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Generalisation of Relations Between Quantity Variations Through Arithmetic Sequences in Functional Thinking

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Function material is one of the most important materials that must be understood by students. However, most students have difficulty understanding the concept of function. These difficulties will have a significant impact on student learning outcomes if not immediately addressed and solved. Student difficulties about function can be minimised by developing students' functional thinking, starting at an early age. Functional thinking is a type of representational thinking that focuses on the relationship between two (or more) variations of quantities. The purpose of this study is to find out how students think functionally through arithmetic sequences. Research data was collected through test sheets and interviews. The results showed that students' functional thinking worked through arithmetic sequences through several stages of functional thinking components. These include 1) understanding the problem, 2) determining recursive patterns, 3) covariational thinking, and 4) generalising the relationship between quantity variations. The stages are carried out by students sequentially to produce a generalisation of relationships between quantity variations through arithmetic sequences.

Keywords: *Function, Functional Thinking, Material Sequence, and Arithmetic Sequence.*

Introduction

A function is one of the mathematics materials in junior high schools (SMP). The basic competencies of SMP/MTs in the 2013 curriculum require students to: 1) understand patterns and use them to deduce and make generalisations (conclusions), 2) use patterns and generalisations to solve problems, and 3) conduct experiments to find empirical opportunities from real problems and present them in the form of tables and graphs. NCTM (2000) also states that students in grades 6-8 should be able to: 1) understand patterns, relations, and functions; 2) represent and analyse mathematical situations and structures using algebraic symbols; 3) use mathematical models to represent and understand quantitative relationships, and 4) analyse the change in various contexts. By looking at the demands of the curriculum above, the function material is one of the materials that must be understood by students. The concept of function can also be used as the basic student competence to support the success of subsequent student learning, such as calculus and algebra (Akkoç, 2003). This is in line with the view of Subanji (2011), Dubinsky & Wilson (2013), and Ronda (2009), who state that strong knowledge of the concept of function is important to support success in learning calculus, advanced mathematics, or science.

The concept of function is not a concept easily understood by students (Chazan,1996 in Clement, 2001; Warren et al., 2006). Most students experience difficulties in representing and interpreting functions. According to Tanişli (2011) and Schwartz & Yerushalmy (1992), research findings also showed that many students experienced misconceptions about functions and difficulties in representing the use of algebraic notation. Most students had difficulty in completing the general expression of $y = 2x - a$ and $y = 3x - a$. These difficulties and misconceptions will have a large impact on student learning outcomes if not immediately addressed.

Functional thinking is an important aspect of school mathematics learning (Stephens, et al, 2012; Tanişli, 2011; Warren, et al, 2006). According to Smith (2003) and Akkoç & Tall (2002), functional thinking is a type of representational thinking that focuses on the relationship between two (or more) covariant quantities. Blanton et al, (2015) indicate that functional thinking involves generalising the relationship between covariant quantities and reasoning as well as representing this relationship through natural language, algebraic notation (symbols), tables, and graphs.

Literature Review

The topic of functional thinking in mathematics learning has been widely studied. For example, Blanton & Kaput (2004), Brizuela, et al. (2015), Blanton, et al. (2015), Muir & Livy, 2015), Stephens et al. (2012), Tanişli (2011), Warren (2005), Warren, Miller & Cooper (2013), Warren et al. (2006), and dan Wilkie (2014) conducted research on elementary students and showed that they were able to generalise and represent relationships. Doorman, et al (2012), Blanton, et al. (2015), Stephens et al. (2017), Warren et al. (2006), and Wilkie (2014, 2016) developed learning which could explore functional thinking. Next, McEldoon (2010) developed an assessment of the ability of elementary students in functional thinking, especially in students' abilities to find corresponding rules in function tables. Allday (2018) researched student behaviour significant for developing an intervention. The results of the study above use the relationship between two variations of quantity. There is still no research that uses the relationships among three variations of quantity. Thus, the researchers intend to detect how students generalise the relationship between 3 variations of quantity guided by Smith's research (Blanton, et al., 2015 and Tanişli, 2011).

According to Blanton, et al. (2015), Smith, (2017), and Tanişli, (2011), students' functional thinking can be investigated through a number of ways. First, a recursive pattern regards finding a pattern in a sequence of numbers given previous numbers. Second, covariational thinking is focused on the relationship of changes in each variable (for example, variable x increases by one and variable y increases by 2) or the relationship between variant quantities. Third, correspondence regards the generalised relationship between variant quantities. These three items are usually referred to as the components of functional thinking. The components of functional thinking were used by Tanişli (2011) in his research. The results showed that first, students used a recursive approach and looked for a recursive pattern in investigating the function table. They identified the value of the dependent variable as a pattern without considering the independent variables. The students identified changes in sequential pattern values. Second, students defined the relationship between two quantities as multiplication and addition. The relationship was represented in two ways: first, the relationship was determined in writing using words; second, it was explained in a semi-symbolic form using familiar mathematical symbols, i.e. numbers (1, 2, 3, etc.) and mathematical operations (i.e. +, -, x). Third, students worked on a function table with general expressions of y = 2x + a and y = 3x + a.

In this research, the functional thinking component is used to identify students' functional thinking in solving the "sequence" problem. The sequence material was chosen by the researchers based on Payne's (2012) study, which states that one of the task structures that could be used to improve functional thinking was a sequence. Wilkie (2014) says students' functional thinking can be explored by making rules for a sequence of numbers. These rules can be generalised through the relationship between two quantities or numbers, for example:

$$2 \underset{+3}{\rightarrow} 5 \underset{+3}{\rightarrow} 8 \underset{+3}{\rightarrow} 11 \quad 14 \quad 17 \ldots$$

From the sequence of numbers above, students can recognise that each number (or item) is more than 3 from the previous number or "add 3 to the next item". This is an example of a "recursive pattern". However, when students are asked to find the 100^{th} item in the sequence of numbers, they must generalise to find correspondence by involving a relationship (covariation) between the position of an item and the item itself, which is explained in the following sequence of numbers:

↑2	5	↑8	11	14	↑17...
1st	2nd	3rd	4th	5th	6th

× 3 then −1

In this case, there will be a generalisation of "multiply the positions of an item with 3, then subtract with 1" (Wilkie, 2014). Thus, the "sequence and series" material can be used to explore students' functional thinking. Meanwhile, the purpose of this paper is to find out how students think functionally through arithmetic sequences.

Methods

This research was conducted on the 9th-grade students in a junior high school located in the middle of Pekanbaru, Riau. The school was chosen as the research sample because it has student input with above-average competence and with an indication of functional thinking. Besides, based on the relevant references regarding the importance of functional thinking in early grades, secondary-level students should be able to apply functional thinking as well (Blanton, et al., 2015; Tanişli, 2011; Warren, et al, 2006). Of the 60 students given the problem-solving test sheet, six students were chosen as internal samples because the results of the student work had functional thinking indicators.

Research data was collected through a test sheet and interviews. The test sheet was in the form of descriptive problems. Meanwhile, interviews were conducted in an unstructured way by adjusting the students' answers to the problem. The test sheet was given to 60 students with a 30-minute time allocation to complete the test sheet. The test sheet contained 1 problem with 4 alternative answers, i.e. (n + 4) + (2m + 9); (4n - 2) + (8m - 3); 3n + (6m + 1); and (2n + 2) + (4m + 5). Each answer had several variant quantity relationships and each student was given the freedom to choose alternative answers. In this task, an independent variable was represented by "the number of rows", and dependent variables were represented by "the number of tables" and "the number of chairs". The test sheet was given to students (Figure 1).

Before conducting the research, the researchers discussed with the 9th grade teacher about whether the students would be able to solve the questions or not. Then, the researchers discussed the most appropriate time to conduct the research. The teacher distributed the sheets while the researchers acted as observers. The researchers classified the collected students' answer sheets based on the functional thinking component and selected 6 students as the research subjects from students' answer sheets. The research subjects were interviewed according to the answer sheets. Before conducting the interviews, the researchers asked permission for the willingness of the research subjects to be interviewed. Interviews were recorded using a cell phone. The interview time was adjusted to the number of students' answers. The purpose of the interviews was to clarify and detect functional thinking components that were not seen in the answer sheet.

Qualitative data analysis was conducted interactively and lasted continuously until the data was saturated. Data saturation was indicated by the absence of new data and information. The stages of data analysis were: first, transcribing audible data and interviews, scanning students' answers, sorting and compiling data into certain types based on data characteristics, and performing data reduction. Data reduction was intended to select, focus, abstract, and formulate raw data. Second, coding or categorising data was processed by taking collected written data or images, segmenting the sentences or pictures into categories, and labelling these categories with specific terms. Third, describing the structure of students' functional thinking was based on data categorisation. The last stage involved drawing conclusions based on data analysis from the test sheets and the interview results (Miles et al., 1994).

Results

This research investigates the functional thinking of junior high school students in solving a mathematics problem about "sequence and series". Students' functional thinking was identified based on the work of Blanton et al., (2015); Smith, (2003); and Tanişli, (2011). Research involved 1) determining recursive patterns, 2) covariational thinking, and 3) generalising the relationship between quantities. The research findings based on the test and interviews are presented below:

Understanding a Problem

Understanding a problem regards students' initial basis in problem-solving. Once students can understand the problem, they will be able to solve the problem accurately and correctly. This is in accordance with the work of Polya (1973), who indicates there are four stages of problem-solving. One of the problem-solving steps is to understand the problem. Understanding the problem is an activity that includes understanding various issues involving the problem, such as what is already known, what the purpose is, whether there is adequate information, what data is available, what the conditions are, and whether the problem at hand is similar to other problems that have been solved before. At this stage, students can take several steps needed to understand the problem, such as sketching pictures, recognising the notation used, grouping data, and so on. For example, Chandra (see figure 2), one of the research subjects, completed the understanding the problem stage by writing down what was already known, i.e. writing down "It is known that the number of the 1st row and the 3rd row is 10 tables, the number of chairs 2 times the number of tables per row. Tables and chairs in the 1st row, 2nd row, 3rd row, and so on are natural numbers. Then there is the addition of 3 chairs and 1 table per row." It can be seen that Chandra focused on the number of tables in the 1st and 3rd rows of 10 tables (natural numbers). The number of tables in the 1st row was 4, and the number of tables in the 3rd row was 6. Thus, regarding the number of tables, there was a sequence of numbers of 4, 5, 6, and so on. The number of chairs was 2 times the number of tables per row. Thus, regarding the number of chairs, there was a sequence of numbers of 8, 10, 12, and so on. Chandra also wrote that there was an addition of 1 table and 3 chairs per row.

On the other hand, William (Figure 3) completed the understanding of the problem stage by writing "Chair = 2x table, then the table in the 2nd row = table in the 1st row + table in the 3rd row of 10 tables." Then, William calculated the number of tables in the 2nd row by giving an expression of $2y = x + z$, then $2y = 10$ and $y = 5$, so the number of tables in the 2nd row was 5. In the test sheet, it was known that the number of tables was 10, so William gave four possibilities: a) 3 + 7 = 10; b) 4 + 6 = 10; c) 1 + 9 = 10; and d) 2 + 8 = 10. Thus, to find the number of tables, 4 sequences of numbers were obtained: a) 3, 5, 7, and so on; b) 4, 5, 6, and so on; c) 1, 5, 9, and so on; and d) 2, 5, 8, and so on. Then, to find the number of chairs, 4 sequences of numbers were also obtained: a) 6, 10, 14, and so on; b) 8, 10, 12, and so on; c) 2, 10, 18, and so on; and d) 4, 10, 16, and so on.

Thus, Chandra preferred one solution in solving the problem, while William used all alternative answers, i.e. the four solutions in solving the problem.

Determining a Pattern in a Sequence of Numbers

In finding a pattern, the students focused on relationships in the order of values. Students focused on the change of the new value from the previous value. In the process, the students wrote "$b = U_2 - U_1$". This method is used for identifying patterns in a sequence of numbers from the number of tables and the number of chairs. This method is usually called recursive relations or "recursive patterns" (Tanisli, 2011). Table 1 presents the illustration.

Suppose Chandra wrote a sequence of numbers for the number of tables with "$U_1 = 4$, $U_2 = 5$, $U_1 = 6$" and obtained $b = 5 - 4 = 1$. Then, to identify the number of chairs, he wrote "$U_1 = 8$, $U_2 = 10$, $U_1 = 12$" and obtained $b = 10 - 8 = 2$. Figure 4 presents Chandra's work.

On the other hand, William (Figure 5), without showing the completion process, wrote the pattern of the number of tables as "pattern + 4"; "pattern + 3"; "pattern + 2"; "pattern + 1" and pattern of the number of chairs as "pattern + 8"; "pattern + 6"; "pattern + 4", "pattern + 2". There were 4 patterns of the number of tables and 4 patterns of the number of chairs because William gave 4 possibilities in solving the problem.

In determining the recursive pattern, they focused more on the order of values in the dependent variable, regardless of the independent variables.

Covariational Thinking and Generalising the Relationship Between Quantities

The student work process showed that the students related the quantity of the number of rows to the quantity of the number of tables. They related the quantity of the number of rows to the quantity of the number of chairs, and related the quantity of the number of rows, the number of tables, and the number of chairs. Then, the students generalised the relationships into the appropriate function forms. Table 2 presents an illustration of one of the answer keys.

For example, Nicholas (Figure 6) related the quantity of the number of rows to the number of tables by writing "b = 1". In contrast, the relationship between the quantity of the number of rows and the number of chairs was written with "b = 2". Nicholas used the formula "U_n = a + (n-1) b". Then, he generalised the relationship between the number of rows and the number of tables in the function of "U_n = 4 + n", and the relationship between the number of rows and the number of tables in the function of "U_n = 9 + 2n". Next, the relationship between the number of rows, the number of chairs, and the number of tables were in the function of "Un (table + chair) = (4 + n) + (9 + 2n)". However, Nicholas did not differentiate between the number of tables variable and the number of chairs variable.

In functional thinking, students begin to understand a given problem by writing what is known and what is being asked (Polya, 1973). Then, the students identified and organised data about the problem. The students solved the problem by choosing one solution and four solutions. There were three stages of students' functional thinking in solving the mathematical problem about "sequence and series". First, students identified a recursive pattern. In identifying the recursive pattern, the students focused on the dependent variable. The students realised that the dependent variable had a sequence of patterns. This is in line with the opinion of Warren et al (2006); the order of values in the table can help the functional relationship. Second, in identifying the relationship between a quantity and another quantity, the students related 3 variant quantities: relating the number of rows to the number of tables, the number of rows to the number of chairs, and the number of rows, the number of tables, and the number of chairs. In the relationship between these variations, there is a change in value between the placement of an item and the item itself. In this case, students used addition, subtraction, and multiplication in identifying the relationship between a quantity and another quantity.

According to Tanişli (2011), the relationship between two quantities is defined by multiplication and addition, represented in writing. Third, in generalising the relationship between quantity variations, most students use algebra (n + 4), (4n - 2), 3n, and (2n +2) for the relationship between many rows of tables and many tables. The relationship between many rows of chairs and a lot of chairs can be obtained through generalisation (2m +9), (8m - 3), (6m + 1), and (4m + 5). The relationship between many tables and chairs is (n + 4) + (2m +9); (4n - 2) + (8m - 3); 3n + (6m + 1); and (2n +2) + (4m + 5). Tanişli's (2011) research states that students define the relationship between quantities in writing by using familiar mathematical words and symbols like numbers (i.e. 1, 2, 3, etc.) and mathematical operations (i.e., +, -, x).

Conclusion

Thus, the way of functional thinking of students through the arithmetic sequence is carried out in several stages: 1) understanding the problem, 2) determining the recursive pattern, 3) covariational thinking, and 4) generalising the relationship between quantity variations. In this study, there are still weaknesses, namely the questions provided are not validated by a team of experts, so there are some students who feel confused in the process of completion and many wrong answers. Thus, for future research, appropriate questions should be designed to explore students' mental processes in connecting two or more variations in quantity.

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Figure 1. Problem-Solving Test Sheet

Al Fatih Pekanbaru School will hold a Students' Parents/Guardians meeting at the Multipurpose Building. In the building, the tables and the chairs will be arranged, in which the tables and the chairs are arranged in the following ways:

The table in the 1st row, chair in the 1st row ...

The table in the 2nd row, chair in the 2nd row ...

The table in the 3rd row, chair in the 3rd row ...

To tables in the n^{th} row and chairs in the m^{th} row:

The arrangement of the tables and the chairs:
- The number of tables in the 1st row and the 3rd row is 10 tables.
- The number of chairs two times the number of tables per row.
- The number of tables in the 1st row, 2nd row, to m^{th} row, and the number of chairs in the 1st row, 2nd row, to n^{th} row form a natural number pattern.

But when the meeting is going to start, the provided table and the chairs are insufficient, so the committee must add three chairs and one table in each row. Based on the above problems:

1. Determine the number of tables in the 20th row?
2. Determine the number of chairs in the 20th row?
3. Determine the formula to determine the number of tables in the n^{th} row?
4. Determine the formula to determine the number of chairs in the m^{th} row?
5. Determine the formula to determine the number of tables and the number of chairs in the nm^{th} row?

Figure 2. Chandra in Understanding the Problem

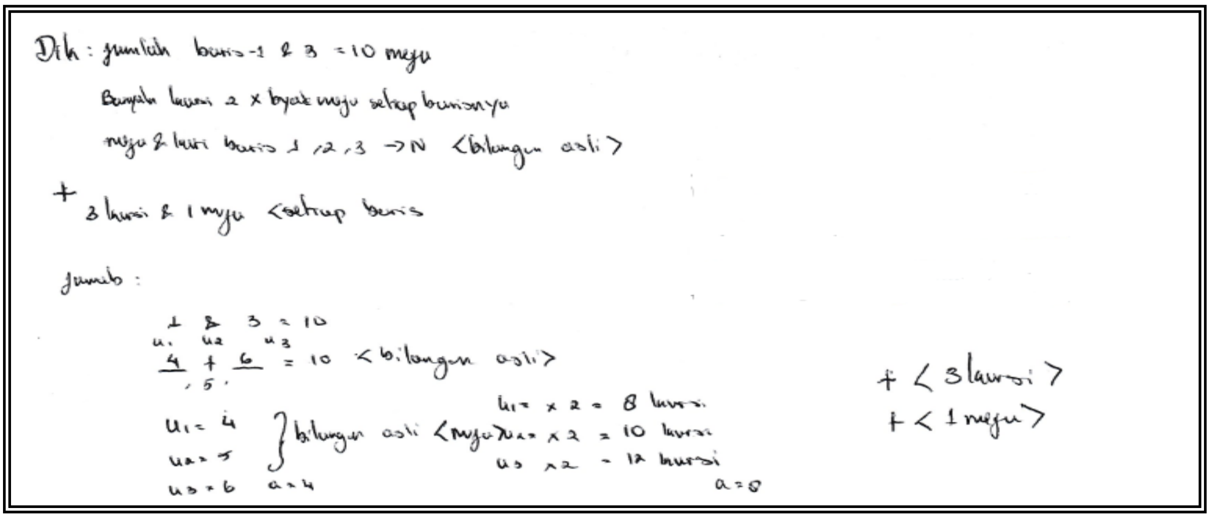

Figure 3. William in Understanding the Problem

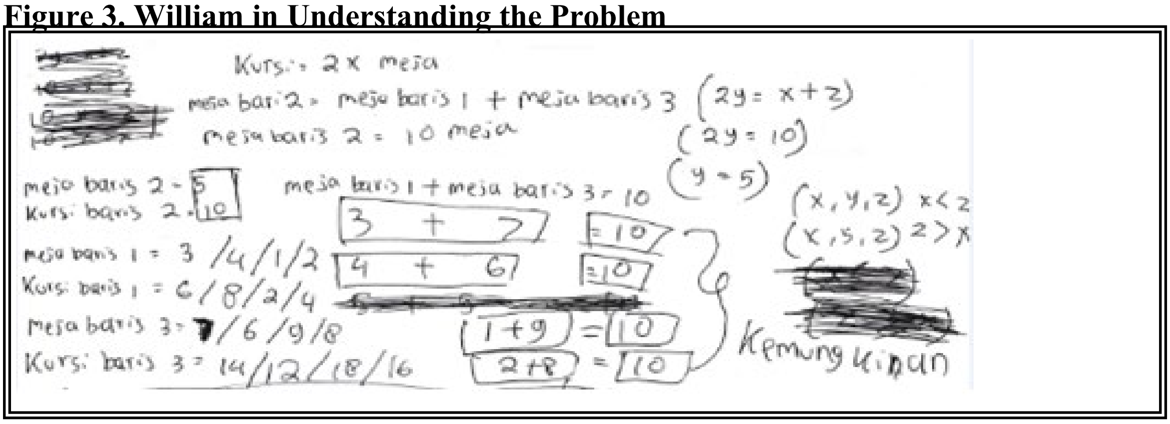

Figure 4. Chandra in Finding Patterns

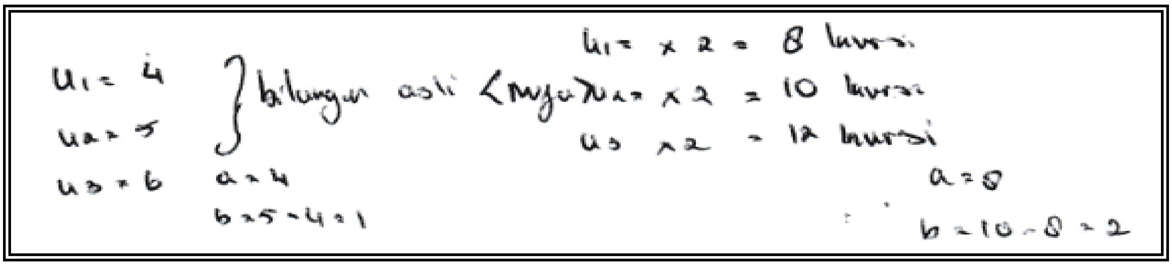

Figure 5. William in Finding Patterns

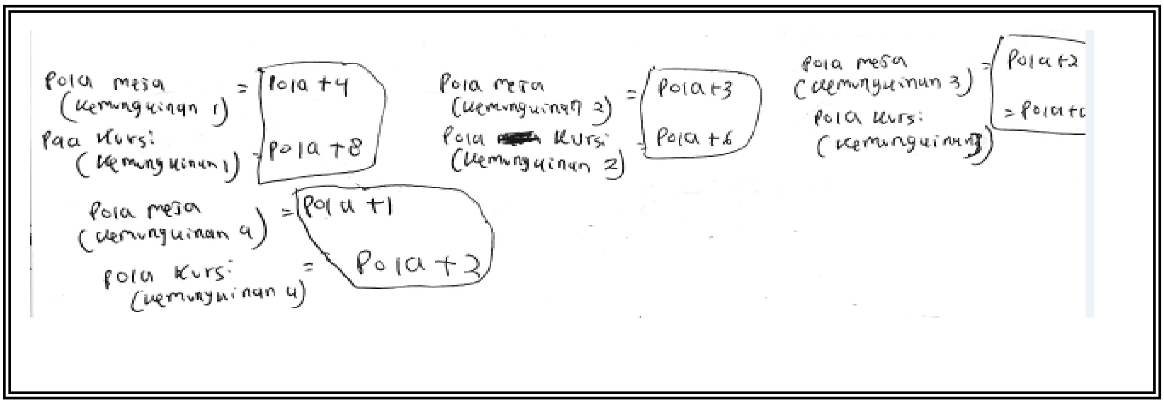

Figure 6. Nicholas in Relating and Generalising between Quantities

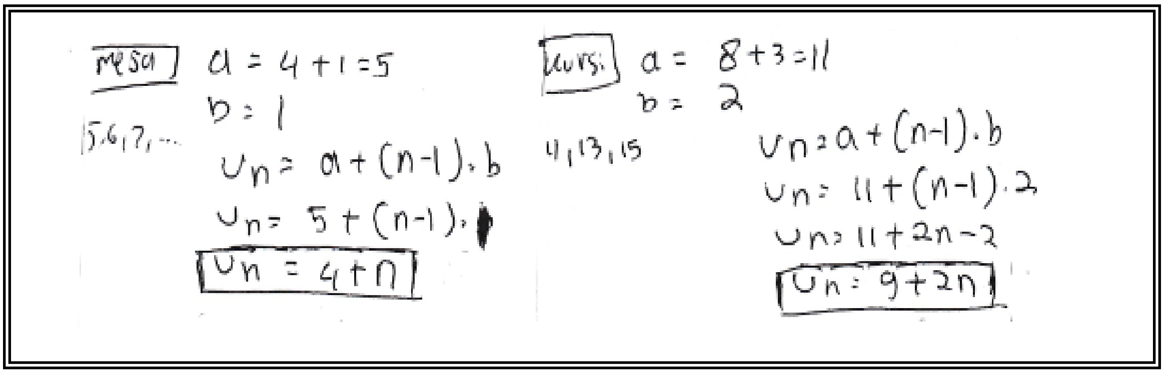

Table 1. Recursive Pattern

The Row of Tables	The Row of Chairs	The Number of Tables	The Number of Chairs
1	1	5	10
2	2	6	12
3	3	7	14
4	4	8	16
.	.	.	.
.	.	.	.
.	.	.	.
(n-1)	(m-1)	f(n-1) } a	f(m-1) } b
n	m	f(n)	f(m)

Table 2. Relationship and Generalisation Between Quantities

The Row of Tables	The Number of Tables	The Row of Chairs	The Number of Chairs
1	**(1+4) = 5**	**1**	**(2.1+9) = 11**
2	**(2+4) = 6**	**2**	**(2.2+9) = 13**
3	**(3+4) = 7**	**3**	**(2.3+9) = 15**
4	**(4+4) = 8**	**4**	**(2.4+9) = 17**
.	.	.	.
.	.	.	.
.	.	.	.
n	**(n+4)**	**m**	**(2m+9)**

$k = (n+4) + (2m+9)$

k = the number of tables and chairs in the row of *n* = *m*

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