the translation process stages. For determining the equivalence stage, some students did not do it. For unpacking the source, constructing the target, and determining equivalence stages were done in the same way.

There were two different characteristics at this translation process, namely numerically (finding pattern) and algebraically (making the formula of the function). In the group of subject 1, the students did the preliminary coordination stage by determining the pattern that is done by S1 and S2. While on the subjects of group 2, students performed a preliminary coordination stage by determining the function formula that is done by S3 and S4. The two characteristics of the mathematical representation translation of each subject group are presented in Table 4.

Table 4. Characteristics of Mathematical Representation Translation From Verbal to Graph

Subject	Characteristics of Translation	Translation Process
S1 and S2	Numerically	When conducting preliminary coordination, the subjects made the pattern that occurred related to the relationship between the distances of the toy car to time.
S3 and S4	Algebraically	When doing preliminary coordination, the subjects made a mathematical equation to determine the function formula related to the relationship between the distances of a toy car to time.

The first characteristic was the translation done by the subjects when doing preliminary coordination by creating a linked pattern of toy car distance from the edge of the room to changing times in every second. Subjects determine the distance of the cars from the edge of the room when the time is 0 seconds is 2 cm, then at the time of 1 second is 7cm and at 2 seconds is 12 cm and at 3 seconds is 17 cm. The subjects determined the distance of the toy cars from the edge of the room when the time 0 second, 1 second, 2 seconds, 3 seconds. From the pattern result obtained, the subjects concluded that the more time increased, the further the distance of the toy cars from the edge of the room was. Patterns can help students to see the regularity of change from one variable to another. To facilitate students in determining the component graph. Janvier (1987) referred to it as a measuring activity. Students create patterns numerically as an intermediary representation to construct a graph representation.

The second characteristic was the translation process done by the subjects when doing preliminary coordination by determining the function formula of the relationship between the distance of toy cars from the edge of room and time. The subject connects with the previously understood concept and performs a series of algebraic operations to obtain the mathematical equation. Students understand that constructing a graph must change the verbal representation into a mathematical equation. By the formula of the function, students can determine the components of the graph, i.e. the axis coordinates, point coordinates, gradient, domain, and range. This finding supports the statement of Krawec (2014) that translating texts (verbal), involves the transformation of text (verbal) information into understandable forms. Here the two students involve a transformation to a symbolic form of mathematical equations. Students connect with understood concepts. And students perform algebraic operations to obtain the mathematical equations. Janvier (1987) calls it modelling activity.

Students determine the pattern numerically and the function formula as an intermediate representation to construct a graph representation. This shows that the translations of verbal representations to graphs occur indirectly (Janvier, 1987) or globally (Duval, 2006) or through intermediaries (Bosse et al., 2011) are numerical in the form of patterns and symbolic formulas of functions. The intermediate representation of numerical in representation translations from verbal into graphs in this study supports Bosse et al. (2011), which states that the translation from verbal to graph through an intermediate representation of tables is also a numerical representation. While the representation of symbolic intermediaries in the form of function in verbal translation to the graph, in this study adds the statement of Bosse, et al. (2011) that the translation from verbal to graph through an intermediate representation in the form of tables.

The results showed that the process of representation mathematical translation from verbal to the graph by students occurs indirectly that can be through numerical or symbolic (mathematical equations) intermediaries. This intermediate representation causes the complexity and degree of difficulty of translation from verbal to graph. This corresponds with Bosse et al. (2011), who state that the use of intermediate representation is one of the difficult factors of the translation process. The results of this study can also add information from research Gurbuz and Sahin (2015), Bal (2015), Dundar (2015), and Çelik and Sağlam-Arslan (2012) that one of the causes of many students experiences the limited ability of verbal representation of translation to graph is one of the characteristics of representation translation from verbal to the graph at the preliminary coordination stage involving other representations, i.e., numerical and symbolic.

The ability of students to translate between mathematical representations is very important in understanding mathematical concepts such as functions. To understand the concept of function, students must be able to understand the information presented in various representations and to translate between multiple representations (Birgin, 2012). This shows that one of the factors in understanding the concept of function is the ability to translate from among representations in function representation. Besides, NCTM (2000) also sets the standard of representation that is expected to be mastered in learning, one of which can select, apply, and translate between mathematical representations to solve problems.

Conclusion

Based on the results of this study, it is expected that teachers can be used as a consideration of designing appropriate learning, especially functions. So it can optimise the ability and minimise students' difficulties in translating mathematical representation. To optimise translation skills, teachers should present a problem in one type of representation, and students are asked to present it in various other forms of representation. Based on the study result of mathematical representation translation from verbal to the graph of Junior High School students can be concluded as follows:

1. The process of mathematical representation translation from verbal to graph occurs indirectly. The process of mathematical representation translation from verbal to graph is generally carried out through four stages, namely: unpacking the source, preliminary coordination, constructing the target, and determining equivalence.
2. The characteristics of representation translation from verbal to graph are numerically and algebraically. Numerically, the student translates the verbal representation to the graph by determining the pattern at the preliminary coordination stage. Algebraically/symbolically, the student translates the verbal representation to the graph by determining the function formula (mathematical equations). That states the relationship between the dependent variable and the independent variable at the preliminary coordination stage.

Recommendations

This study is still limited to the representation translation from verbal to the graph. It has not been able to describe the reverse translations, nor for other types of representation, It is still open for further study.

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